Truthful and Fair Lateral Transshipment in Multi-Retailer Systems

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Abstract

We consider a multi-retailer system where the sellers are connected with each other over a network and the transactions with the consumers happen on a platform. Since the demands and supplies to the sellers (e.g., the retailers on the platform) are stochastic in nature, supplies can be either in excess or in deficit. Lateral transshipment of such products is beneficial for the retailers since otherwise, the excess supply leads to wastage and the deficit in bad reputation. Only the sellers know their excess (which can be salvaged at a price or transshipped to another seller) and the deficit (which can be directly procured from a supplier or transshipped from another seller), both of which have information that is private. We propose a model that allows the lateral transshipment at a price and design mechanisms such that the sellers are incentivized to *voluntarily participate* and be *truthful*. Experimenting on different types of network topologies, we find that the sellers who are at more *central* locations in the network get an unfair advantage in the classical mechanism that aims for *economic efficiency*. We, therefore, propose a modified mechanism with tunable parameters which can ensure that the mechanism is more *equitable* for non-central retailers. Our synthetic data experiments show that such mechanisms do not compromise too much on efficiency, and also reduce *budget imbalance*.

Keywords: lateral transshipment, multi-retailer systems, mechanism design, fairness, individual rationality.

1 Introduction

Modern markets work as an integration of online and in-store inventories. They provide a platform for the interaction between distributed retail partners or service providers and the consumers. The consumers can shop, compare, and purchase the products on these platforms. Each retailer has multiple stores and warehouses at different geographical locations to service in a wide area. Due to the uncertainty in demand, these stores order and keep the supplies in their inventory in advance. If the estimated demand is less or more than the realized demand, the stores face stock-out or excess supply. For perishable goods like dairy products, baked goods, fruits, flowers etc., the loss due to excess supply is enormous due to its limited shelf life. Similarly, in the case of urgently needed products, such as spare parts, the firms keep them available in inventory on a high inventory handling cost to reduce the lengthy downtime. However, they may not use that product during their lifetime. Excess inventory results in increased inventory holding costs, and shortage of supply results in poor service level and lost revenue. Collaboration among the retailers can improve efficiency, alleviate the loss and reducing the costs in inventory management in supply chains.

For multiple stores of a parent firm at different geographical locations, the objective is to maximize the collective revenue and service level. In such cases, lateral transshipment is one of the most efficient ways to reduce the loss and is practised by various firms. Perishable foods are vulnerable to loss. A significant food loss in transportation and distribution networks is due to the rejection of perishable food shipments, which are wasted if another buyer can not be found quickly. A study about the Swedish bread industry by Ghosh and Eriksson (2019) says that suppliers faced 30% returns on total volume produced in the period 2011–2015, and had to bear the cost of bread rejections, collection, and disposal. For perishable items, Nakandala et al. (2017) proposed a lateral transshipment model which embodies spoilage costs in the total inventory costs. Ekren et al. (2021) design sustainable food supply chain networks for e-commerce food companies by applying lateral inventory sharing policies between organizations. Wang et al. (2021) consider uncertainties of demand and distribution of relief supplies during disaster using lateral transshipment. For clinical trial supply chains, Zheng et al. (2021) propose an optimal inventory allocation using replenishment, reverse replenishment, and lateral transshipment among the distribution centers and clinics. Another set of applications of lateral transshipment is in case of industries that require to stock ready–for–use spare parts of the advanced technical system to be able to quickly respond to a breakdown of a system to avoid the lengthy and expensive downtime costs (van Wijk et al., 2019). There exist a variety of such contexts, including chemical plants (Vereecke and Verstraeten, 1994), airline industry (Kilpi and Vepsäläinen, 2004), power-generating plants (Kukreja et al., 2001) and many more.

In this paper, we explore the use of *lateral transshipment* between different parties of same echelon, e.g., retailers, wholesalers. We use the term transshipment to mean lateral transshipment.

Researchers have considered the problem of transshipment allocation and profit distribution between multiple retailers under a parent firm as a cooperative game and proposed the effective use of profit distribution methods such as the Shapley value, the nucleolus, and the τ-value from cooperative game theory (Kemahlıoğlu-Ziya and Bartholdi III, 2011; Lozano et al., 2013).

On the other hand, in multi-retailer systems, when the retailers are independent agents or are competitors and try to optimize their objectives, providing enough incentive to the parties for collaboration becomes challenging, which leads to inefficiency in the supply chain. Another challenge in inter-firm transshipment is the distribution of the profit generated by the transshipment among the retailers of different firms. The monetary amount paid by a retailer to the other retailer on the exchange of the transshipped product is known as the transshipment price. Rudi et al. (2001) examine the transshipment between two independently located sellers. Comparing multiple assumptions about the central and decentralized environment, Rudi et al. (2001) show that the decentralized system can be coordinated by an appropriately set predetermined per unit transshipment prices. Hezarkhani and Kubiak (2010) suggested using Nash bargaining solution to coordinate transshipment prices between two retailers, which depends on the retailers' inventory decisions. Shao et al. (2011) study a vertically and horizontally decentralized supply chain and examine the conflict of incentives when the transshipment price is set either by retailers or by the manufacturer. They show that if the manufacturer has control, it will set a high transshipment price to stimulate retailers' incentive to stock inventory. However, if the retailers have control, they may choose a low transshipment price to suppress inventory and limit the manufacturer's ability to extract profit. In that case, the manufacturer may be worse off from transshipment.

