CS 698L: Parallel Patterns

Swarnendu Biswas

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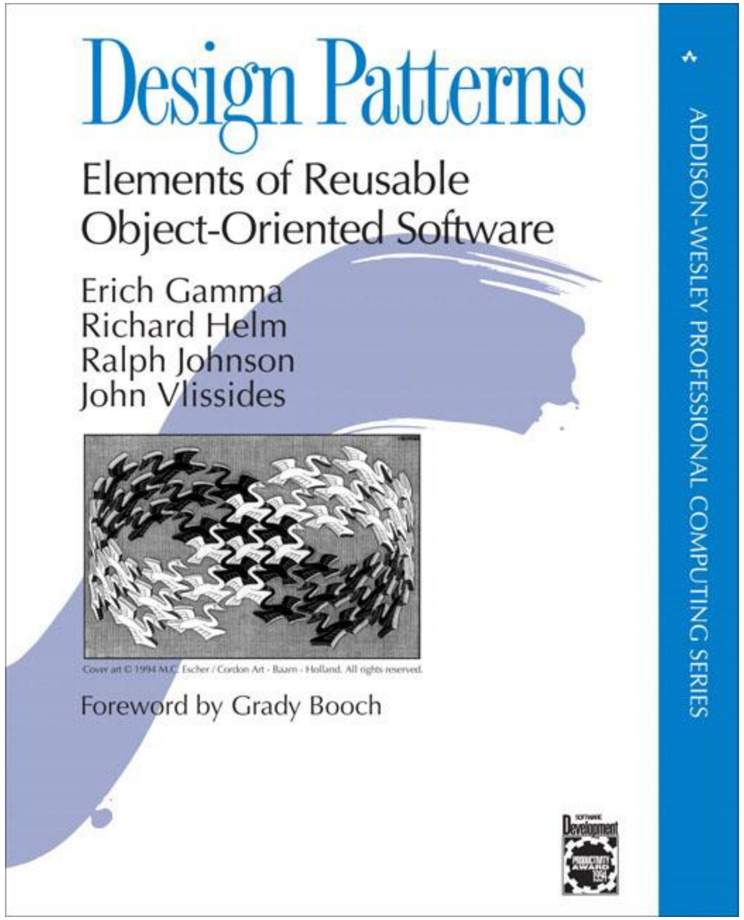
CSE, IIT Kanpur

Content influenced by many excellent references, see References slide for acknowledgements.

Parallel Programming Patterns

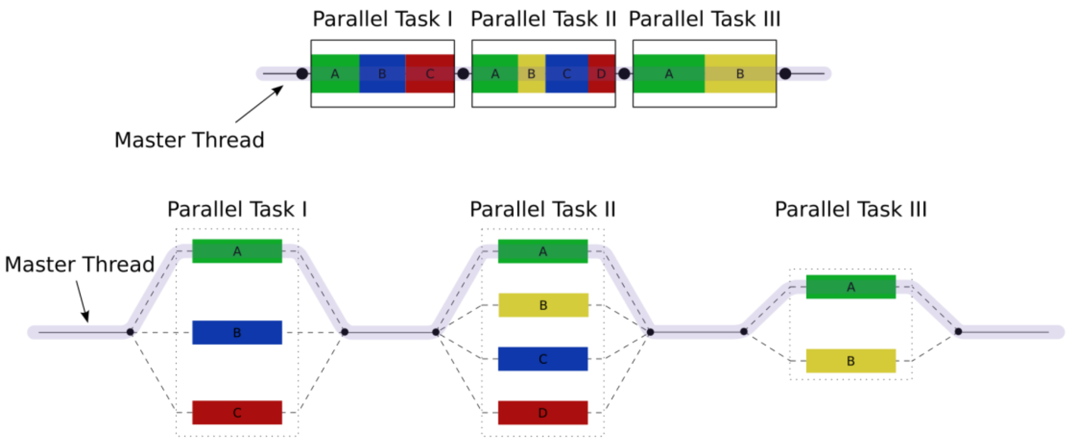
- Patterns codify best practices (remember the Gang of Four book!)
- Parallel pattern
  - Recurring combination of task distribution and data access that solves a problem in parallel algorithm design



Control Pattern: Fork-Join

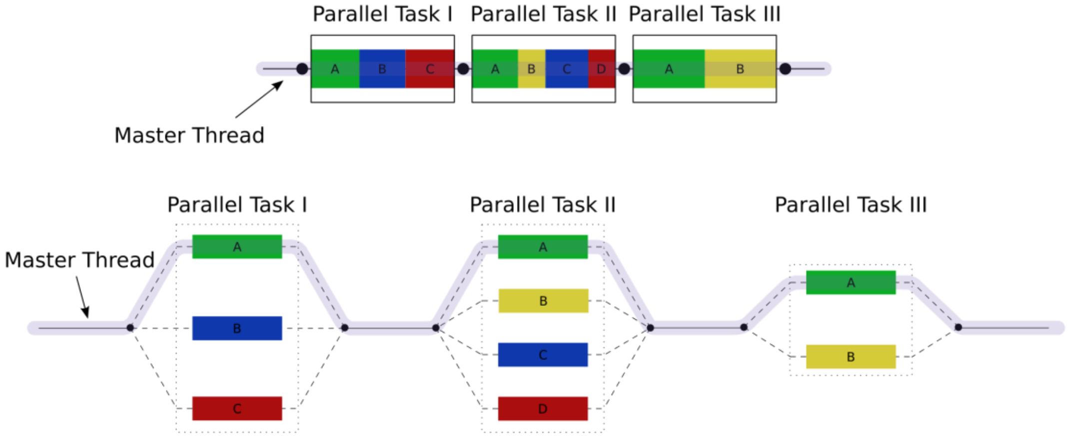
- Forks control flow into multiple parallel flows and joins later
  - OpenMP's parallel construct
  - Cilk Plus-style spawn and sync

- Is a join and a barrier same?



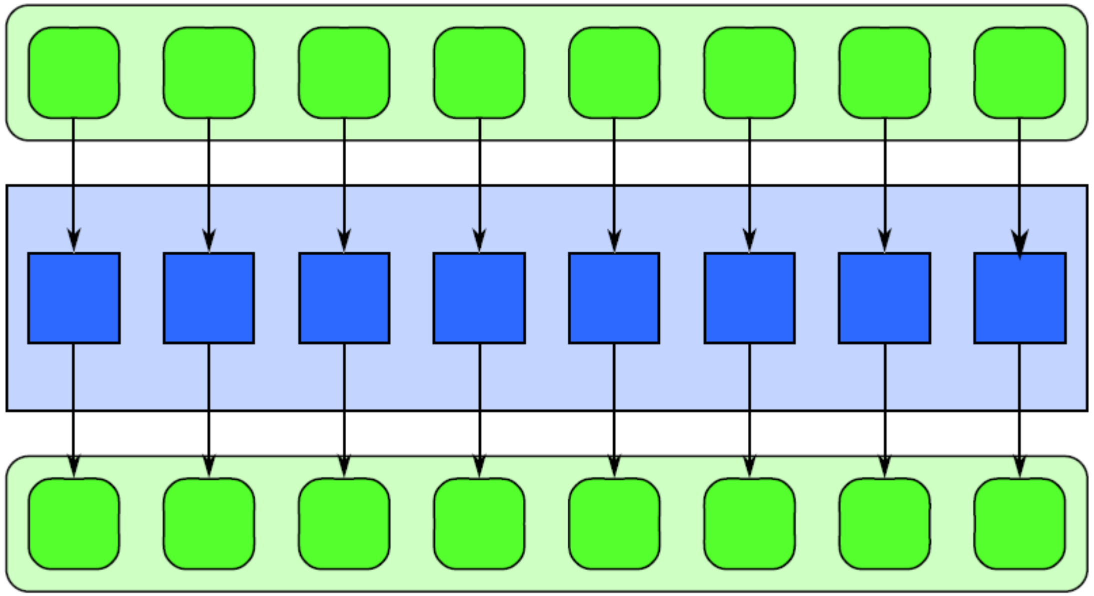
Control Pattern: Fork-Join

- Forks control flow into multiple parallel flows and joins later
  - OpenMP's parallel construct
  - Cilk Plus-style spawn and sync
- Barriers are different
  - Operates on threads, and all threads continue after the barrier
  - Only one thread continues after a join



Control Pattern: Map

- A function is applied to all elements of a collection, usually producing a new collection with the same shape as the input



- Loop bodies are independent
- Loop count is known in advance
- Elemental function must not have side-effects (i.e., pure)

Control Pattern: Map

```
>>> a = [1, 2, 3, 4, 5, 6]
>>> print(list(map(lambda x: x*x, a)))
[1, 4, 9, 16, 25, 36]
```

- Similar in flavour to SIMD model

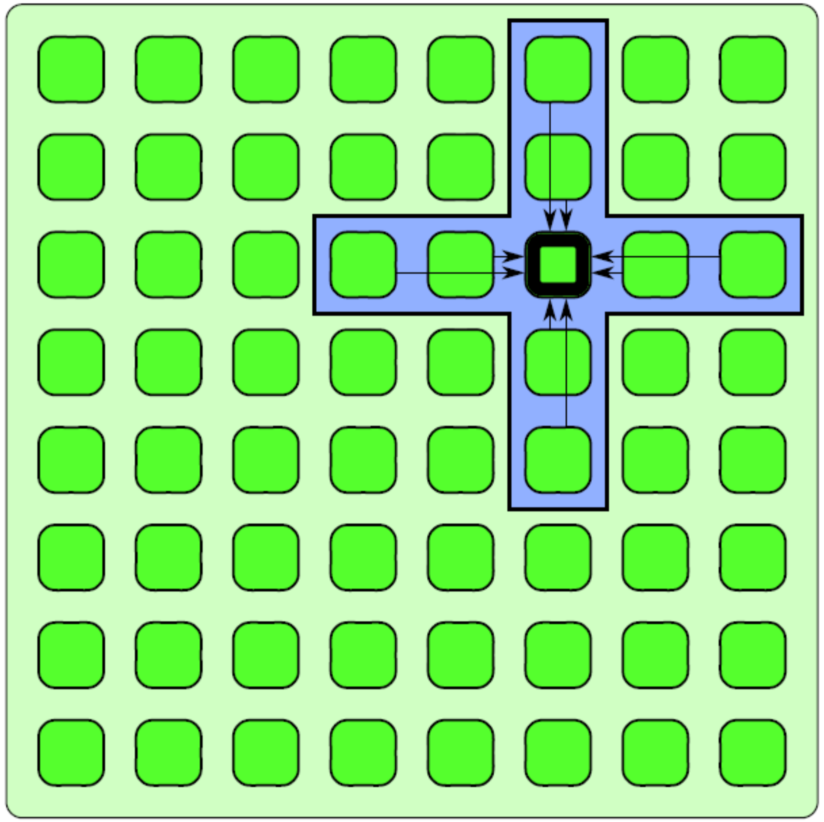
Linear Algebra Operations

- SAXPY operation $y = Ax + y$
- Similarly DAXPY, CAXPY, ZAXPY
- Very frequently used in linear algebra such as Gaussian elimination

Control Pattern: Stencil

- Elemental function can access more than one element

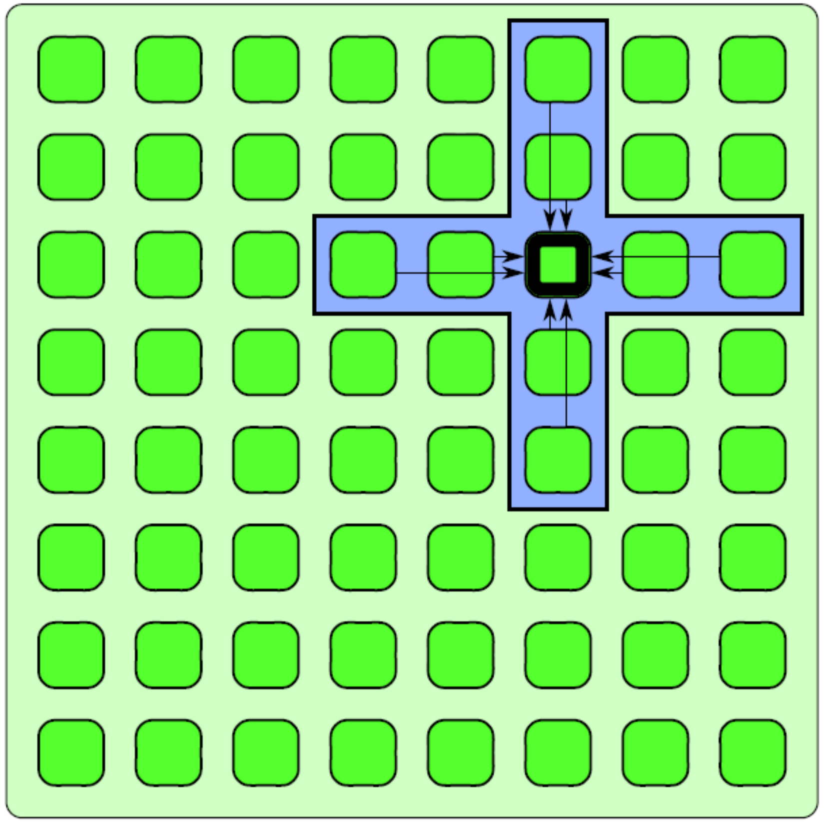
- Is stencil and map similar?



