CS698L: Data Dependence Analysis

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Content influenced by many excellent references, see References slide for acknowledgements.

How to Write Efficient and Scalable Programs?

Good choice of algorithms and data structures

- Determines number of operations executed

Code that the compiler and architecture can effectively optimize

- Determines number of instructions executed

Proportion of parallelizable and concurrent code

- Amdahl's law

Sensitive to the architecture platform

- Efficiency and characteristics of the platform
- For e.g., memory hierarchy, cache sizes

Role of a Good Compiler

Try and extract performance automatically

Optimize memory access latency

- Code restructuring optimizations
- Prefetching optimizations
- Data layout optimizations
- Code layout optimizations

Parallelism Challenges for a Compiler

- On single-core machines
  - Focus is on register allocation, instruction scheduling, reduce the cost of array accesses
- On parallel machines
  - Find parallelism in sequential code, find portions of work that can be executed in parallel
  - Principle strategy is data decomposition – good idea since this can scale

Can we parallelize the following loops?

```
do i = 1, 100
  A(i) = A(i) + 1
enddo
```

```
do i = 1, 100
  A(i) = A(i-1) + 1
enddo
```

Data Dependences

S1 a = b + c

S2 d = a * 2

S3 a = c + 2

S4 e = d + c + 2

Data Dependences

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S4 e = d + c + 2

Execution constraints

- S2 must execute after S1
- S3 must execute after S2
- S3 must execute after S1
- S3 and S4 can execute in any order, and concurrently

Data Dependence

- There is a data dependence from S1 to S2 if and only if
  - Both statements access the same memory location
  - At least one of the accesses is a write
  - There is a feasible execution path at run-time from S1 to S2

Types of Dependences

Flow (true)

```
S1   X = …
S2   … = X
```

Anti

```
S1   … = X
S2   X = …
```

Output

```
S1   X = …
S2   X = …
```

Input

```
S1   … = a/b
S2   … = b * c
```

Bernstein's Conditions

- Suppose there are two processes P_1 and P_2
- Let I_i be the set of all input variables for process P_i
- Let O_i be the set of all output variables for process P_i
- P_1 and P_2 can execute in parallel (denoted as $P_1 || P_2$) if and only if
  - $I_1 \cap I_2 = \Phi$
  - $I_2 \cap O_1 = \Phi$
  - $O_2 \cap O_1 = \Phi$

Bernstein's Conditions

- Suppose there are two processes P_1 an
- Let I
  varia
- Let O_i be the set of all output variables for process P_i

- P_1 and P_2 can execute in parallel
  only
  - $O_2 \cap O_1 = \Phi$

Two processes can execute in parallel if they are flow-, anti-, and output-independent

Bernstein's Conditions

- Suppose there are two processes P_1 ar…
- Let I… vari…
- Let O_i be the set of all output variables for process P_i

- P_1 and P_2 can execute in parallel … only
  - $O_2 \cap O_1 = \Phi$

Two processes can execute in parallel if they are flow-, anti-, and output-independent

- If P_i || P_j, does that imply P_j || P_i?
- If P_i || P_j and P_j || P_k, does that imply P_i || P_k?

Find Parallelism in Loops – Is it Easy?

- Need to analyze array subscripts
- Need to check whether two array subscripts access the same memory location

Dependence in Loops

```
    for i = 1 to 50
S1    A[i] = B[i-1] + C[i]
S2    B[i] = A[i+2] + C[i]
    endfor
```

- Unrolling loops can help figure out dependences

```
S1(1)    A[1] = B[0] + C[1]
S2(1)    B[1] = A[3] + C[1]
S1(2)    A[2] = B[1] + C[2]
S2(2)    B[2] = A[4] + C[2]
S1(3)    A[3] = B[2] + C[3]
S2(3)    B[3] = A[5] + C[3]
......
```

Dependence in Loops

```
    for i = 1 to 50
S1    A[i] = B[i-1] + C[i]
S2    B[i] = A[i+2] + C[i]
    endfor
```