In a setting with more than two retailers, finding the efficient allocation for transshipment between non-cooperative players is challenging. Anupindi et al. (2001) gave a two-stage framework where in the first stage, the players are non-cooperative and decides the amount of supply to order individually. After the demand realization, the decision for the amount of transshipment is made in the next step. Assuming that each retailer's actual residual supply and demand is complete knowledge, Anupindi et al. (2001) provide a dual price allocation for transshipments and prove that the solution is in the *core*.[1] Following Anupindi et al. (2001), Granot and Sošić (2003) proposed a three-stage model, where each retailer has the opportunity to decide how much of her residual supply/demand she would like to share with others and strategically reports the residuals before the allocation is decided. Granot and Sošić (2003) prove that, for transshipment games, there are no allocation rules based on dual prices that can ensure complete sharing of the residuals. Yan and Zhao (2015) proposed a model and mechanism for coordination among the manufacturer and retailers. At first, each retailer decides whether to participate in the transshipment allocation in the future and pays a participation fee accordingly. After demand realization, the retailers have leftover or unmet demands.

[1] An allocation is said to be in *core* if it is efficient and provides coalitional rationality, which means no group of retailers can collude and get more benefit than that in the given allocation (Davis and Maschler, 1965).

They strategically choose and report the amount of the residual they want to share with other retailers. In the next step, the efficient allocation between the participants is done, and the net profit is given to both the parties instead of distributing it among both of them. The mechanism leads to only a grand coalition inducing complete residual sharing, with an appropriately set participation fee.

To make a moral decision concerning the distribution of residuals, one way is to consider the well-known Aristotle's principle of distributive justice (Moulin, 2003): "Equals should be treated equally, and unequals unequally, in proportion to the relevant *similarities* and *differences*." Hornibrook et al. (2009) developed a behavioral theoretical approach to fill the gap between fairness, justice, and the supply chain relationships between the buyers and sellers to achieve a fair allocation of resources such as time, effort, and money. Fearne et al. (2012) provide a study to measure fairness in supply chain trading relationships and show the importance of understanding fairness in sustainable supply chains.

It is worth noting that there is a dearth of models in the extant literature that can simultaneously consider the efficiency and profit distribution goals in transshipment allocation between non-cooperative retailers. Some of the literature assumes that the retailers share their complete private information and then examine methods for a cooperative game to find transshipment allocation and price distribution between the retailers. The other kind of literature considers additional participation fees, which may result in a negative payoff for retailers and, therefore, retailer's lack of interest in participating in transshipment. We describe our problem setting and contributions in the following section.

Brief problem description and contributions

This paper aims to provide a realistic model for the modern online market. These markets provide a platform for the interaction between retailers and consumers. We consider a platform where the consumer selects a product and places an order from a retailer. Due to uncertainty in demands by the consumers, the retailers often face unsold stock and returns or more than expected demand from the consumers. The retailers are rational agents and connected on a transshipment network. The objective is to transship the products from over-stocked locations to under-stocked to minimize the resource wastage, total holding cost and increase social welfare. The challenge is that every retailer has private information consisting of multiple components called the types of a retailer, such as the amount of stock she has and the price at which she bought the products from outside market or manufacturer. The platform needs to know the actual types of every retailer to find an efficient allocation. This makes the setting a competitive game between the retailers where the retailers may strategically choose to lie and misreport their actual types to maximize their payoff. At the same time, the platform must ensure that every retailer will always get a non-negative benefit from participating in the transshipment process. The goal is to guarantee the voluntary participation of every retailer and also incentivize them to report true types to achieve collaborative welfare.

We consider the partition of the transactions on a platform in periods and propose a game-theoretic model. At the beginning of each period, the retailers individually estimate the demand and decide the amount of inventory to order. They privately place the order from outside sources. At the end of the period, the retailers may face excess or shortage of the products. We propose the `Weighted Value Transshipment` (`WVT`) mechanism, which is applied during a sub-period before the end of a period. The mechanism has a pre-defined and publicly known contract for allocation and profit distribution. The platform asks the retailers to announce their individual multi-dimensional types and decides the transshipment allocation and price using the mechanism.

The contribution of the paper are summarized in the following points:

- The proposed mechanism incentivizes the retailers to report their types truthfully. (Theorem 1)
- The mechanism ensures that the retailer can never get worse off by participating in the lateral transshipment hence guarantees their voluntary participation. (Theorem 2)
- We define a notion of fairness among the retailers. Experiments on a classical mechanism for truthful resource allocation, considering different network topologies as the transshipment network, show that the classical mechanism leads to unfair advantages to more central retailers in the transshipment network.