Control Pattern: Stencil

- Elemental function can access more than one element
  - Generalization of map

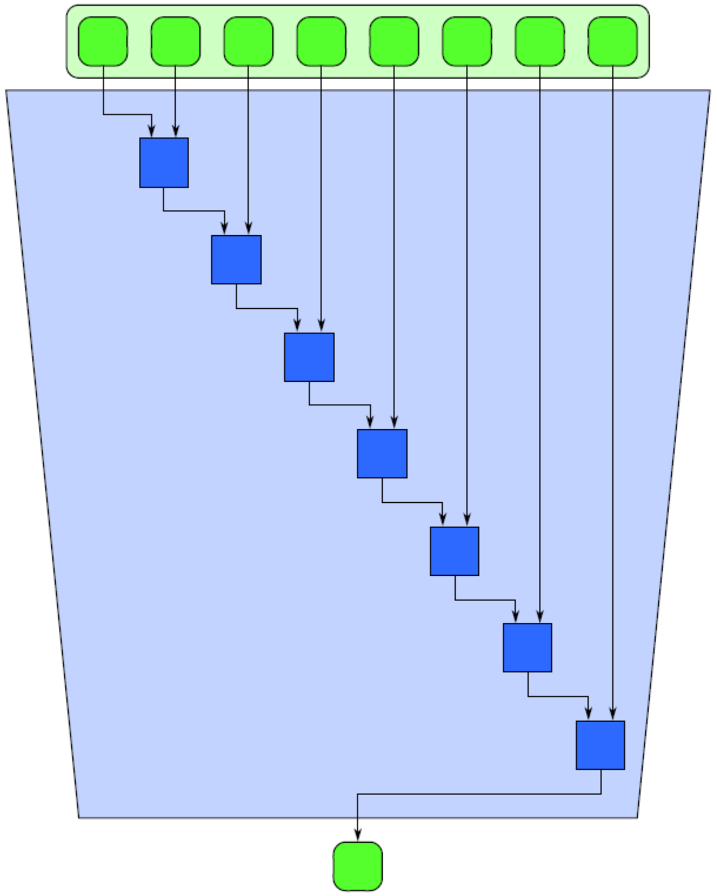
- A **convolution** uses the stencil pattern but combines elements linearly using a set of weights



Control Pattern: Reduction

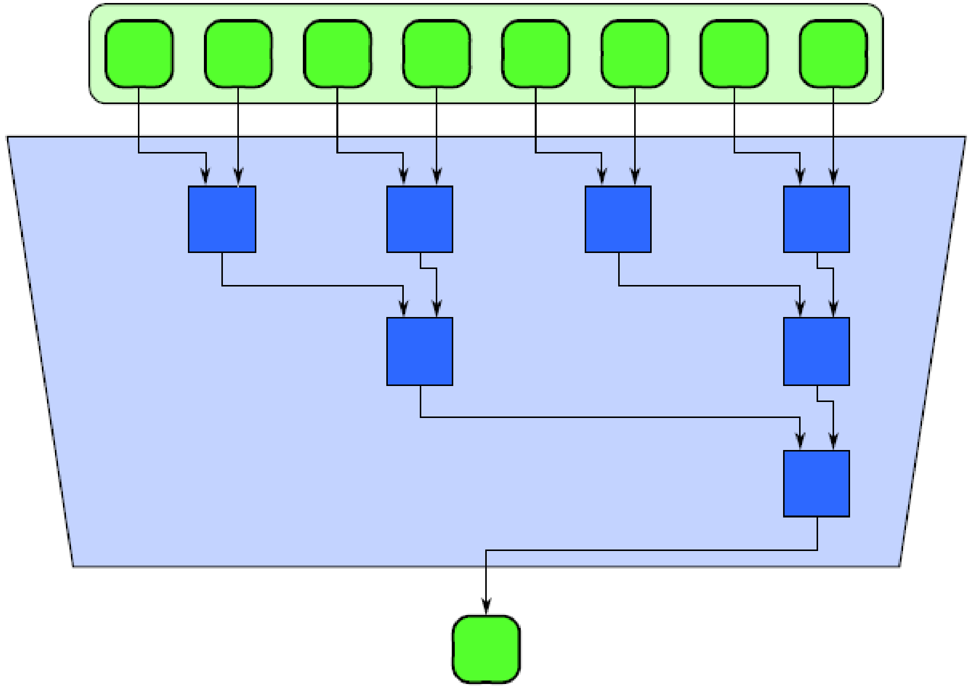
- Combines every element in a collection into a single element using an associative **combiner function**
- Many different orderings are possible

```
double add_reduce(const double a[], size_t n ) {
  double r = 0.0; // initialize with identity
  for (int i = 0; i < n; ++i) {
    r += a[i];
  return r;
}
```

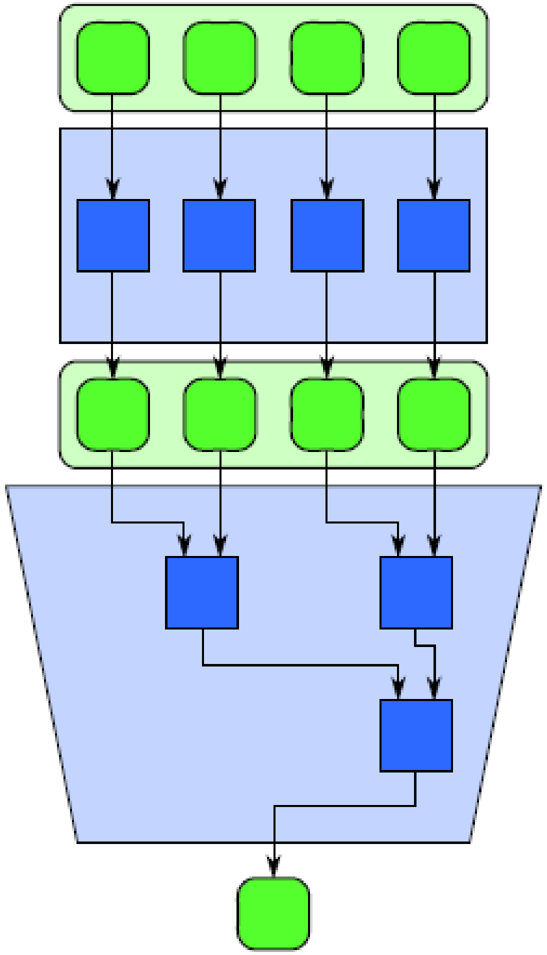


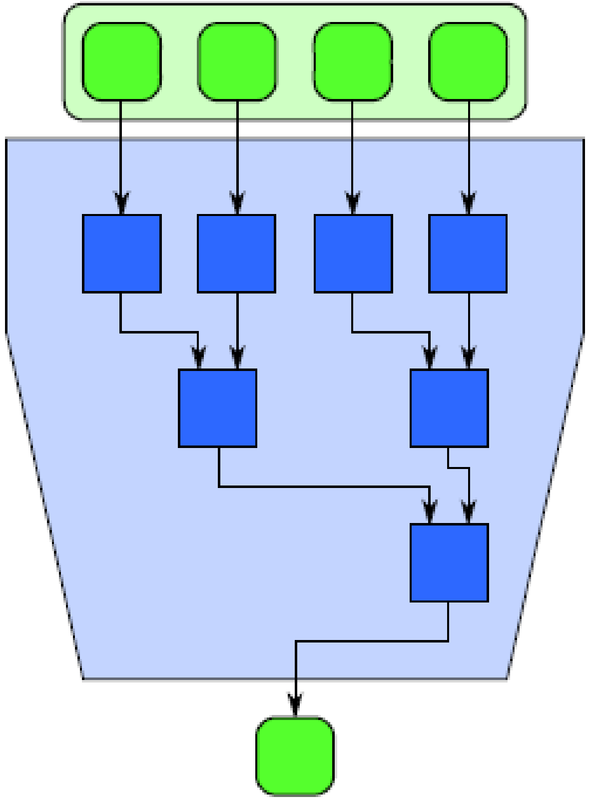
Control Pattern: Reduction

- Several choices for parallel reduction
  - Tree reduction
    - Could have local workers perform serial reduction, and then have a shallow tree to reduce results from workers