- large loop bounds
- loop bounds may not be known at compile time

- Unrolling loops can help figure out dependences

```
S1(1)    A[1] = B[0] + C[1]
S2(1)    B[1] = A[3] + C[1]
S1(2)    A[2] = B[1] + C[2]
S2(2)    B[2] = A[4] + C[2]
S1(3)    A[3] = B[2] + C[3]
S2(3)    B[3] = A[5] + C[3]
```

Dependence in Loops

- Parameterize the statement with the loop iteration number

```
      DO I = 1, N
S1      A(I+1) = A(I) + B(I)
      ENDDO

      DO I = L, U, S
S1      ...
      ENDDO
```

Normalized Iteration Number

For an arbitrary loop in which the loop index I runs from L to U in steps of S, the *normalized iteration number i* of a specific iteration is equal to the value $(I - L + 1)/S$, where I is the value of the index on that iteration

Iteration Vector

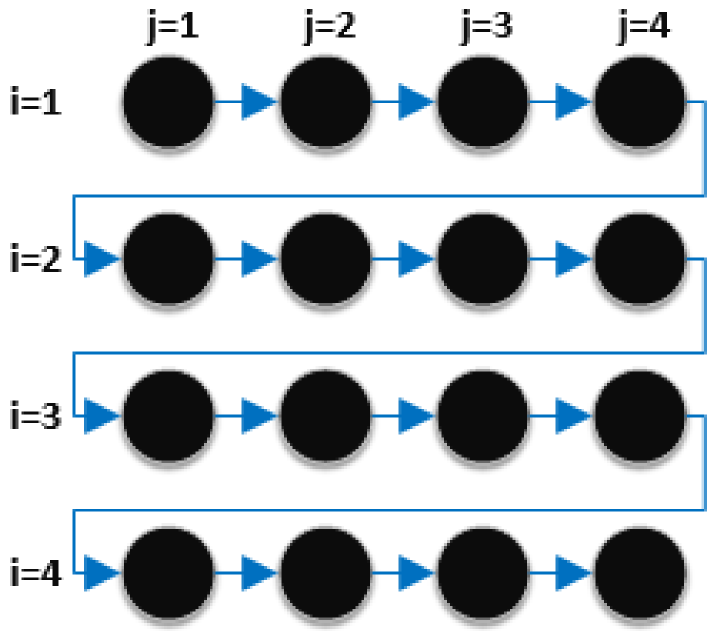
Given a nest of n loops, the *iteration vector* ***i*** of a particular iteration of the innermost loop is a vector of integers that contains the iteration numbers for each of the loops in order of nesting level.

The iteration vector ***i*** is $\{i_1, i_2, ..., i_n\}$ where i_k, $1 \le k \le n$, represents the iteration number for the loop at nesting level k.

Iteration Space Graphs

- Represent each dynamic instance of a loop as a point in the graph
- Draw arrows from one point to another to represent dependences

```
      for (i = 0; i < 4; i++)
        for (j = 0; j < 4; j++)
S1:       a[i][j] = a[i][j-1] * x;
```



Iteration Space Graph

- Dimension of iteration space is the loop nest level
- Not restricted to be rectangular

```
for i = 1 to 5 do
  for j = i to 5 do
    A(i, j) = B(i, j) + C(j)
  endfor
endfor
```

Lexicographic Ordering of Iteration Vectors

- Assume i is a vector, i_k is the k^{th} element of the vector i, and i[1:k] is a k-vector consisting of the leftmost k elements of i

- Iteration i precedes iteration j, denoted by i < j, if and only if
  i. i[1:n-1] < j[1:n-1], or
  ii. i[1:n-1] = j[1:n-1] and $i_n < j_n$