(Figure 1).

- The proposed mechanism has tunable parameters that can ensure a more equitable outcome for smaller or non-central retailers as shown in the experiments (Figure 2b)
- Experiments on synthetic data show that the mechanism does not compromise too much on efficiency (Figure 2a) and reduces the budget imbalance (Figure 2c).

2 Model Descriptions and Assumptions

Define $\mathcal{R} = \{1, 2, \cdots, n\}$ to be the set of retailers of a product[2] available on the platform. The minimum trade volume is referred to as *one unit*. Assume that the retailers interact with each other once in every period. The retailers are myopic and want to maximize their payoff in each period. At the beginning of the period, each retailer independently estimates the demand that she will receive in that period, checks the stock she has, and decides the quantity of product to buy from the manufacturers. Retailer $i \in \mathcal{R}$ orders Q_i units from a manufacturer at per unit cost b_i. As the period begins, demands are realized for all the retailers. Each retailer $i \in \mathcal{R}$ receives the demand D_i on the platform and sells the product at per unit selling price p_i to the consumers. After satisfying demand with her inventory, each retailer decides how much of her excess supply or deficit (excess demand) she wants to share with others and reports her residuals to the platform. The platform computes the allocation of the residuals by transshipment among retailers. At the end of the period, each retailer i faces a penalty cost ρ_i for per unit of unmet demand. The unsold product in i's inventory has a salvage value s_i.

The order quantity Q_i, buying cost b_i, and the salvage value s_i are the private information of the retailer i and are represented as a tuple $\mathcal{Z}_i = (Q_i, b_i, s_i)$, which we call the *type* of retailer i. Let $\mathcal{Z} = [\mathcal{Z}_i]_{i \in \mathcal{R}}$ denote the vector of the agents' types. The set of all type vectors is denoted by Θ. The tuple $\mathcal{Z}_i$ is unknown to the platform and the other retailers in $\mathcal{R}$. As the interaction with the customers happens on the platform, the platform knows the realized demands D_is and selling prices p_is. We assume that the platform also knows the penalty costs ρ_i for all $i \in \mathcal{R}$, and the locations of the retailers on the transshipment network, hence the per unit transportation costs between them. We represent the per unit transportation cost between retailers i and k as τ_{ik}. Let $\mathbb{A}$ be the set of all possible transshipment allocations for a product. The transshipment allocation, $\mathcal{A} \in \mathbb{A}$ can be represented as a matrix $[a_{ik}]$ for $i, k \in \mathcal{R}$, s.t., $a_{ik} \in \mathbb{R}_{\geqslant 0}$ is the quantity of products to be transshipped from retailer i to k. The retailer k earns p_k per unit from selling the products that are transshipped from i to her and satisfying the previously unmet demand. The platform computes the share of transshipment profit, $\mathfrak{p}_{ik}$ paid by k to i, for each unit of transshipment from retailer i to k. We assume that the receiving retailer k pays τ_{ik} per unit as the transportation cost. Each retailer $i \in \mathcal{R}$ has a value v_i for the transshipment allocation, representing the revenue she gets over a period after the transshipment happens.[3] The value v_i is the total earning through transshipment from and to i and the total salvage value i gets from unsold inventory minus the total penalty cost for the unmet demand. Mathematically,

$$\begin{aligned} v_i(\mathcal{A}, \mathfrak{p}, \mathcal{Z}_i) = &\sum_{k \in \mathcal{R} \setminus \{i\}} a_{ik}(\mathfrak{p}_{ik}) + \sum_{l \in \mathcal{R} \setminus \{i\}} a_{li}(p_i - \mathfrak{p}_{li} - \tau_{li}) \\ &+ \Big((Q_i - D_i)^+ - \sum_{k \in \mathcal{R} \setminus \{i\}} a_{ik}\Big) s_i - \Big((D_i - Q_i)^+ - \sum_{l \in \mathcal{R} \setminus \{i\}} a_{li}\Big) \rho_i \end{aligned} \tag{1}$$

The total revenue i gets is the sum of the direct revenue (before transshipment) and her valuations for a given allocation and transshipment prices $(\mathcal{A}, \mathfrak{p})$, given by

$$\text{Revenue}_i(\mathcal{A}, \mathfrak{p}, \mathcal{Z}_i) = (min\{D_i, Q_i\}\ p_i - b_i Q_i) + v_i(\mathcal{A}, \mathfrak{p}, \mathcal{Z}_i) \tag{2}$$

[2]Multiple products are handled separately in this model.

[3]The valuation for a retailer for a product is independent of that for another product. The total valuation can be calculated as the sum of the values she gets for each product.