Fusing Map and Reduce

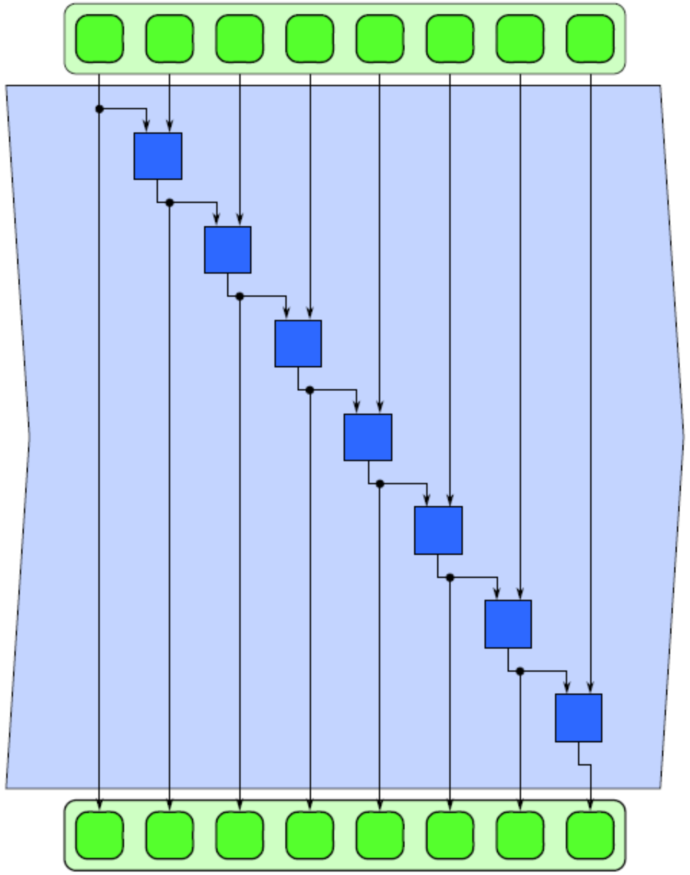




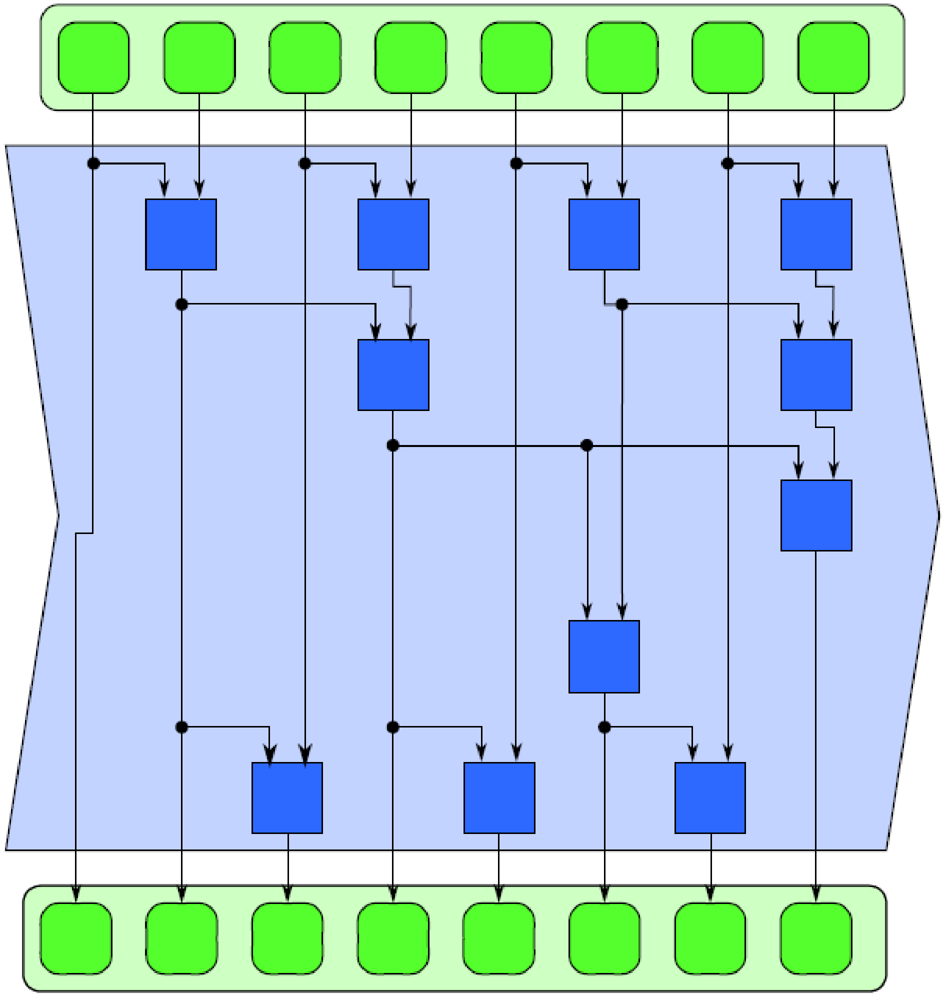
Control Pattern: Scan

- Scan computes all partial reductions of a collection
- For every output position, a reduction of the input up to that point is computed

```
void add_iscan(const float a[],
         float b[], size_t n) {
  if (n>0)
    b[0] = a[0];
  for (int i = 1; i < n; ++i)
    b[i] = b[i-1] + a[i];
}
```



Control Pattern: Scan



sum_arr = f(arr)

```
int arr[8] = {10, 1, 4, 2, 9, 5, 7, 8}
```

```
int sum_arr[8] = {10, 11, 15, 17, 26, 31, 38, 46}
```

Definition of Inclusive Prefix Scan

$[x_0, x_1, x_2, \dots, x_{n-1}]$

array of n elements

$\oplus$

binary associative operator

$[x_0, (x_0 \oplus x_1), (x_0 \oplus x_1 \oplus x_2), \dots, (x_0 \oplus x_1 \oplus \dots \oplus x_{n-1})]$

Exclusive Prefix Scan

$[x_0,\ x_1,\ x_2,\ \ldots,\ x_{n-1}]$

$\oplus$

I

Identity

$[I,\ x_0,\ (x_0 \oplus x_1),\ \ldots,\ (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})]$

A Problem

- Assume we have a 100-inch sandwich to feed ten people
- We know how many inches each person wants

```
3, 5, 2, 7, 28, 4, 3, 0, 8, 1
```

- How do we cut the sandwich quickly and distribute?

Solution to the Problem

- Method 1: Cut the sandwich sequentially starting from say left
- Method 2: Calculate prefix sum and cut in **parallel**

```
3, 8, 10, 17, 45, 49, 52, 52, 60, 61
```

Sequential Inclusive Prefix Scan

```
output[0] = arr[0]
for (int i = 1; i < n; i++) {
  output[i] = output[i-1] + arr[i];
}
```

How can Inclusive Prefix Scan be Parallelized?

```
output[0] = arr[0]
for (int i = 1; i < n; i++) {
  output[i] = output[i-1] + arr[i];
}
```

loop-carried dependence

A **Naïve** Parallel Prefix Sum

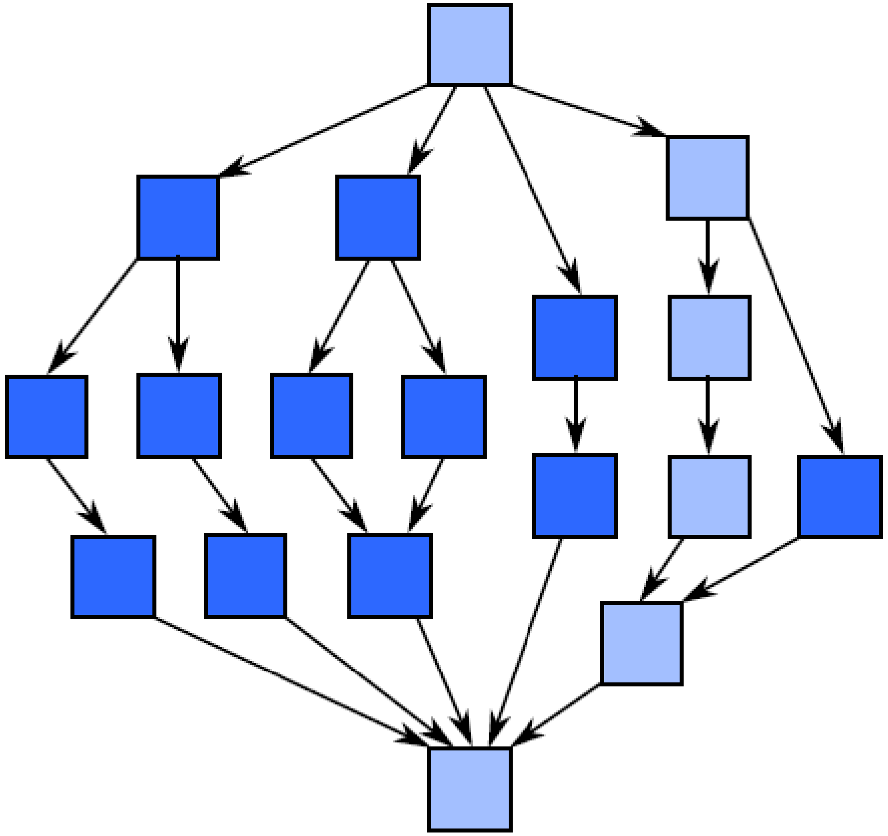
- Use one thread to compute each output element
  - The thread adds up all the previous elements needed for the output

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$
…

Analysis of Parallel Algorithms

- T_p = Execution time of a parallel program with p processors
- **Work**
  - Total number of computation operations performed by the p processors
  - Time to run on a single processor (T_1)
- **Span**
  - Length of the longest series of sequential operations or the critical path
  - Time taken to run on infinite processors (T_∞)

Work-Span Model



Analysis of Parallel Algorithms

- **Cost**
  - Total time spent by **all** processors in computation (pT_p)

Cost $\geq$ Work

$$pT_p \geq T_1$$

Execution time $\geq$ Span

$$T_p \geq T_\infty$$

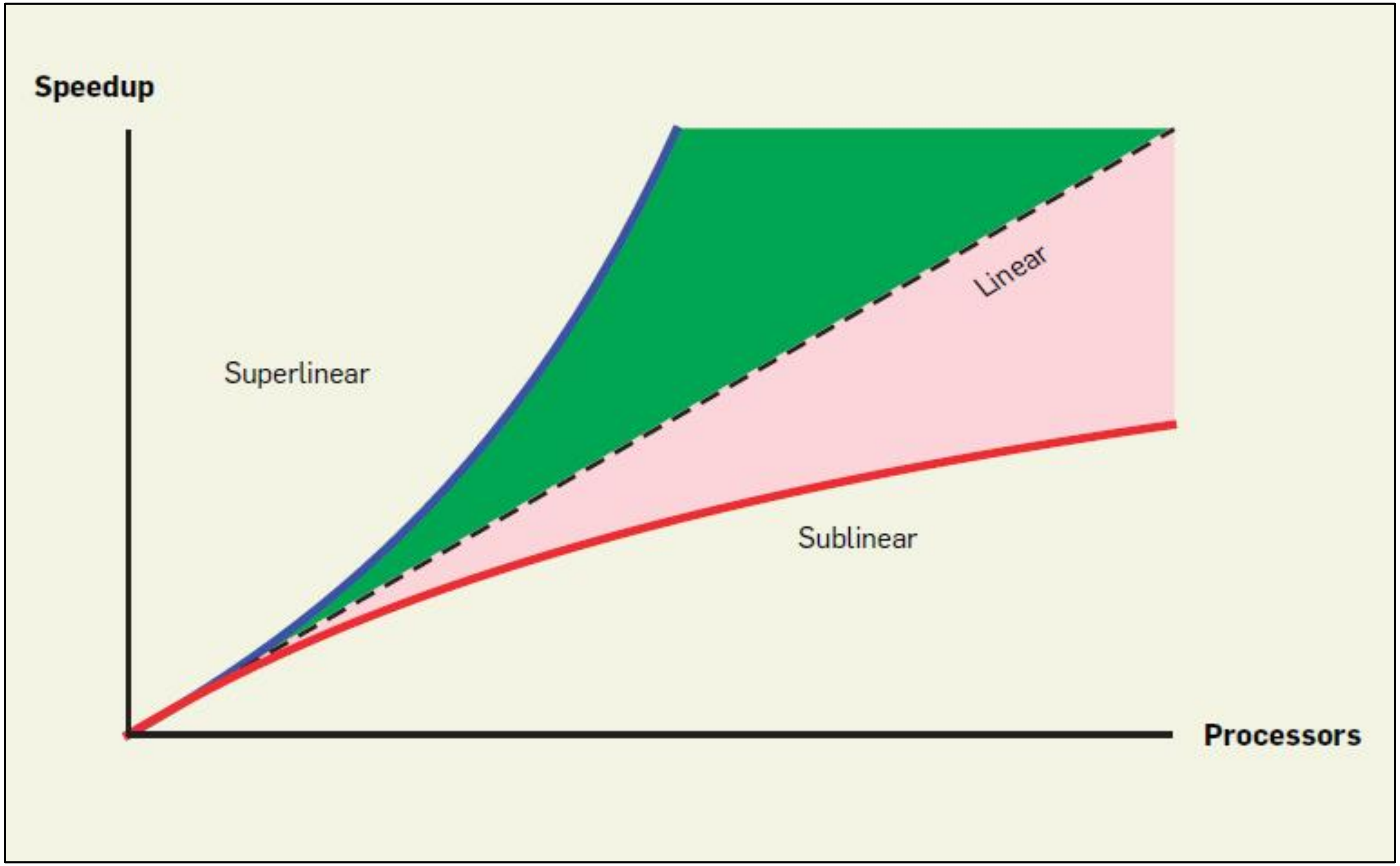
Analysis of Parallel Algorithms

- **Speedup (S_p)**
  - Total time spent by all processors in computation (pT_p)

$$\text{Speedup} = \frac{T_1}{T_p} \leq \frac{T_1}{T_\infty}$$

$$\text{Speedup} = \frac{T_1}{T_p} \leq p$$

Speedup



Other Metrics

- **Efficiency**
  - Speedup per processor $\frac{S_p}{p} = \frac{T_1}{pT_p}$

- **Parallelism**
  - Maximum possible speedup given any number of processors $\frac{T_1}{T_\infty}$

Sequential Inclusive Prefix Scan

```
output[0] = arr[0]
for (int i = 1; i < n; i++) {
  output[i] = output[i-1] + arr[i];
}
```

Asymptotic complexity $O(n)$

Work = $O(n)$

Span = $O(n)$

A Naïve Parallel Prefix Sum

- Use one thread to compute each output element
  - The thread adds up all the previous elements needed for the output

```
y0 = x0
y1 = x0 + x1
y2 = x0 + x1 + x2
…
```

