CS738: Advanced Compiler Optimizations

Static Single Assignment (SSA)

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Agenda

- SSA Form
- Constructing SSA form
- Properties and Applications

SSA Form

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- *Static Single Assignment* – A variable is assigned only once in program text
  - May be assigned multiple times if program is executed

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  - Directly access points where relevant data flow information is available

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- Control flow like traditional programs
- Some *magic* is needed at *join* nodes

Example

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i = i + 1;
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j = i * 5;
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if (i < 20)
  i = i + j;
else
  j = j + 2;
print i, j;
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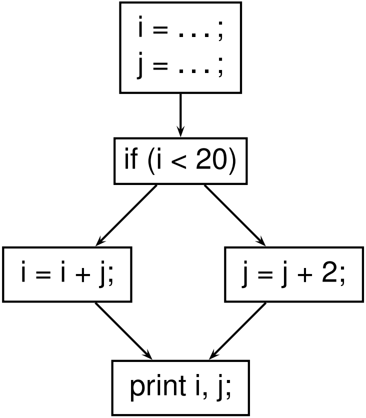

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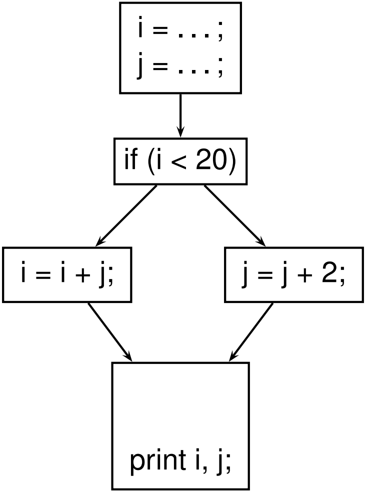

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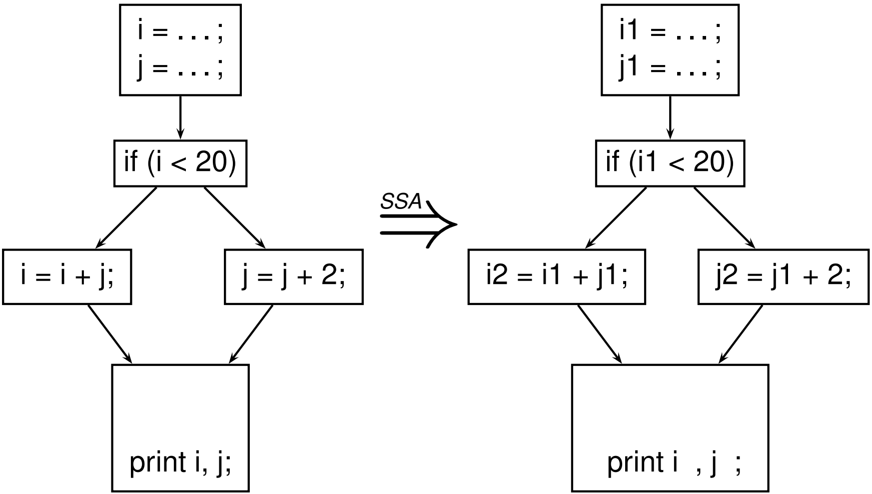

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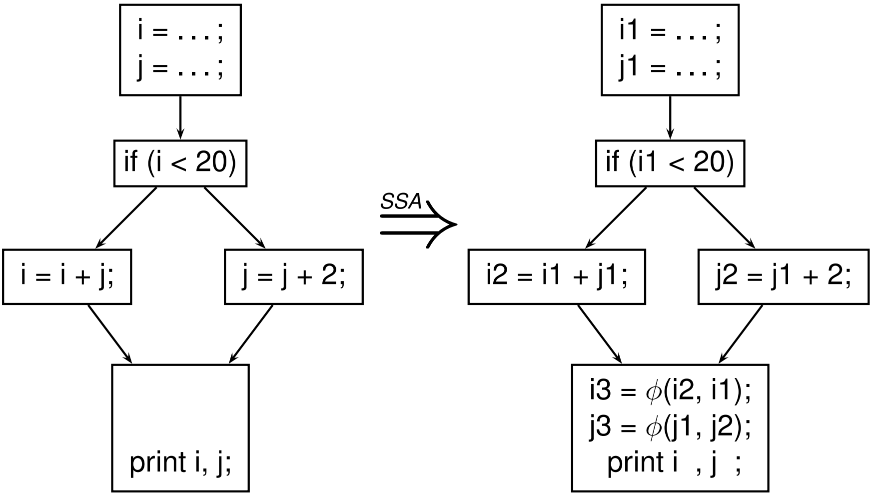