In the settings where the agents' valuations are private and the mechanism does not have any additional structures (e.g., payments in our context), only dictatorial mechanisms are truthful (Gibbard, 1973; Satterthwaite, 1975). This negative result holds irrespective of whether agents' preferences are *ordinal* (representable as an order relation over the outcomes) or *cardinal* (agents have a real number to represent the intensity of the preference). Note that in our setup, the agents' preferences are cardinal. A complementary analysis by Roberts (1979, Thm 7.2) shows that a dictatorship result reappears under certain mild conditions in a *quasi-linear* setting (which is our current setting) unless transfers (of utility) are allowed. Therefore, use of transfers in some form is inevitable to ensure truthfulness of the agents. In this paper, the mechanism decides the allocation $\mathcal{A}$, the transshipment price $\mathfrak{p}$, which directly affects the valuations of retailers; and determines payments or transfers $\mathcal{P} = (\mathcal{P}_i, i \in \mathcal{R})$ for each of the retailers.

We assume that every retailer wants to maximize her valuation and also wants to pay less. The net payoff or utility[4] of a retailer is assumed to follow a standard *quasi-linear form* (Shoham and Leyton-Brown, 2008):

$$u_i((\mathcal{A}, \mathfrak{p}, \mathcal{P}), \mathcal{Z}_i) = v_i(\mathcal{A}, \mathfrak{p}, \mathcal{Z}_i) - \mathcal{P}_i. \tag{3}$$

As the utility function depends on $\mathcal{Z}_i$, which is the private information of the retailer i, the platform needs the retailers to report the $\mathcal{Z}_i$s to the mechanism designer that decides the allocation. This leaves an opportunity for a retailer to misreport her private information and get a better allocation. A mechanism needs to carefully design the allocations and payments on the face of such strategic behavior of the retailers. We use the notation $\mathcal{X}$ to denote the pair $(\mathcal{A}, \mathfrak{p})$ and the set of all possible $\mathcal{X}$ as $\mathbb{X}$. To distinguish, we denote $\hat{\mathcal{Z}}_i$ as the announced information and $\mathcal{Z}_i$ as the true information of i. Therefore, a mechanism in this setting is defined as a function formally defined as follows.

Definition **1** (Transshipment Function). *A* Transshipment Function (TF) *is a mapping $f : \Theta \to \mathbb{X} \times \mathbb{R}^m$ that maps the reported type vector to an allocation, transshipment price and payment for each retailer. Hence, $f(\mathcal{Z}) = (\mathcal{X}(\mathcal{Z}), \mathcal{P}(\mathcal{Z}))$, where $\mathcal{X}$ is the function which computes the allocation and the transshipment price, and $\mathcal{P}$ is the payment function.*[5]

The TF defines two payments as its output: the transshipment price indicates the price at which the transaction between the source and destination retailers happen, while the payment indicates the side-payment to satisfy other desirable properties, e.g, truthfulness, individual rationality (defined in the next section). We will use $v_i(\mathcal{X}(\mathcal{Z}))$ or $v_i(\mathcal{X})$ as a shorthand for $v_i(\mathcal{X}(\mathcal{Z}), \mathcal{Z}_i)$ when the arguments are obvious from the context. In the next section, we formally define the desirable properties of a transshipment mechanism.

2.1 Desirable Properties

The following property ensures that every retailer i is incentivized to reveal her private information $\mathcal{Z}_i$, truthfully.

Definition **2** (Dominant Strategy Truthfulness). *A mechanism $f = (\mathcal{X}(\cdot), \mathcal{P}(\cdot))$ is* truthful in dominant strategies *if for every $\mathcal{Z}_i, \mathcal{Z}'_i \in \mathbb{Z}^m_{\geqslant 0} \times \mathbb{R}^m_{\geqslant 0} \times \mathbb{R}^m_{\geqslant 0}, i \in \mathcal{R}$*

$$v_i(\mathcal{X}(\mathcal{Z}_i, \mathcal{Z}_{-i})) - \mathcal{P}_i(\mathcal{Z}_i, \mathcal{Z}_{-i}) \geqslant v_i(\mathcal{X}(\mathcal{Z}'_i, \mathcal{Z}_{-i})) - \mathcal{P}_i(\mathcal{Z}'_i, \mathcal{Z}_{-i}). \quad \forall \mathcal{Z}_{-i}$$

The inequality above implies that if the true information of agent i is $\mathcal{Z}_i$, the allocation and payment resulting from reporting it 'truthfully' maximizes her payoff *irrespective of the reports of the other agents.*

The following property ensures that it is always weakly beneficial for every rational retailer to participate in such a mechanism.

[4]We will only consider the valuation component of the revenue in the utility of the agent since the direct revenue ($min\{D_i, Q_i\}\ p_i - b_i Q_i$) is insensitive to the mechanism, which decides the allocation and payments.

[5]We overload the notation $\mathcal{X}$ and $\mathcal{P}$ to denote both functions and values of those functions, since their use will be clear from the context.

Definition 3 (Individual Rationality). *A mechanism* $f = (\mathcal{X}(\cdot), \mathcal{P}(\cdot))$ *is* individually rational *if for every* $\mathcal{Z}_i$*, and* $i \in \mathcal{R}$

$$v_i(\mathcal{X}(\mathcal{Z}_i, \mathcal{Z}_{-i})) - \mathcal{P}_i(\mathcal{Z}_i, \mathcal{Z}_{-i}) \geqslant 0. \quad \forall \mathcal{Z}_{-i}$$

In the next section, we present our mechanism.