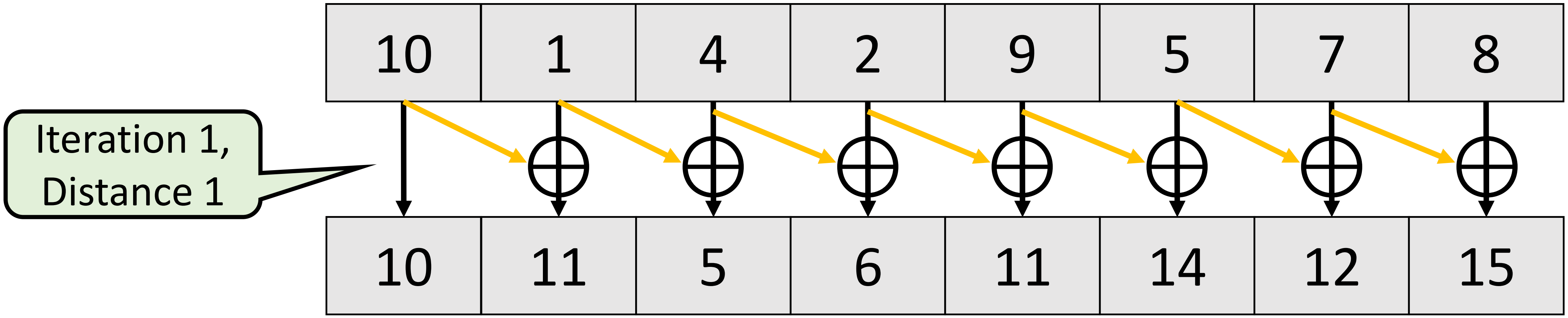
- Work $= 1 + 2 + 3 + \ldots + n = \dfrac{n(n+1)}{2}$

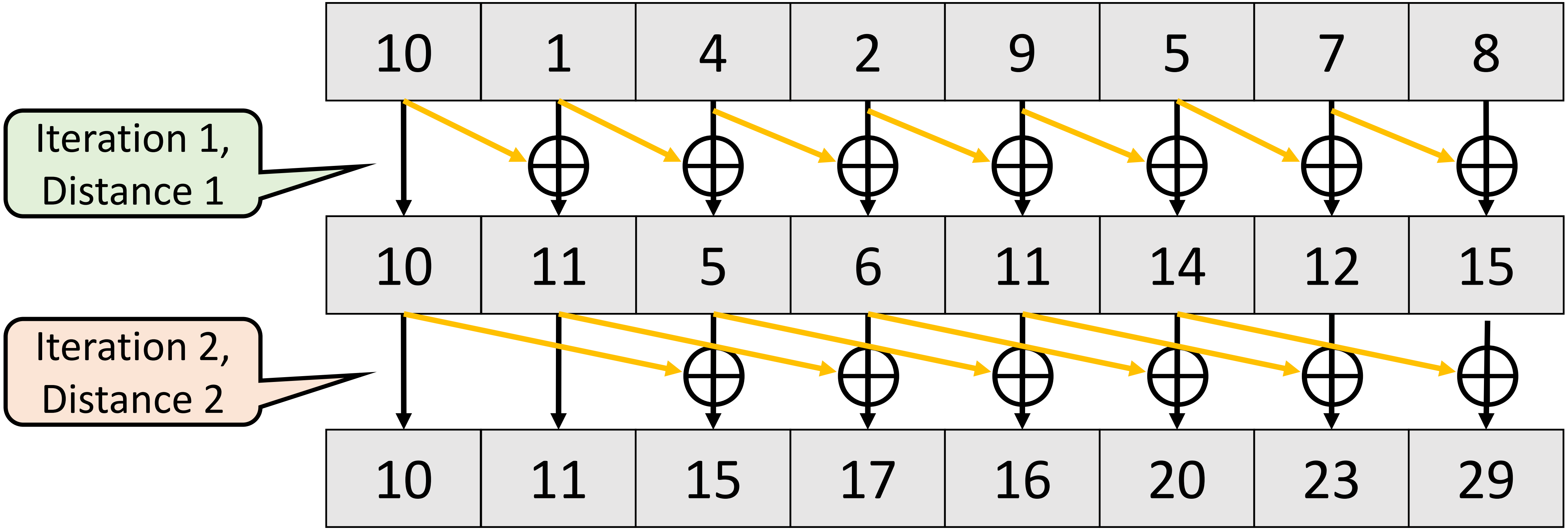
  $= O(n^2)$ operations

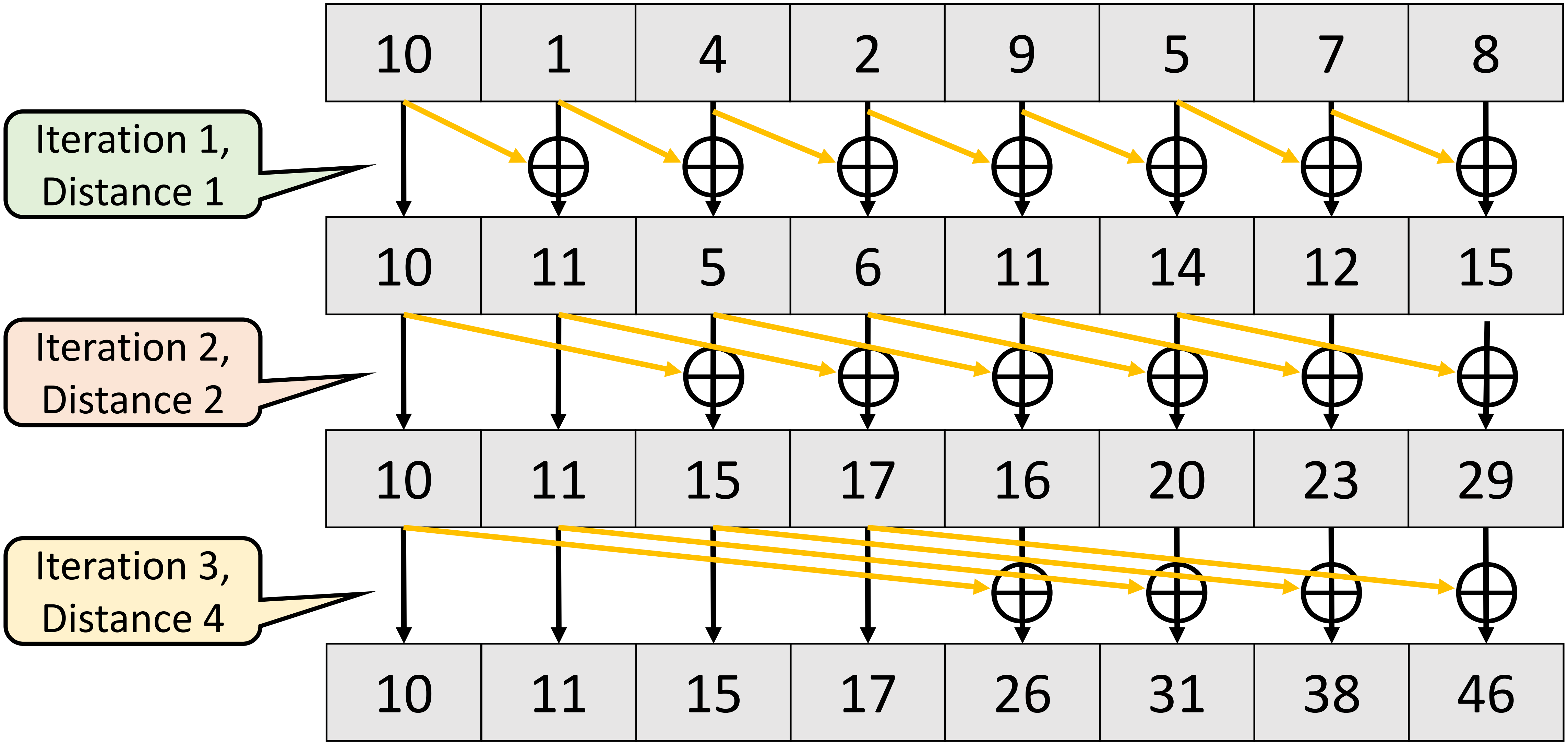
Parallel Inclusive Prefix Sum

10	1	4	2	9	5	7	8

threads: p
(here p == n, and n = 8)


10
1
4
2
9
5
7
8
Iteration 1,
Distance 1
10
11
5
6
11
14
12
15


10 1 4 2 9 5 7 8
Iteration 1, Distance 1
10 11 5 6 11 14 12 15
Iteration 2, Distance 2
10 11 15 17 16 20 23 29


10 1 4 2 9 5 7 8
Iteration 1, Distance 1
10 11 5 6 11 14 12 15
Iteration 2, Distance 2
10 11 15 17 16 20 23 29
Iteration 3, Distance 4
10 11 15 17 26 31 38 46

Algorithm Efficiency

- # of iterations: log n
- First iteration: (n-1) additions
- Second iteration: (n-2) additions
- Third iteration: (n-4) additions
- Last iteration: (n – n/2) additions
- Total additions $= (n-1) + (n-2) + (n-4) + \ \ldots + \left(n - \frac{n}{2}\right)$

$$= n \log n - \left(1 + 2 + 4 + \cdots + \frac{\mathrm{n}}{2}\right)$$

$$= n \log n - (n-1) = O\,(n \log n)$$

Algorithm Efficiency

- **Work** = O $(n \log n)$
  - Remember Work for the sequential algorithm was O(n)
  - For large n, $\log n$ can be a non-trivial factor

Hillis and Steele

```
for i = 0 to ⌈log n-1⌉ do
    for j = 2^i to n-1 in parallel do
        A [j] = A[j] + A[j-2^i]
```

Asymptotic complexity O(log n)