Formal Definition of Loop Dependence

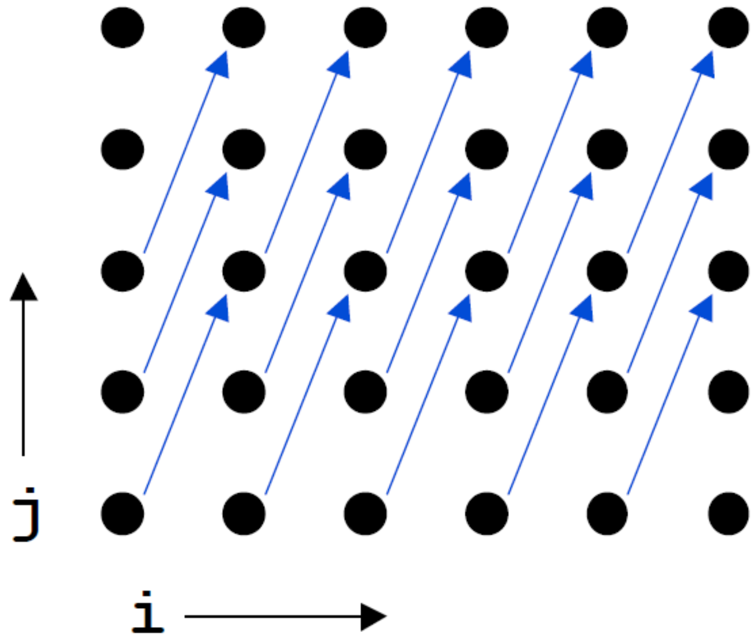
There exists a dependence from statement S1 to statement S2 in a common nest of loops if and only if there exist two iteration vectors i and j for the nest, such that

i. $i < j$ or $i = j$ and there is a path from S1 to S2 in the body of the loop,
ii. statement S1 accesses memory location *M* on iteration i and statement S2 accesses location *M* on iteration j, and
iii. one of these accesses is a write.

Distance Vectors

- For each dimension of an iteration space, the distance is the number of iterations between accesses to the same memory location

```
do i = 1, 6
  do j = 1, 5
    A(i, j) = A(i-1, j-2) + 1
  enddo
enddo
```



- Distance vector: (1, 2)

outer loop

inner loop

Distance Vectors

- Suppose that there is a dependence from statement S1 on iteration i of a loop nest and statement S2 on iteration j, then the *dependence distance vector* d(i,j) is defined as a vector of length *n* such that $\boldsymbol{d}(\boldsymbol{i},\boldsymbol{j})_k = \boldsymbol{j}_k - \boldsymbol{i}_k$.

- A vector (d1, d2) is positive if (0,0) < (d1, d2), i.e., its first (leading) non-zero component is positive

Direction Vectors

- Suppose that there is a dependence from statement S1 on iteration i of a loop nest of *n* loops and statement S2 on iteration j, then the *dependence direction vector* is *D*(*i,j*) is defined as a vector of length *n* such that

$$D(i,j)_k = \begin{cases} - \; if \; D(i,j)_k < 0 \\ 0 \; if \; D(i,j)_k = 0 \\ + \; if \; D(i,j)_k > 0 \end{cases}$$

Distance and Direction Vectors

- Suppose that there is a dependence from statement S1 on iteration i of a loop nest of n loops and statement S2 on iteration j, then the *depe*... n such...

In any valid dependence, the leftmost non-"0" component of the direction vector must be "+"

$$D(i,j)_k = \begin{cases} - \; if \; D(i,j)_k < 0 \\ 0 \; if \; D(i,j)_k = 0 \\ + \; if \; D(i,j)_k > 0 \end{cases}$$

Distance and Direction Vector Example

```
    DO I = 1, N
      DO J = 1, M
        DO K = 1, L
S1        A(I+1,J,K-1) = A(I,J,K) + 10
        ENDDO
      ENDDO
    ENDDO
```

Distance and Direction Vector Example

```
    FOR I = 1, 5
      DO J = 1, 5
S1      A(I,J) = A(I,J-3) + A(I-2,J) + A(I-1,J+2) + A(I+1,J-1)
      ENDFOR
    ENDFOR
```

Reordering Transformations

- A reordering transformation does not add or remove statements from a loop nest
  - Only reorders the execution of the statements that are already in the loop