3 The Proposed Mechanism

According to Rawl's theory of distributive justice (Rawls, 1971),"Social and economic inequalities are permissible if they confer the greatest benefit to the least-advantaged members of society." To provide equal opportunity to every retailer in transshipment by removing the bias of their network position, and to achieve a better payoff-fairness, we assign a weight w_i to each of retailer $i \in \mathcal{R}$. The weights are dependent on their position in the network, e.g., a function of some centrality measure of i.

Allocation function: The allocation function is a weighted utilitarian function, where the objective is to maximise the weighted sum of valuations given by the following optimization problem (OP).

$$\begin{aligned}
\underset{\mathcal{X}=(\mathcal{A},\mathfrak{p})\in\mathbb{X}}{\text{argmax}} \quad & \sum_{i,k\in\mathcal{R}} w_i \; v_i(\mathcal{X}, \mathcal{Z}_i) \\
\text{s.t.} \quad & \sum_{k\in\mathcal{R}\setminus\{i\}} a_{ik} \leqslant (Q_i - D_i)^+ \quad \forall i \in \mathcal{R} \\
& \sum_{l\in\mathcal{R}\setminus\{i\}} a_{li} \leqslant (D_i - Q_i)^+ \quad \forall i \in \mathcal{R} \\
& \mathfrak{p}_{ik} \leqslant p_k + \rho_k - \tau_{ik} \quad \forall i, k \in \mathcal{R} \\
& \mathfrak{p}_{ik} \geqslant s_i \quad \forall i, k \in \mathcal{R} \\
& a_{ik} \geqslant 0, \quad \forall i, k \in \mathcal{R}
\end{aligned} \tag{4}$$

The first set of constraints in (4) ensures that the total transshipment from every retailer i to others is not more than the excess supply $(Q_i - D_i)^+$. Similarly, the second set of constraints ensures that the total transshipment to every retailer i from other retailers is not more than the unmet demand $(D_i - Q_i)^+$. The third and fourth set of constraints bound the transshipment price $\mathfrak{p}_{ik}$ to make sure that it is beneficial to every i and k. For every unit of the transshipment from i to k, the retailer k does not face the unmet demand and hence is not charged with the penalty cost ρ_k, which would have been charged in the absence of transshipment. Retailer k also earns p_k from the sale of the transshipped product, but pays the transportation cost τ_{ik}. Hence the retailer k earns total profit of $p_k + \rho_k - \tau_{ik}$ from per unit a_{ik} and the third set of constraints ensures that $\mathfrak{p}_{ik}$ is not more than the profit earned by k if the transshipment happens; otherwise, it is better for her not to buy this unit of transshipment. The fourth set of constraints ensures that every retailer i gets the price $\mathfrak{p}_{ik}$ from the transshipment which is at least as much as she earns if the transshipment did not happen. As the retailer i gets per unit salvage value s_i in the absence of transshipment, the lower bound for $\mathfrak{p}_{ik}$ is s_i. Note that OP (4) is solved by the mechanism designer who can only access the reported types $\hat{\mathcal{Z}}$. Denote the optimal solution of OP (4) by $\mathcal{X}^*(\hat{\mathcal{Z}})$. Also, denote the solution of a similar optimization problem when retailer i does not participate in the transshipment by $\mathcal{X}^*_{-i}(\hat{\mathcal{Z}}_{-i})$.

Payment function: For every agent i, the payment is the marginal contribution of i in social welfare for other retailer, and is given by

$$\mathcal{P}_i := \begin{cases} \frac{1}{w_i}\left(\sum\limits_{\ell\in\mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}^*_{-i}) - \sum\limits_{\ell\in\mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}^*) \right) & w_i > 0 \\ 0 & w_i = 0 \end{cases} \tag{5}$$

Algorithm 1 Weighted Value Transshipment(WVT) Mechanism in every period

The retailers independently order quantities, Q from the manufacturer. The demand realization on the platform results in residual supply/demand of the products.

1: Every retailer $i \in \mathcal{R}$ reports her type $\hat{\mathcal{Z}}_i$ to the platform, where $\mathcal{Z}_i = (Q_i, b_i, s_i)$.
2: Using the realized demand D, reported information $\hat{\mathcal{Z}}$, and weights w, the platform computes the TF $f(\hat{\mathcal{Z}}) = (\mathcal{X}^*(\hat{\mathcal{Z}}), \mathcal{P}^*(\hat{\mathcal{Z}}))$ where $\mathcal{X}^*$ and $\mathcal{P}^*$ are given by Equation (4) and (5) respectively.
3: **Output:** $\mathcal{X}^*(\hat{\mathcal{Z}})$ and the payment vector $\mathcal{P}^*$.

The mechanism is succinctly presented in Algorithm 1.