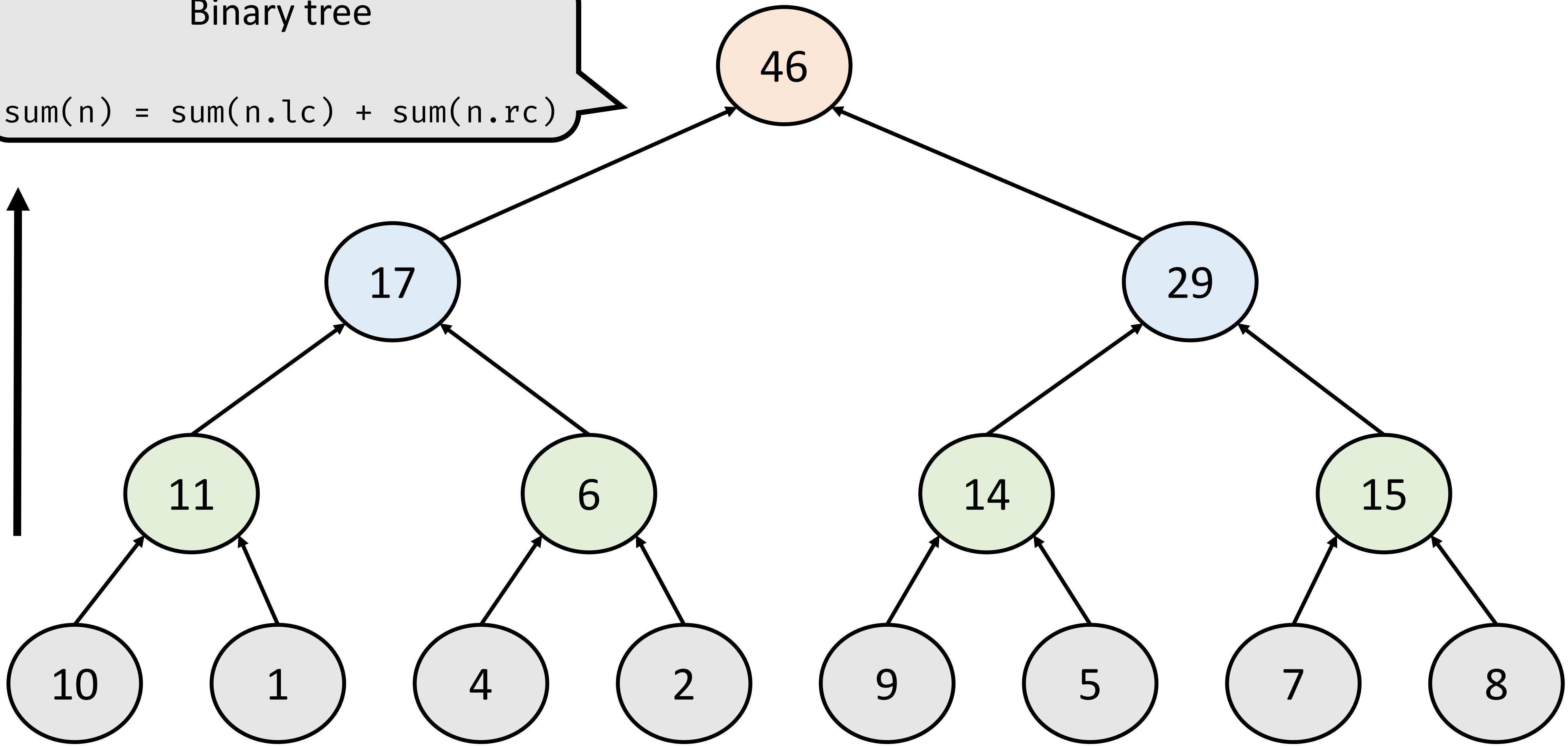
Algorithm With Improved Work-Efficiency

Guy Blelloch

10	1	4	2	9	5	7	8

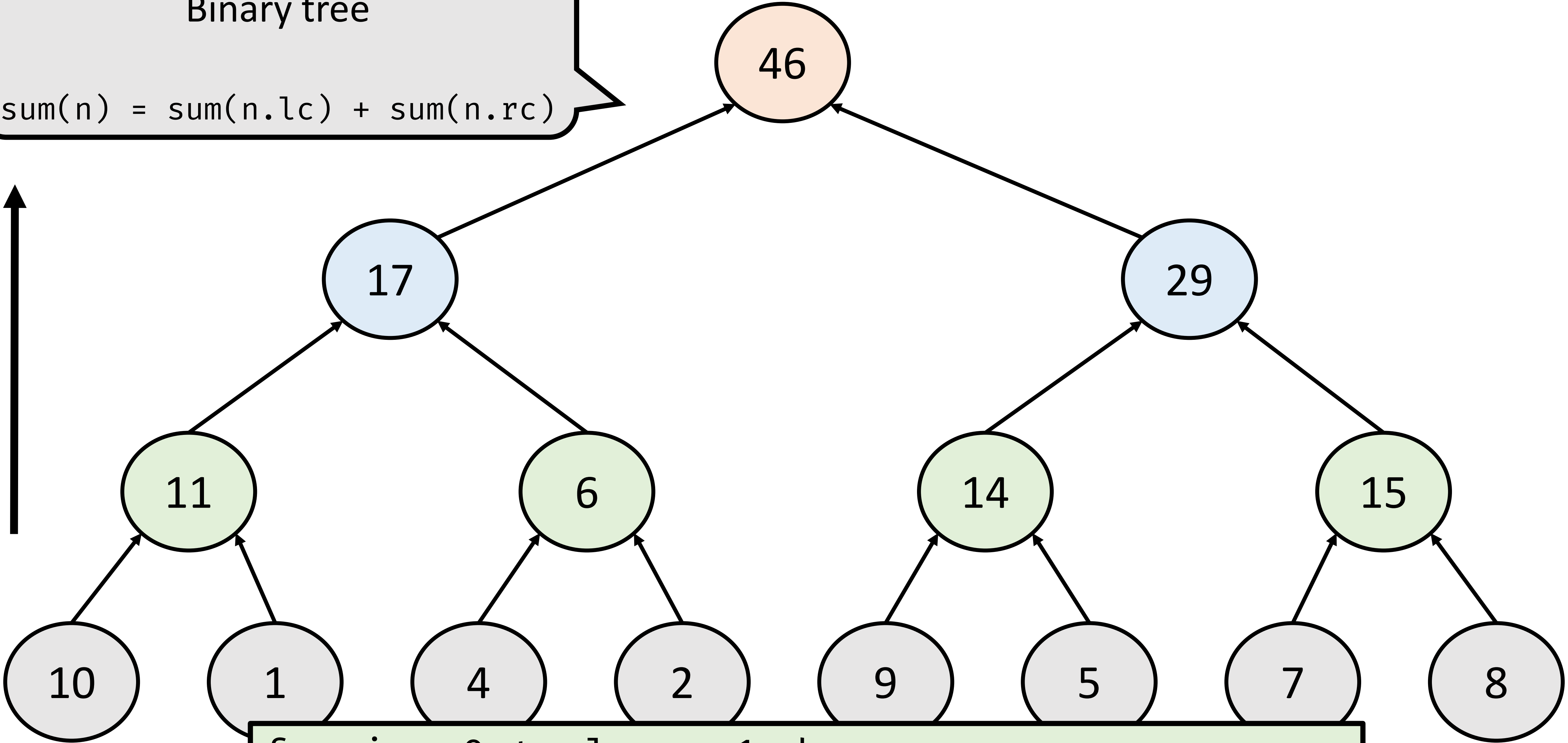
Binary tree

```
sum(n) = sum(n.lc) + sum(n.rc)
```