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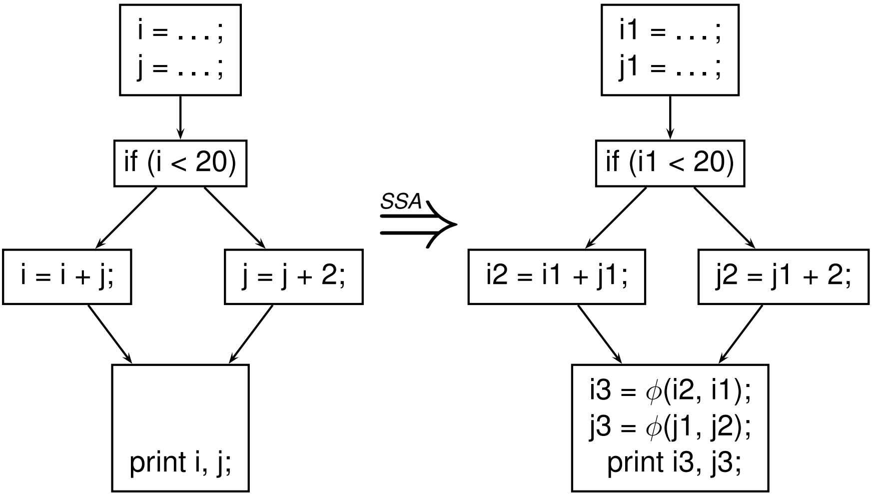

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- Examples?

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- ϕ is a conceptual entity
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- No direct translation to machine code
  - typically mimicked using "copy" in predecessors
  - Inefficient
  - Practically, the inefficiency is compensated by dead code elimination and register allocation passes

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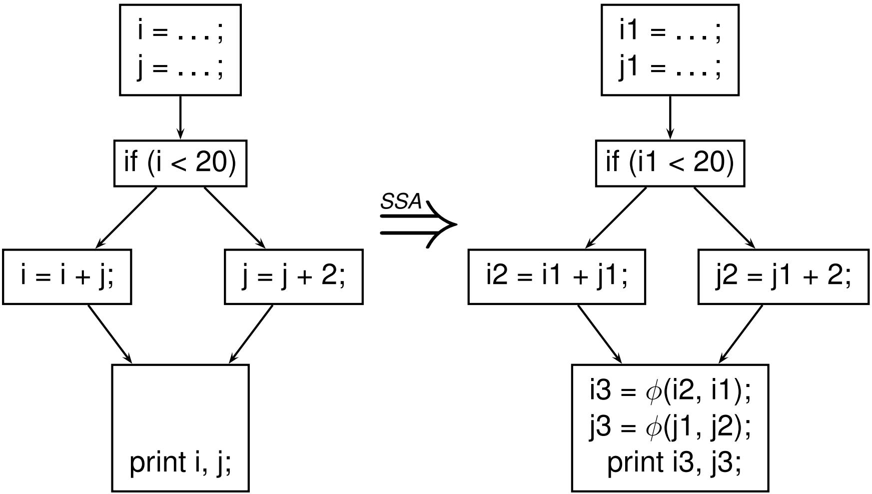
  - Ordering of ϕ arguments according to the edge ordering is important

SSA Example (revisit)



Construction of SSA Form

Assumptions

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Computing Dominators

- Equation

$$\mathrm{DOM}(n) \quad = \quad \{n\} \cup \left(\bigcap_{m \in \mathrm{PRED}(n)} \mathrm{DOM}(m) \right),$$
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where N is the set of all nodes, n_{Entry} is the node corresponding to the *Entry* block.