The allocation and payment decisions given by Equations (4) and (5) resemble the affine maximizer rule (Roberts, 1979). However, there are two significant observations that makes the proposed mechanism different in the current setup. First, the allocation decision here considers not only the volume of the item transshipped but also the price at which the transshipment occurs - each of which has its own constraints to be satisfied. Secondly, the types of the agents in the classic affine maximizer rule are single dimensional, i.e., the value of that agent. In our setting, the type of the agent i has three components: ordered quantity Q_i, purchase price b_i, and salvage price s_i. Hence, to show that properties truthfulness and individual rationality holds in such a *multidimensional* setting is a non-trivial exercise.

From the first two constraints in OP (4), the mechanism ensures that no retailer is asked to transship more than the residuals she reports to the platform. We assume that the platform can view the information of sale of product and the sale price for each of the retailers as these transactions are performed on the platform. So, we assume that the mechanism can be implemented as a contract between the platform and the retailers, which they can not break. This implies:

- The retailers can not refuse to accept the transshipment allocated by the mechanism. For example, if a retailer i is asked to buy a product from l, she has to buy it, and if she is asked to sell the product to k, she has to sell that to k. In case the retailer i had initially misreported her type information and does not have the product in her inventory to sell to k, then the retailer will have to buy the product from outside market (assumed to be at a much higher price) to fulfil the commitment.
- At the end of the transshipment, the platform can verify if the transshipped units of the products are sold by the receiving retailer or not. If not, then it is assumed that she has violated the contract.

In the next section, we present the theoretical guarantees of the mechanism.

4 Theoretical Guarantees

We first show the truthfulness of the WVT mechanism. We prove this in a few steps. First, we observe that the valuation function of every retailer i is independent of the per unit price b_i at which i buys the product from the manufacturer. The constraints in OP (4) are independent on b_is. Hence, the following lemma is immediate.

LEMMA **1.** *No retailer $i \in \mathcal{R}$ can get a better utility by misreporting the purchasing cost b_i.*

Our following result proves that the dependency of the optimal $\mathfrak{p}$ is restricted to a few parameters.

LEMMA **2.** *The optimal transshipment price $\mathfrak{p}^*_{ik}$ computed by OP (4) between any pair of retailers $i, k \in \mathcal{R}$, depends only on p_k, τ_{ik}, ρ_k and s_i.*

Proof. From the third and fourth set of constraints in OP (4), for per unit of transshipment from retailer i to k, the transshipment price $\mathfrak{p}_{ik} \in (s_i, p_k + \rho_k - \tau_{ik})$. We claim that the optimal transshipment price $\mathfrak{p}^*$

has the following property

$$\mathfrak{p}^*_{ik} = \begin{cases} p_k + \rho_k - \tau_{ik}, & \text{if } w_k < w_i \\ s_i & \text{otherwise} \end{cases} \tag{6}$$

The correctness of the Equation (6) can be proved case-wise as follows:

Case 1: $w_k < w_i$
Suppose for contradiction, the optimal price is $\mathfrak{p}^*_{ik} = (p_k + \rho_k - \tau_{ik}) - \epsilon$, where $\epsilon > 0$. The retailer i gets $a_{ik}(p_k + \rho_k - \tau_{ik}) - \epsilon)$ (first component of Equation (2)), and the retailer k gets $a_{ik}(\epsilon)$ (second component of Equation (2)) by the transshipment from i to k. As the weighted valuations are added in the objective function of OP (4), both the components are added after multiplying the weights of the retailers: $w_i a_{ik}(p_k + \rho_k - \tau_{ik} - \epsilon) + w_k a_{ik}\epsilon$.

An increase in $\mathfrak{p}^*_{ik}$ by ϵ is consistent with the constraints. If $w_i > w_k$ then by updating $\mathfrak{p}^*_{ik}$ to $\mathfrak{p}^*_{ik} + \epsilon$, the weighted sum of the components becomes $w_i a_{ik}(p_k + \rho_k - \tau_{ik}) + 0$. This leads to the contradiction that $\mathfrak{p}^*_{ik}$ is optimal, as with $\mathfrak{p}^*_{ik} + \epsilon$, the value of the objective function of OP (4) has $(w_i - w_k)(a_{ik}\epsilon)$ increase.

Case 2: $w_k \geqslant w_i$
Suppose for contradiction, the optimal transferred price is $\mathfrak{p}^*_{ik} = s_i + \epsilon$, where $\epsilon > 0$. Similar to the case 1, we claim that, with decrease in $\mathfrak{p}^*_{ik}$ by ϵ will bring $(w_k - w_i)(a_{ik}\epsilon)$ increase in the value of objective function of OP 4. This leads to the contradiction that $\mathfrak{p}^*$ is optimal.

The above two cases, and the Equation (6) includes only p_k, τ_{ik}, ρ_k and s_i. Therefore, the lemma is proved. □

Lemma 2 implies that a retailer i can not change the transshipment price $\mathfrak{p}$ by misreporting Q_i. Our next result shows that misreporting Q_i is never a dominant strategy for any retailer $i \in \mathcal{R}$.