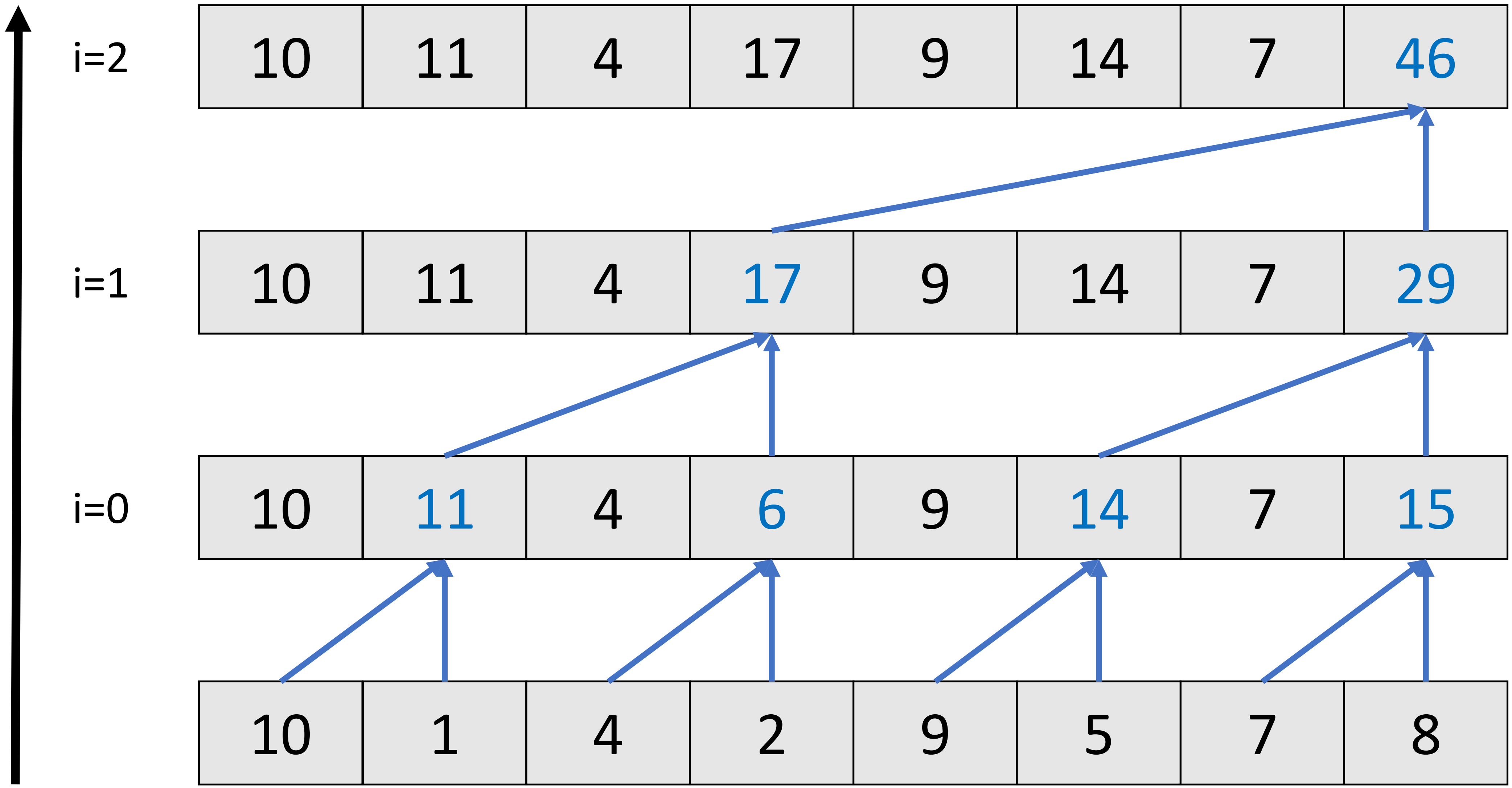


Binary tree

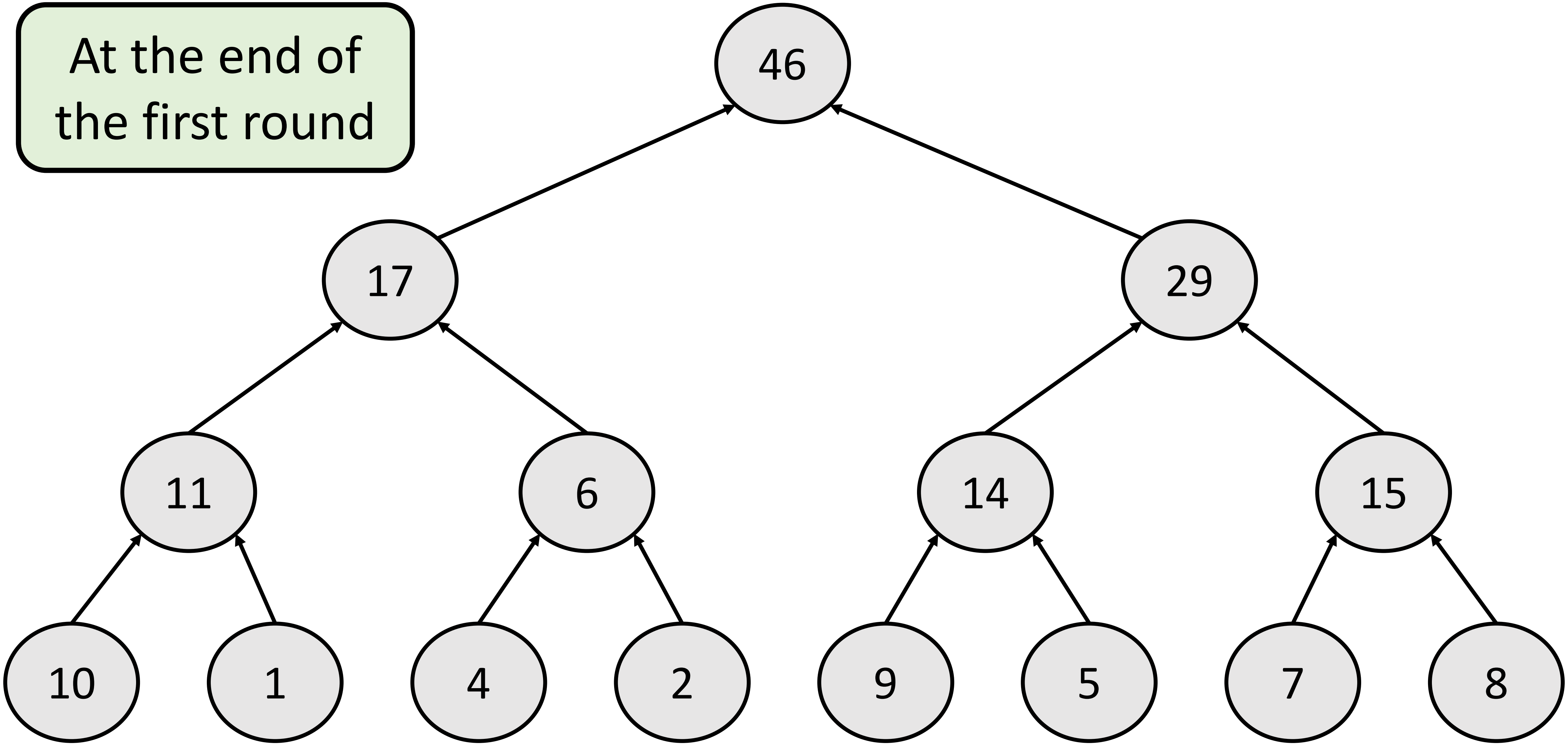
```
sum(n) = sum(n.lc) + sum(n.rc)
```

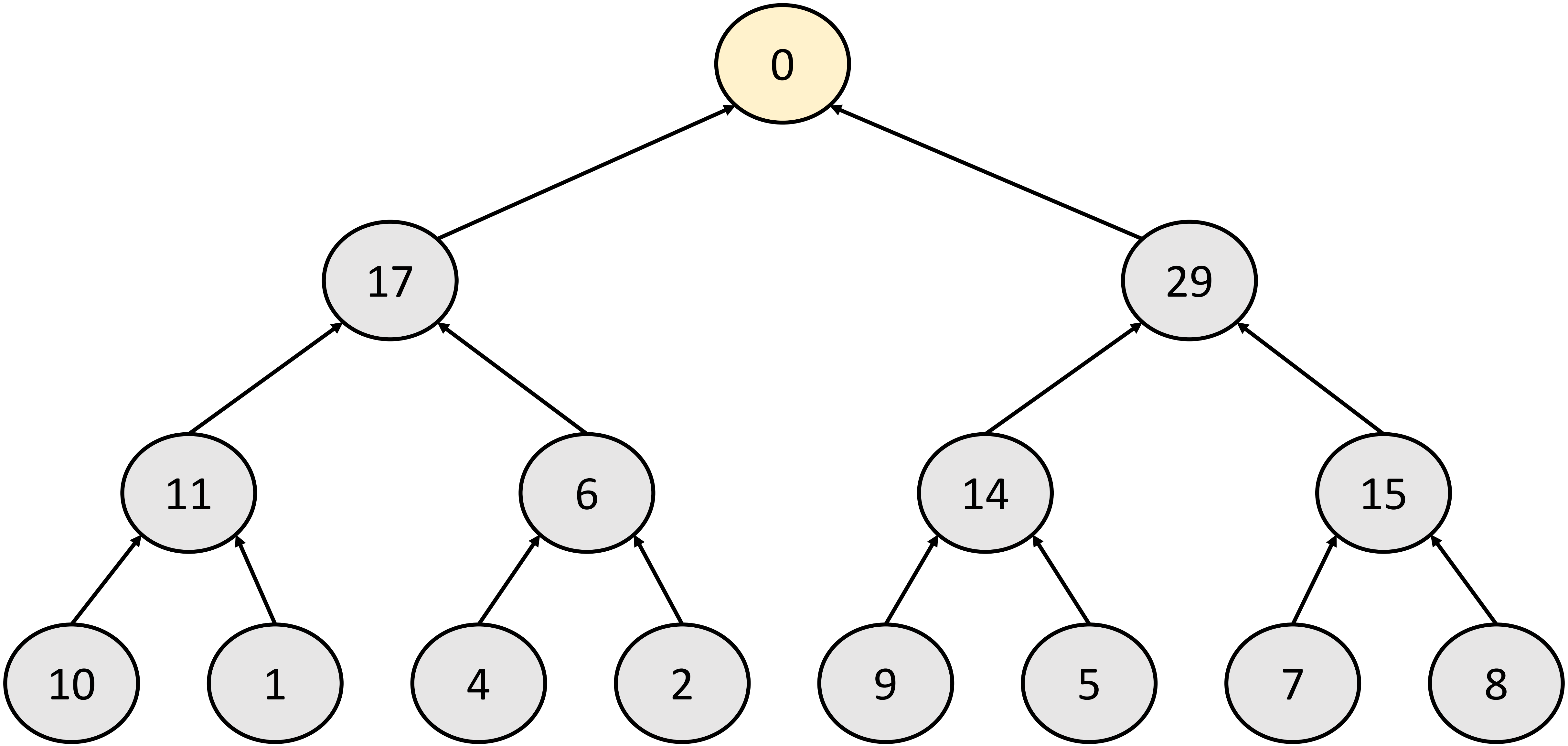


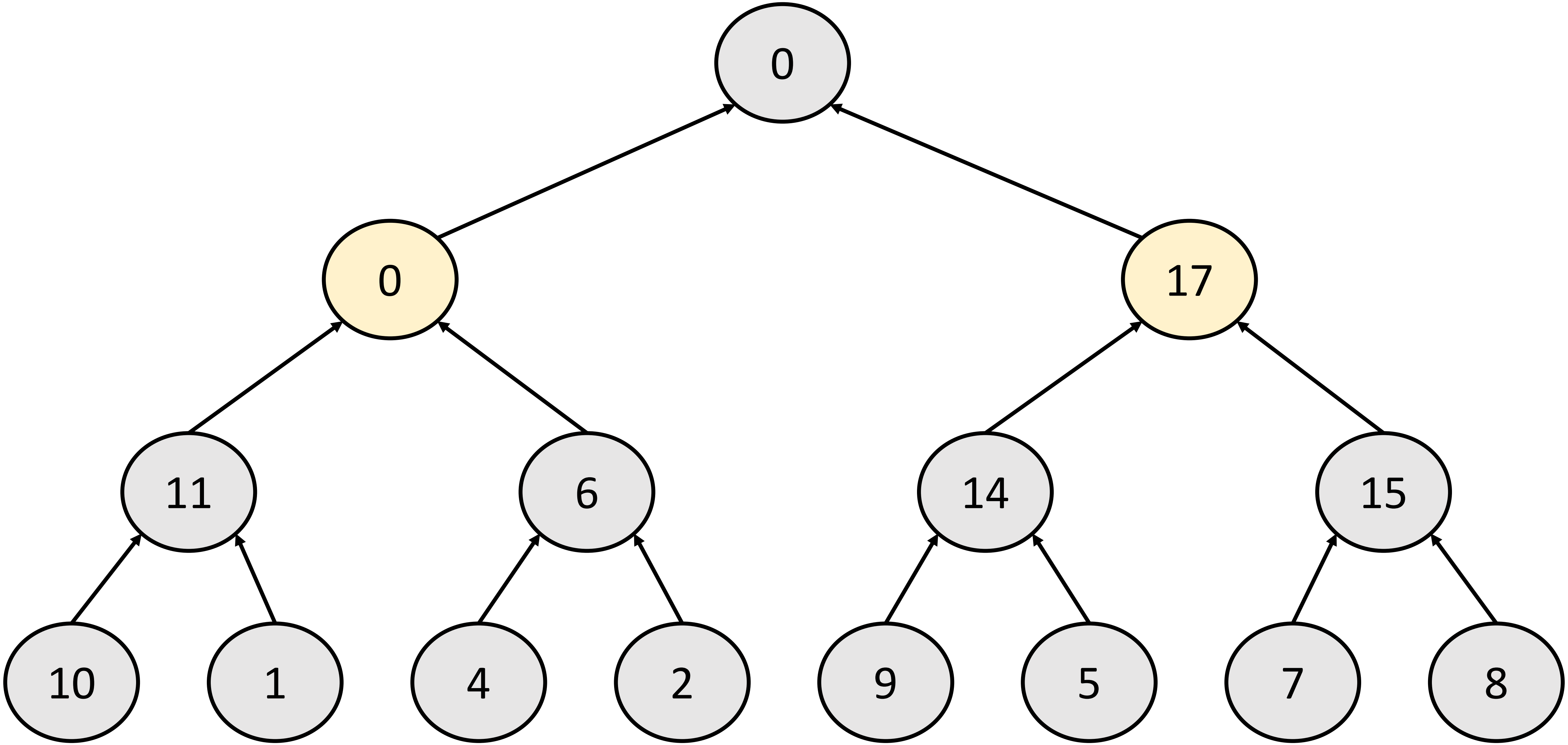

```
for i = 0 to log n-1 do
    for j = 0 to n-1 by 2^(i+1) in parallel do
        a[j+2^(i+1)-1] = a[j+2^i-1] + a[j+2^(i+1)-1]
```


i=2
10 11 4 17 9 14 7 46
i=1
10 11 4 17 9 14 7 29
i=0
10 11 4 6 9 14 7 15
10 1 4 2 9 5 7 8