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- Note that efficient methods exist for computing dominators

Immediate Dominators and Dominator Tree

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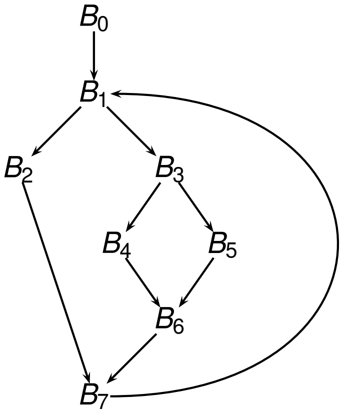
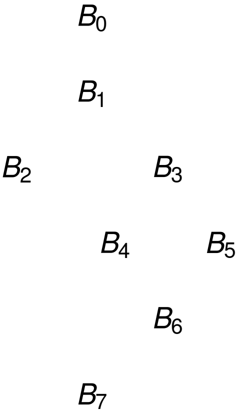
  - A tree showing all immediate dominator relationships

Dominator Tree Example





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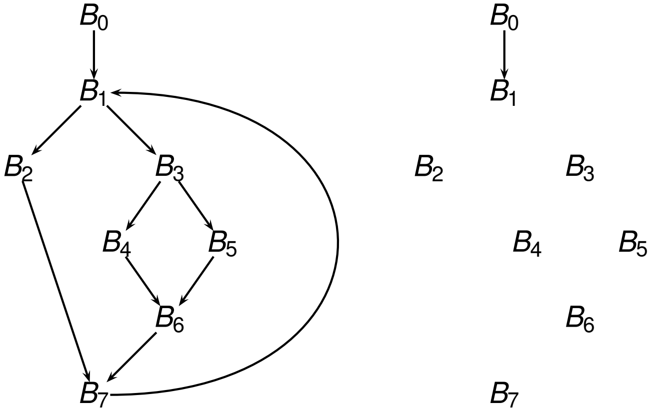

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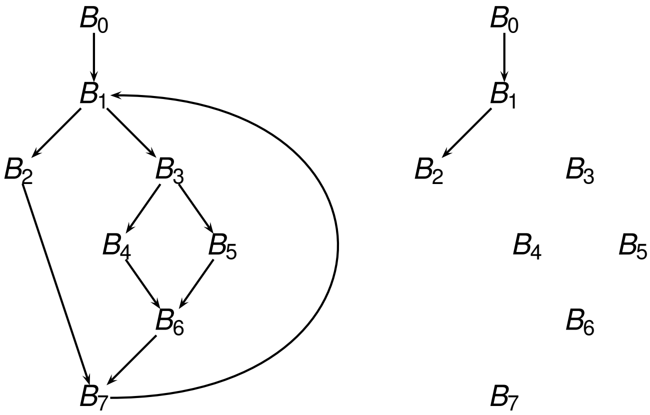

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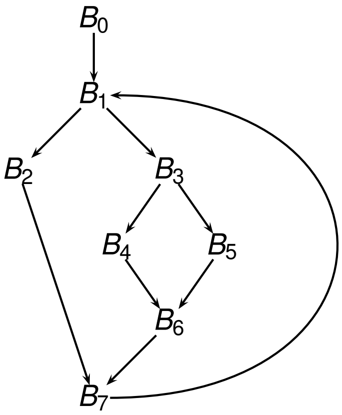

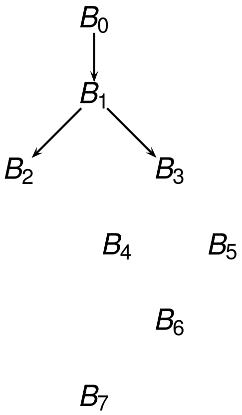

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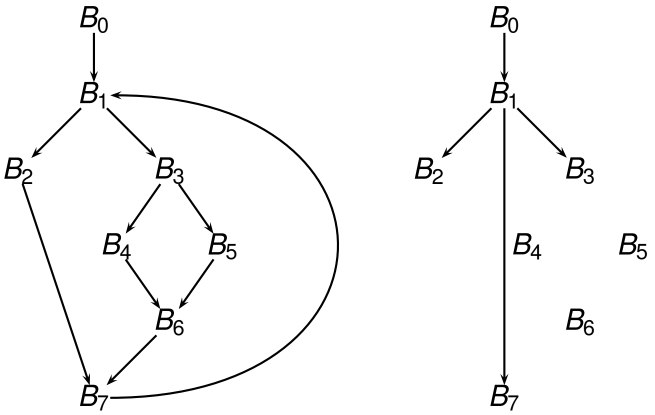