Lemma **3.** *Retailer i can never get better utility by misreporting the quantity of supply, Q_i, $\forall i \in \mathcal{R}$.*

Proof. Suppose a retailer $i \in \mathcal{R}$ reports her inventory level and to be $Q_i^{'}$ while the true level is Q_i. Misreporting the inventory level as $Q_i^{'}$, results in the allocation $\mathcal{X}'$ and allocation when true Q_i is reported is $\mathcal{X}$. There are two possible cases:

Case 1: $Q_i > Q_i^{'}$

1. If $Q_i > Q_i^{'} \geqslant D_i$, as D_i is known to the platform, it is possible that according to the allocation in $\mathcal{X}'$, i has to tranship $(Q_i^{'} - D_i)$ units to some other retailer (say k). Due to misreporting, i misses the opportunity to sell the complete leftover $Q_i - D_i$. Retailer i will have no other option but to get the salvage value of the leftover of $(Q_i - Q_i^{'})$ units, which could have been sold and that would have resulted in increase in valuation by $(Q_i - Q_i^{'})(\mathfrak{p}^*_{ik} - s_i) \geqslant 0$, as $\mathfrak{p}^*_{ik} \geqslant s_i$, for every $k \in \mathcal{R} \setminus \{i\}$.

2. If $Q_i > D_i > Q_i^{'}$, then i hides the actual residual supply of $(Q_i - D_i)$ and reports the residual demand of $(D_i - Q_i^{'})$ units. By misreporting, i asked for the products she already has in excess. It is possible that according to the allocation in $\mathcal{X}'$, the retailer i has to buy $(D_i - Q_i^{'})$ units of a product from some retailer (say, l). The retailer i can not refuse to accept the transshippment and buys the product without actual demand. Hence i can not sell the transshipped excess stock but the only option is to get their salvage value. The platform can verify that the transshipped product is salvaged but not sold, and hence knows that i broke the contract.

   Alternatively, i leaves the opportunity to sell her excess stock of $(Q_i - D_i)$ in $\mathcal{X}'$, which could have been sold and that would have resulted in increase in valuation by $(Q_i - D_i)(\mathfrak{p}^*_{ik} - s_i) \geqslant 0$. The possible decrease in the valuations of i by misreporting is up to $(D_i - Q_i^{'})(\mathfrak{p}^*_{li} + \tau_{li} - s_i) + (Q_i - D_i)(\mathfrak{p}^*_{ik} - s_i) \geqslant 0$.

3. If $D_i \geqslant Q_i > Q_i^{'}$, the supply i asked to be transshipped to her is more than she actually needs. Therefore, i may has to buy $(Q_i - Q_i^{'})$ extra units of the product without actual demand. And, i has no other option but to get the salvage value for it. The platform can verify that the transshipped units are not sold and hence knows that i broke the contract. The possible decrease in the valuation is up to $(Q_i - Q_i^{'})(\mathfrak{p}_{li}^* + \tau_{li} - s_i) > 0$.

Case 2: $Q_i^{'} > Q_i$

1. If $D_i \geqslant Q_i^{'} > Q_i$, the supply i asked to be transshipped to her is less than she actually needs. The allocation $\mathcal{X}'$ can at most transship $(D_i - Q_i^{'})$ units of product (say, from l) to i. Therefore, i has to pay the penalty cost ρ_i for $(Q_i^{'} - Q_i)$ units of remaining unmet demand which results in decrease in the valuation up to $(Q_i^{'} - Q_i)(p_i - \tau_{li} + \rho_i - \mathfrak{p}_{li}^*) \geqslant 0$.
2. If $Q_i^{'} > D_i \geqslant Q_i$, i does not have the excess stock she asked to be transshipped from her, but has unmet demand. Therefore, i misses the opportunity to fulfill the unmet demand of $D_i - Q_i$ units by transshipment (say, from l) to her, which would have been possible if she reports true Q_i.

   Additionally, it is possible that the retailer i has to transship $(Q_i^{'} - D_i)$ to some retailer (say k). In that case, the retailer i has no other option but to buy $(Q_i^{'} - D_i)$ units from outside market (assumed on a higher price H), and then transship to k. The decrease in the valuation can be up to $(Q_i^{'} - D_i)(H - \mathfrak{p}_{ik}^*) + (D_i - Q_i)(p_i - \mathfrak{p}_{li}^* - \tau_{li} + \rho_i) > 0$.
3. If $Q_i^{'} > Q_i > D_i$, the excess stock i asked to be transshipped from her is more than the actual excess stock in her inventory. It is possible that according to the allocation in $\mathcal{X}'$, the retailer i has to transship $(Q_i^{'} - D_i)$ to some retailer (say k). The retailer i has to buy $(Q_i^{'} - Q_i)$ units from outside market on a high price H, and then transship to k. The decrease in the valuation can be up to $(Q_i^{'} - Q_i)(H - \mathfrak{p}_{ik}^*) > 0$.

The optimal strategy for every retailer i is to report true Q_i. □

From the above lemmas, retailer i can only possibly misreport her salvage value s_i. In the following lemma, we show that none of the retailers can gain by such a misreporting.