At the end of the first round




0
17
29
11
6
14
15
10
1
4
2
9
5
7
8


0
0
17
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1
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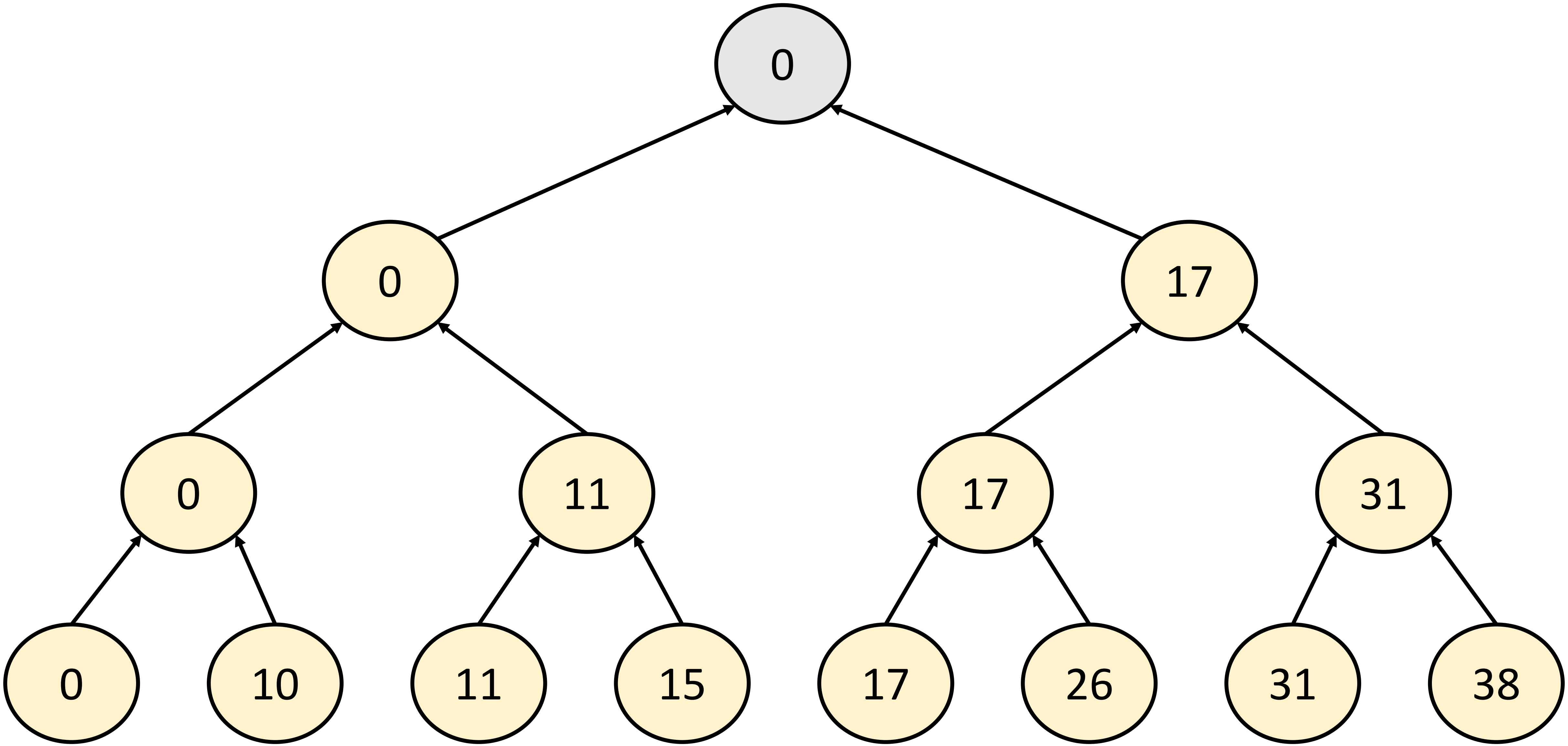
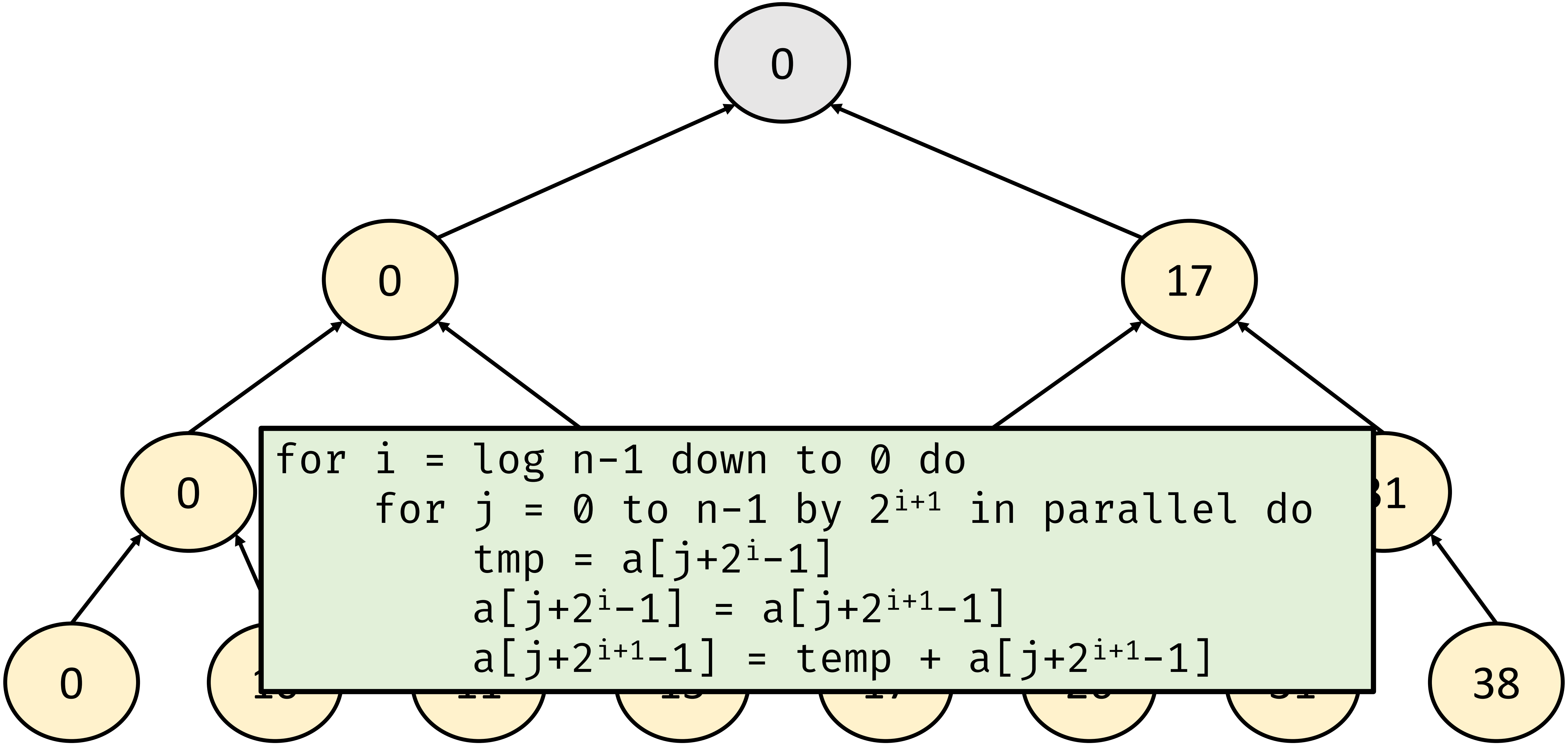
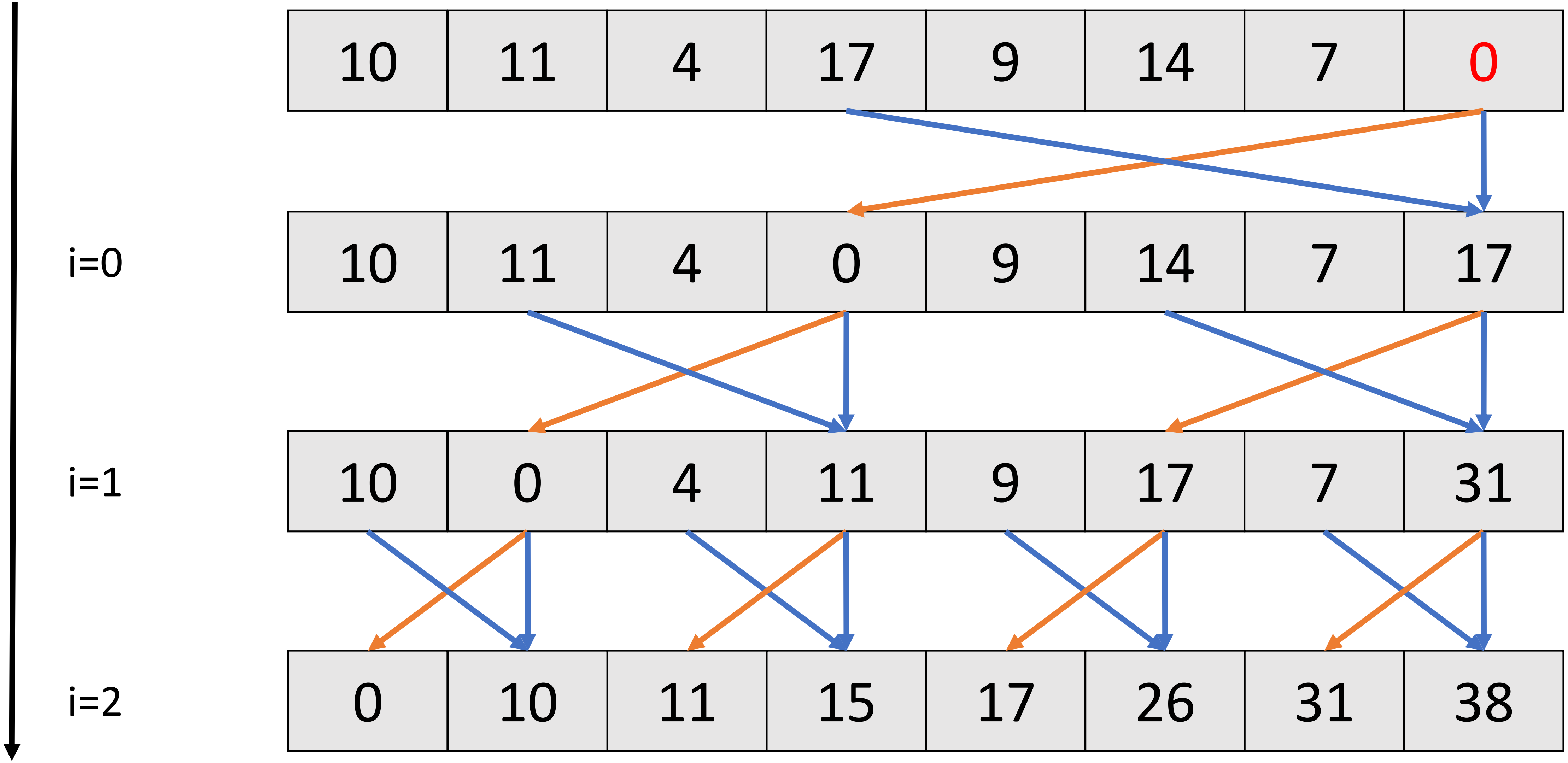
8

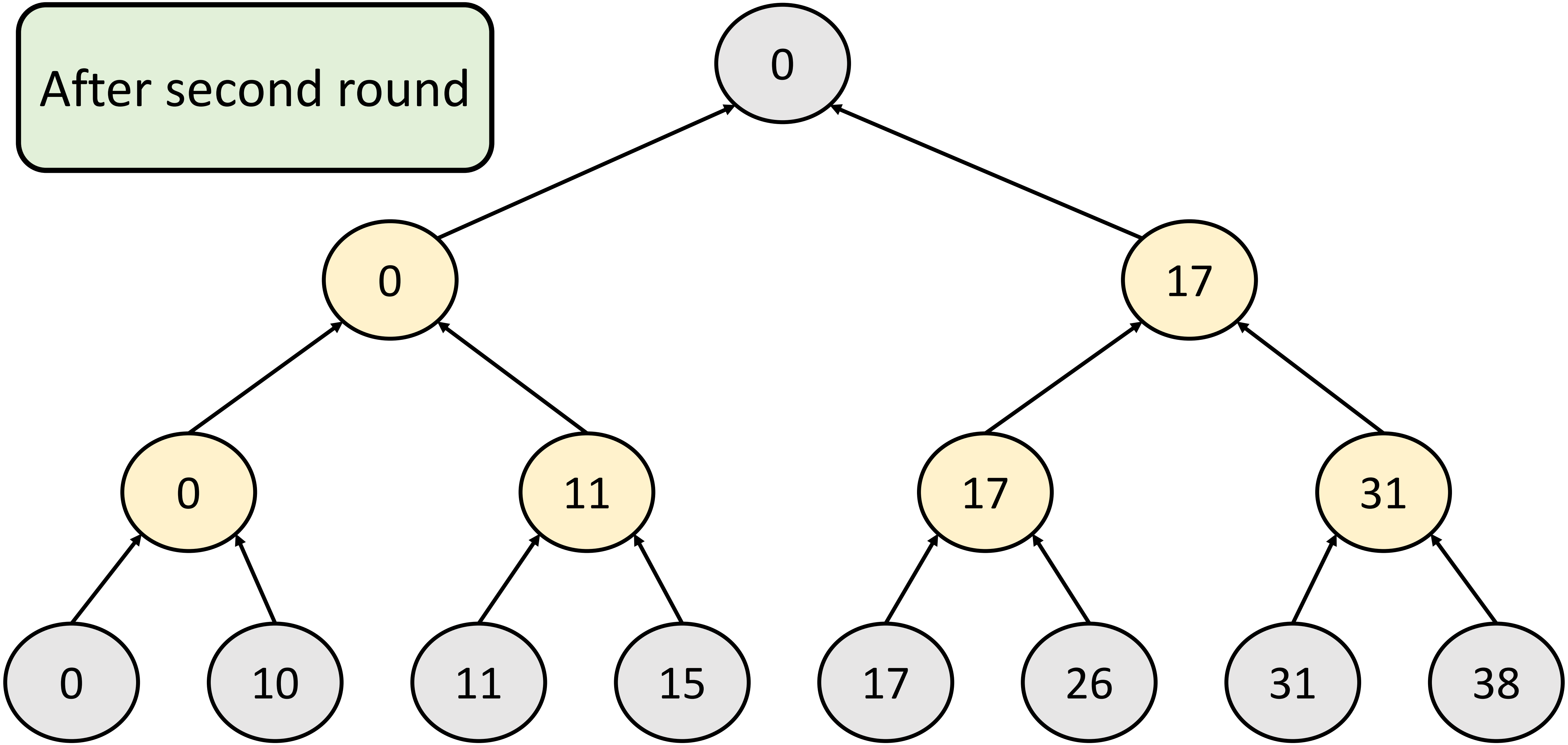

0
0
17
0
11
17
31
0
10
11
15
17
26
31
38




```
for i = log n-1 down to 0 do
    for j = 0 to n-1 by 2^(i+1) in parallel do
        tmp = a[j+2^i-1]
        a[j+2^i-1] = a[j+2^(i+1)-1]
        a[j+2^(i+1)-1] = temp + a[j+2^(i+1)-1]
```


10 11 4 17 9 14 7 0
i=0
10 11 4 0 9 14 7 17
i=1
10 0 4 11 9 17 7 31
i=2
0 10 11 15 17 26 31 38

After second round



Algorithm Efficiency

Asymptotic complexity O(log n)

```
for i = 0 to log n-1 do
    for j = 0 to n-1 by 2^i+1 in parallel do
        a[j+2^(i+1)-1] = a[j+2^i-1] + a[j+2^(i+1)-1]
```

⇩

```
for i = log n-1 down to 0 do
    for j = 0 to n-1 by 2^(i+1) in parallel do
        tmp = a[j+2^i-1]
        a[j+2^i-1] = a[j+2^(i+1)-1]
        a[j+2^(i+1)-1] = temp + a[j+2^(i+1)-1]
```