Do not add or remove statements → Do not add or remove any new dependences

Direction Vector Transformation

- Let *T* be a transformation is applied to a loop nest
  - Does not rearrange the statements in the body of the loop
- *T* is valid if, after it is applied, none of the direction vectors for dependences with source and sink in the nest has a leftmost non-“0” component that is “-”

Validity of Dependence-Based Transformations

- A transformation is said to be valid for the program to which it applies if it preserves all dependences in the program

Loop-Carried Dependences

- S1 can reference the common location on one iteration of a loop; on a subsequent iteration S2 can reference the same location

```
      DO I = 1, N
S1      A(I+1) = F(I)
S2      F(I+1) = A(I)
      ENDDO
```

i. S1 references location *M* on iteration i
ii. S2 references *M* on iteration j
iii. d(i,j) > 0 (that is, contains a "+" as leftmost non-"0" component)

Level of Loop-Carried Dependence

- The *level* of a loop-carried dependence is the index of the leftmost non-“0” of $D(i,j)$ for the dependence.

```
      DO I = 1, 10
        DO J = 1, 10
          DO K = 1, 10
S1          A(I,J,K+1) = A(I,J,K)
          ENDDO
        ENDDO
      ENDDO
```

Utility of Dependence Levels

- A reordering transformation preserves all level-*k* dependences if it
  1. preserves the iteration order of the level-*k* loop
  2. does not interchange any loop at level < *k* to a position inside the level-*k* loop and
  3. does not interchange any loop at level > *k* to a position outside the level-*k* loop.

```
    DO I = 1, 10
S1    A(I+1) = F(I)
S2    F(I+1) = A(I)
    ENDDO
```

```
    DO I = 1, 10
S2    F(I+1) = A(I)
S1    A(I+1) = F(I)
    ENDDO
```

Is this transformation valid?

```
  DO I = 1, 10
    DO J = 1, 10
      DO K = 1, 10
S       A(I+1,J+2,K+3) = A(I,J,K) + B
      ENDDO
    ENDDO
  ENDDO
```

```
  DO I = 1, 10
    DO K = 10, 1, -1
      DO J = 1, 10
S       A(I+1,J+2,K+3) = A(I,J,K) + B
      ENDDO
    ENDDO
  ENDDO
```

Loop-Independent Dependences

- S1 and S2 can both reference the common location on the same loop iteration, but with S1 preceding S2 during execution of the loop iteration.

i. S1 refers to memory location *M* on iteration i
ii. S2 refers to *M* on iteration j and i = j
iii. There is a control flow path from S1 to S2 within the iteration.

```
    DO I = 1, N
S1    A(I+1) = F(I)
S2    G(I+1) = A(I+1)
    ENDDO
```

```
    DO I = 1, 9
S1    A(I) =
S2    ... = A(10-I)
    ENDDO
```

Is this transformation valid?

```
    DO I = 1, N
S1:   A(I) = B(I) + C
S2:   D(I) = A(I) + E
    ENDDO
```

```
    D(1) = A(1) + E
    DO I = 2, N
S1:   A(I-1) = B(I-1) + C
S2:   D(I) = A(I) + E
    ENDDO
    A(N) = B(N) + C
```

Dependence Testing

- Dependence question
  - Can 4*I be equal to 2*I+1 for I in [1, N] ?

```
DO I=1, N
  A(4*I) = …
  … = A(2*I+1)
ENDDO
```

Given (i) two subscript functions f and g, and (ii) lower and upper loop bounds L and U respectively, does $f(i_1) = g(i_2)$ have a solution such that $L \leq \ i_1, i_2 \leq U$?