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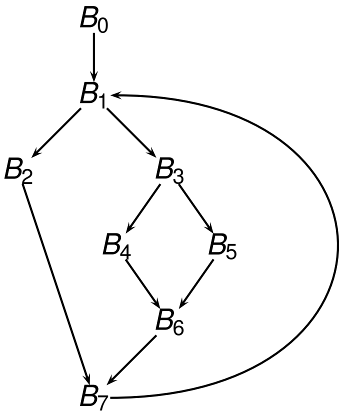

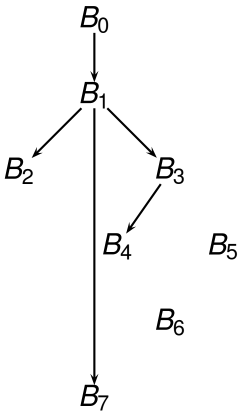

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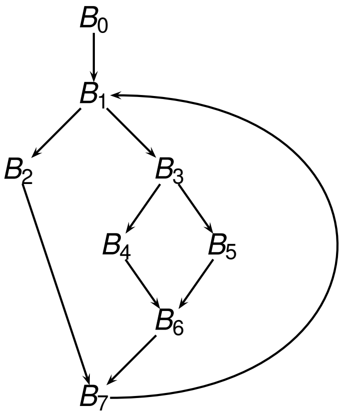

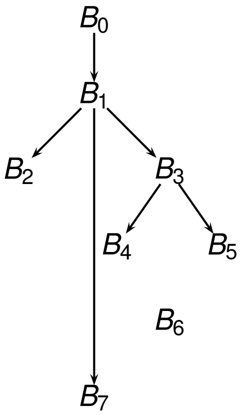

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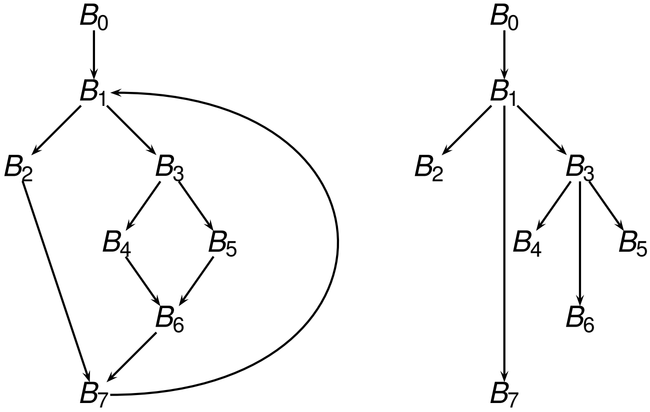

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  - x dominates a predecessor of y AND
  - x does not strictly dominate y
- Denoted DF(x)
- Why do you think DF(x) is important for any x?
  - Think about the information originated in x.

Computing DF

- PARENT(x) denotes parent of node x in the dominator tree.
- CHILDERN(x) denotes children of node x in the dominator tree.
- PRED and SUCC from CFG.

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$$\begin{aligned}
\mathrm{DF}(S) &= \bigcup_{x \in S} \mathrm{DF}(x) \\
\mathrm{DF}^1(S) &= \mathrm{DF}(S) \\
\mathrm{DF}^{i+1}(S) &= \mathrm{DF}(S \cup \mathrm{DF}^i(S))
\end{aligned}$$

- $\mathrm{DF}^+(S)$ is the fixed point of DF^i computation.

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- Why DF^+? Why not only DF?

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    }
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  - Rename ϕ operands through SUCC edge index

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    - If non-ϕ assignment, rename any use of x with the Top mapping of x from the rename stack
    - Push the mapping $x \mapsto x_i$ on the rename stack
    - Replace lhs of the assignment by x_i
    - $i = i + 1$
- For the successors of n
  - Rename ϕ operands through SUCC edge index
- Recursively rename all child nodes in the dominator tree

Renaming Variables (Pseudo Code)

- Rename from the *Entry* node recursively
  - For each variable x, maintain a rename stack of $x \mapsto x_{\text{version}}$ mapping
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  - Rename ϕ operands through SUCC edge index
- Recursively rename all child nodes in the dominator tree
- For each assignment $(x = \ldots)$ in n
  - Pop $x \mapsto \ldots$ from the rename stack