Algorithm Efficiency

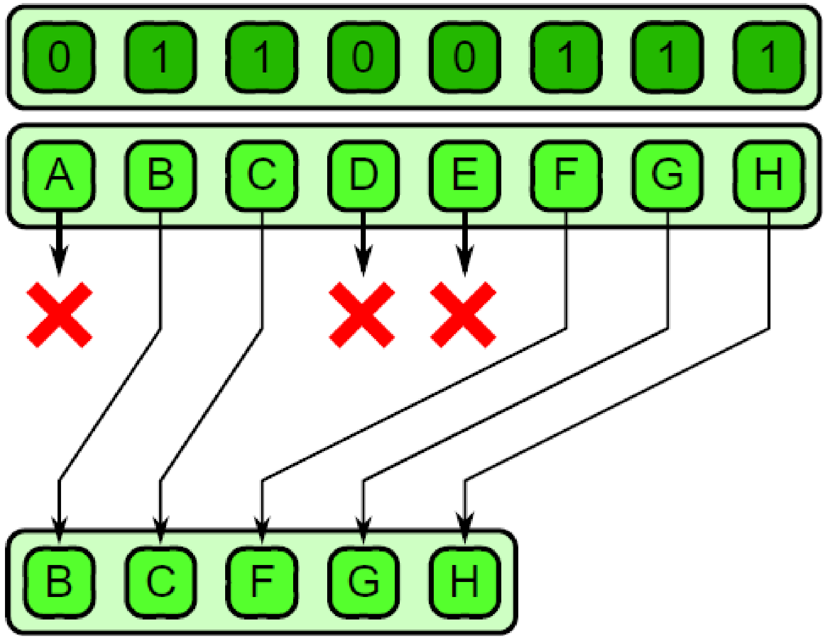
- # of iterations: log n in each pass
- Number of addition operations in first pass: $\frac{n}{2} + \frac{n}{4} + \cdots + 2 + 1$
- Number of addition operations in second pass: $1 + 2 + \cdots + \frac{n}{2}$
- Total additions $= (\mathrm{n} - 1) + (\mathrm{n} - 1) = \mathbf{2}(\mathrm{n} - 1)$

  = O (n)

Benefits from parallelism can overcome the constant factor increase in computation

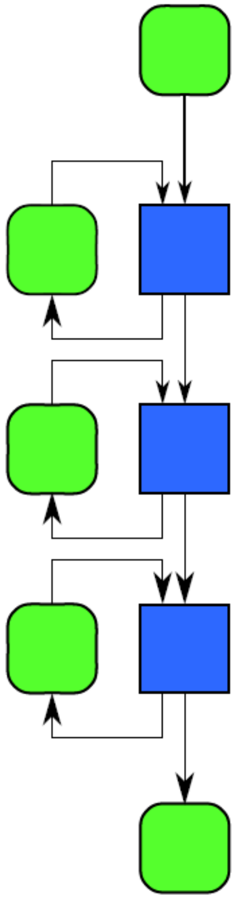
Data Management Pattern: Pack

- Eliminate unused data
  - Helps in reducing required memory bandwidth
  - Retained elements are moved to make them contiguous in memory
- Used in register masks



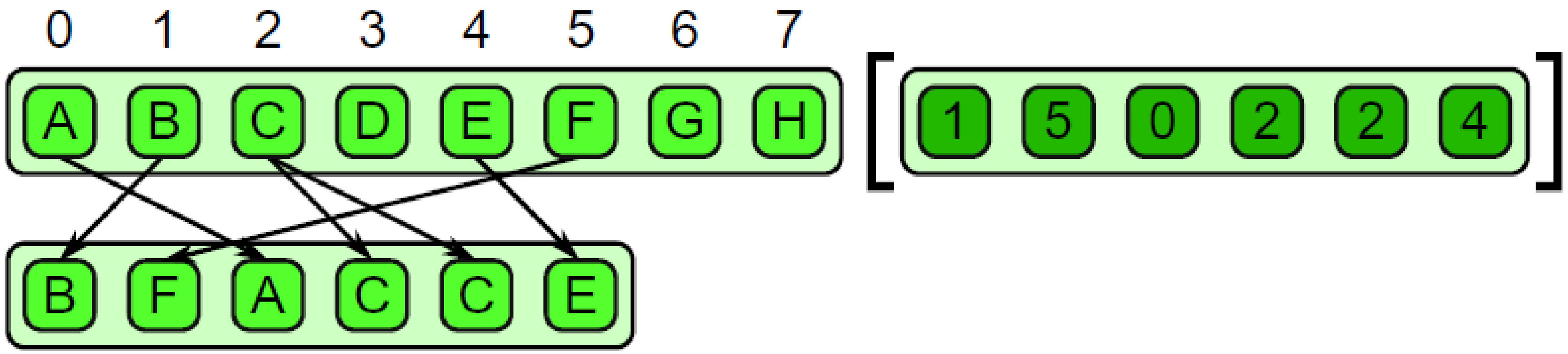
Data Management Pattern: Pipeline

- Connects tasks respecting a producer-consumer relationship
- Used in video encoding for processing incoming frames

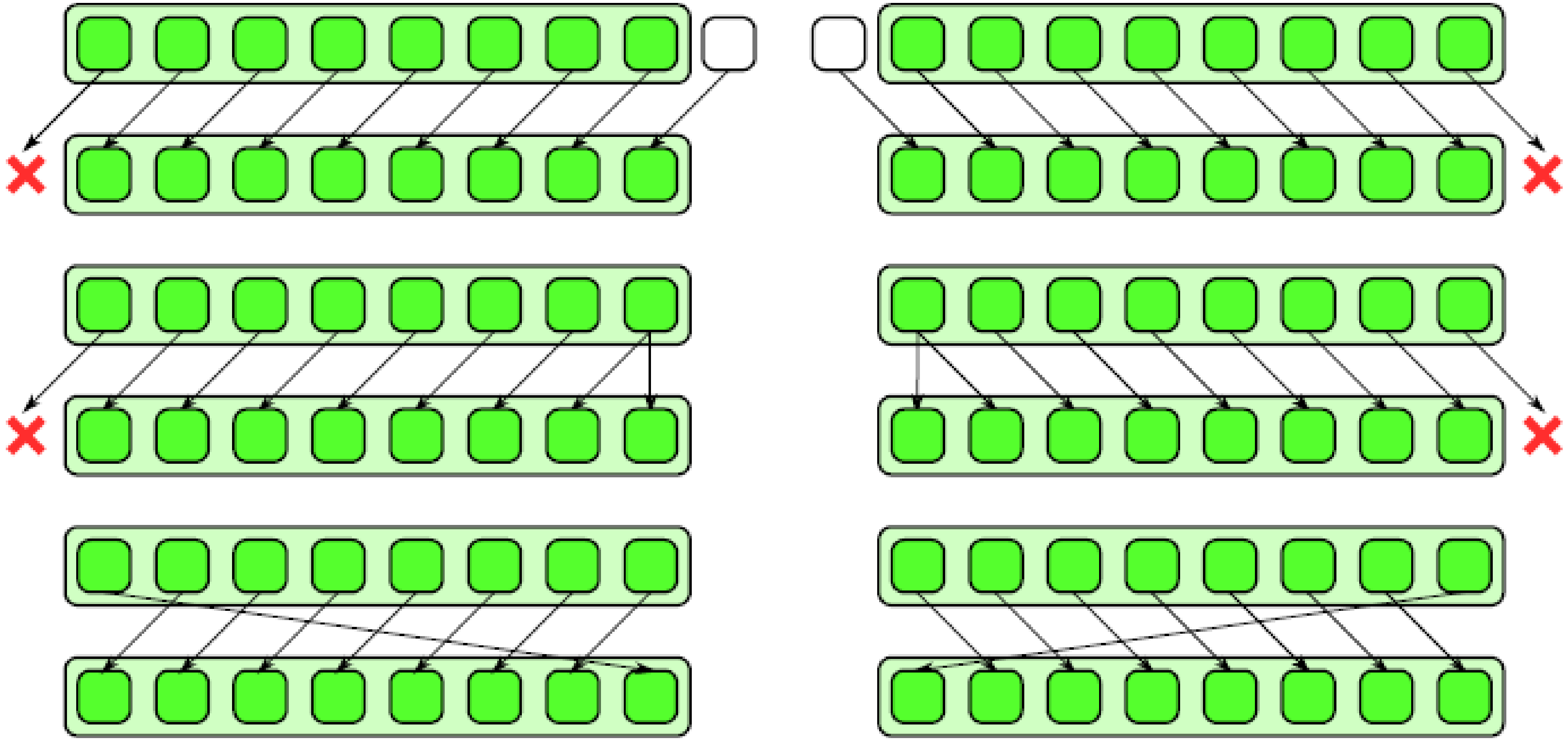


Data Management Pattern: Gather

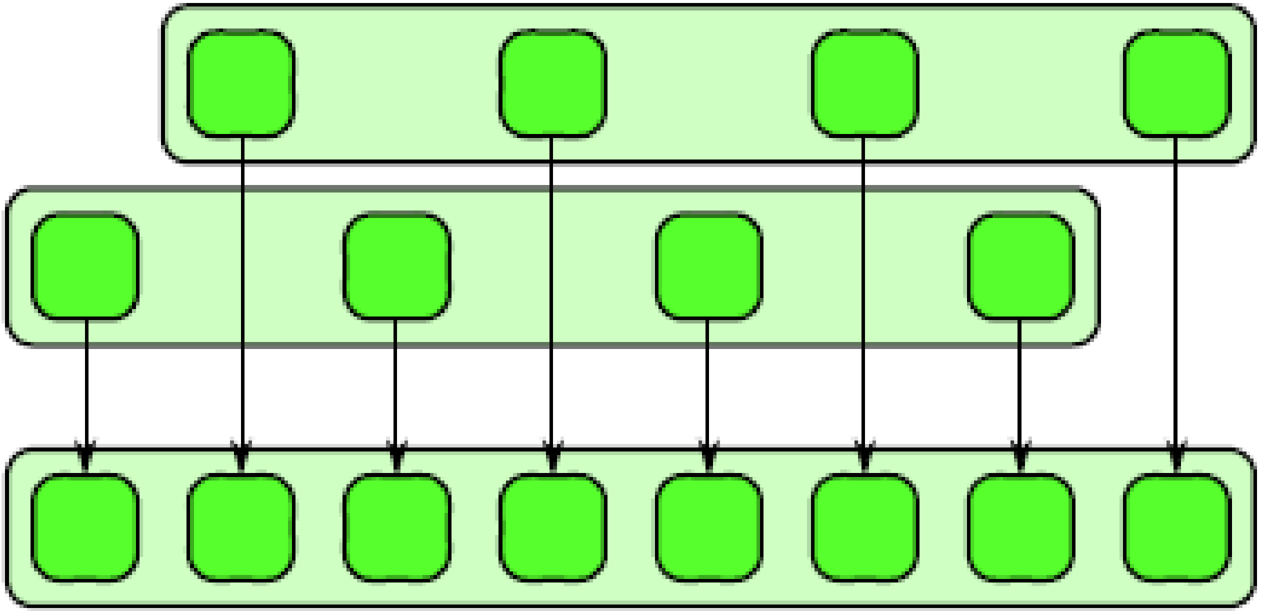
- Collect data based on information from another collection and set of indices
- Left and right shifts are an example of gather operation



Shift Operation

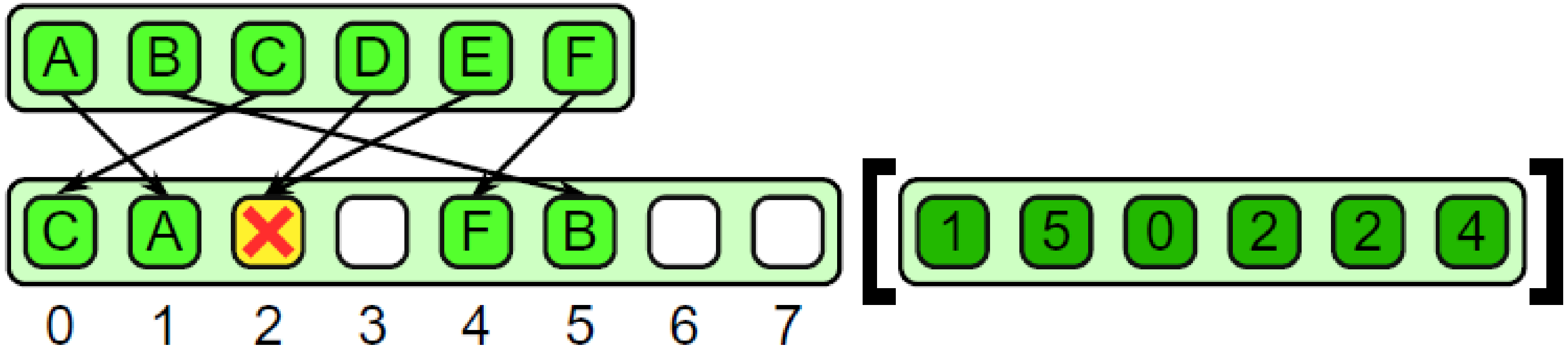


Zip Operation



Data Management Pattern: Scatter

- Inverse of gather, data elements are output



References

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- Yong Cao. Parallel Prefix Sum – Scan.
- G. Blelloch. Prefix Sums and Their Applications.
- Th. Ottmann. Parallel Prefix Computation.