Multiple Loop Nests

```
DO i=1,n
  DO j=1,m
  X(a₁*i + b₁*j + c₁) = …
  … = X(a₂*i + b₂*j + c₂)
  ENDDO
ENDDO
```

- Dependence test

$$a_1 * i_1 + b_1 * j_1 + c_1 = a_2 * i_2 + b_2 * j_2 + c_2$$
$$1 \le i_1, i_2 \le n$$
$$1 \le j_1, j_2 \le m$$

Multiple Loop Indices, Multi-Dimensional Array

```
DO i=1,n
  DO j=1,m
    X(a1*i1 + b1*j1 + c1, d1*i1 + e1*j1 + f1) = …
    … = X(a2*i2 + b2*j2 + c2, d2*i2 + e2*j2 + f2)
  ENDDO
ENDDO
```

- Dependence test

$$
\begin{aligned}
&a_1 i_1 + b_1 j_1 + c_1 = a_2 i_2 + b_2 j_2 + c_2 \\
&d_1 i_1 + e_1 j_1 + f_1 = d_2 i_2 + e_2 j_2 + f_2 \\
&1 \le i_1, i_2 \le n \\
&1 \le j_1, j_2 \le m
\end{aligned}
$$

Multiple Loop Indices, Multi-Dimensional Array

```
DO i=1,n
  DO j=1,m
    X(a1*i1 + b1*j1 + c1, d1*i1 + e1*j1 + f1) = …
    … = X(a2*i2 + b2*j2 + c2, d2*i2 + e2*j2 + f2)
  ENDDO
ENDDO
```

- Dependence test

$$
\begin{aligned}
&a_1 i_1 + b_1 j_1 + c_1 = \ a_2 i_2 + b_2 j_2 + c_2 \\
&d_1 i_1 + e_1 j_1 + f_1 = \ d_2 i_2 + e_2 j_2 + f_2 \\
&1 \le i_1, i_2 \le n \\
&1 \le j_1, j_2 \le m
\end{aligned}
$$

complex

Data Dependence Testing

- Variables in loop indices are integers → Diophantine equations

- Equation $a_1 * i_1 - b_1 * i_2 = c$ has a solution if and only if gcd(a,b) (evenly) divides c
- The Diophantine equation $a_1 i_1 + a_2 i_2 + \cdots + a_n i_n = c$ has a solution iff $\gcd(a_1, a_2, \ldots, a_n)$ evenly divides c

- If there is a solution, we can test if it lies within the loop bounds. If not, then there is no dependence

Dependence Testing Problem

Equivalent to an integer linear programming problem with 2n variables and n+d constraints

An algorithm that finds two iteration vectors that satisfies these constraints is called a dependence tester

This is an NP-complete problem, and so in practice the algorithms must be conservative

Lamport Test

```
    for i = 1 to n
      for j = 1 to n
S1      a[i,j] = a[i-1,j+1]
```

```
    for i = 1 to n
      for j = 1 to n
S1      a[i,2*j] = a[i-1,2*j+1]
```

Other Dependence Tests

- GCD test is simple but not accurate
  - It can tell us that there is no solution
- Other tests
  - Banerjee-Wolfe test: widely used test
  - Power Test: improvement over Banerjee test
  - Omega test: “precise” test, most accurate for linear subscripts
  - Range test: handles non-linear and symbolic subscripts
  - many variants of these tests

```
     for i = 1 to 10
S1     a[i] = b[i] + c[i]
S2     d[i] = a[i-100];
```

Banerjee-Wolfe Test

- If the total subscript range accessed by *ref1* does not overlap with the range accessed by *ref2*, then *ref1* and *ref2* are independent

```
DO j=1,100
  a(j) = …
  … = a(j+200)
ENDDO
```

- Weakness
  - Ranges accessed
    - [1:100], [6:105]
  - No dependence?

```
DO j=1,100
  a(j) = …
  … = a(j+5)
ENDDO
```

Dependence Testing is Hard

- Unknown loop bounds can lead to false dependences
- Need to be conservative about aliasing
- Triangular loops adds new constraints

Why is Dependence Analysis Important?

- Dependence information can be used to drive other important loop transformations
  - For example, loop parallelization, loop interchange, loop fusion

- We will see many examples soon

References

- R. Bryant and D. O'Hallaron – Computer Systems: A Programmer's Perspective.
- R. Allen and K. Kennedy – Optimizing Compilers for Modern Architectures.
- Michelle Strout – CS 553: Compiler Construction, Fall 2007.