LEMMA **4.** *No retailer $i \in \mathcal{R}$ can get a better utility by misreporting the salvage value s_i, $\forall i \in \mathcal{R}$.*

This proof follows standard arguments and is provided in the appendix. Combining Lemmas 1 to 4, we get the following theorem.

THEOREM **1.** *The* `WVT` *mechanism dominant strategy truthful.*

The above theorem implies that irrespective of the reported private information of the other retailers, a given retailer's utility is maximized when she reports her information truthfully. Our next result proves that the retailers are incentivized to participate in the mechanism voluntarily (the proof is deferred to the appendix).

THEOREM **2.** *The* `WVT` *mechanism is individually rational.*

The following section considers the performance of `WVT` for certain metrics that are not captured theoretically.

5 Experimental Results

The theoretical results ensure the truthfulness and participation guarantees of `WVT`. However, certain other social welfare metrics, e.g., fairness of the utilities, surplus in the budget, and efficiency, have not been theoretically captured. To understand how the `WVT` mechanism impacts them, an experimental study is carried out in this section on synthetic datasets.

Our first experiment shows the need of choosing appropriate weights in `WVT`.

5.1 Network position effect on utility

When we discuss about the weights in the `WVT` mechanism, a natural question arises: "*why we need different weights?*" or "*why cannot we use a straightforward mechanism like the Vickrey-Clarke-Groves (VCG) mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973)?*". To answer this, we consider the average utility of the retailers partitioned w.r.t. their network centrality measure. Consider the special case when the weights for every retailer is unity, which reduces `WVT` to the `VCG` mechanism and provides an efficient outcome.

We consider an Erdős–Rényi graph with 10 retailers having the edge forming probability to be 0.7 to emulate the transshipment network. We consider the *closeness* (Bavelas, 1950) centrality as the measure of the retailers' positional impact on the network. The retailers in the network are partitioned in bins and the average value of the utilities of the retailers in each bin is computed.

For every retailer $i \in \mathcal{R}$, we chose the following parameters: per unit price at which the product bought by the retailer from the manufacturer $(b_i) = 15$, price at which the product is sold to the consumers $(p_i) = 30$, cost of transportation on every edge $(\tau_{ik}) = 15$, penalty cost for per unit unmet demand $(\rho_i) = 10$, per unit salvage value of unsold inventory $(s_i) = 10$. We generate the demand (D_i) and the initial inventory level (Q_i) from Normal distribution with mean $(\mu) = 500$ and standard deviation $(\sigma) = 50$, which shows the uncertainty in demand, and the estimated quantity of products to order from the manufacturer. We randomly generate 1200 Erdős–Rényi networks, and for each network, generate 200 instances of D_i and Q_i for every retailer i. Figure 1 shows the average utility plot w.r.t. the centers of the centrality bins. Notice that the plot is increasing in nature even though all the retailers have identical statistical and parametric properties (plots with other centrality measures are also similar). This experiment clearly shows the inequality introduced

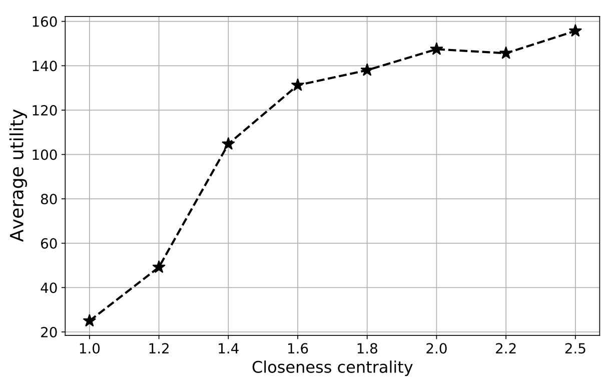


Figure 1: Utility vs centrality plot under `VCG`.

in the net payoff of the retailers due to their network positions and serves as the motivation for the design of the weights. The weights need to be decreasing in the network centrality so that the utilities earned by the retailers having identical statistical and parametric properties are more equalized. This implies that the *equal* retailers are treated more *equally*.

5.2 Fairness and efficiency

Following the definition of *egalitarian* allocation, where every agent gets the same welfare level (Pazner and Schmeidler, 1978; de Clippel, 2012), we define the *unfairness* ($\mathcal{U}$) of a transshipment mechanism f for an input instance $\mathcal{Z}$ as the variance of the utilities of the retailers. Mathematically,

$$\mathcal{U}(f, \mathcal{Z}) = \texttt{var}([u_i(f(\mathcal{Z}), \mathcal{Z}_i)]_{i \in \mathcal{R}}) \tag{7}$$

While it is clear from the discussions of the previous section that the weights need to decrease with centrality measures, it is not clear how the decreasing function should look like. In this section, we attempt to heuristically choose a function and its parameters that we find to reduce the unfairness to a certain extent. The problem of finding an optimal weight that minimizes the unfairness remains an open problem. The weight function we choose is

$$w_i = e^{-ac_i} + b, \tag{8}$$

where c_i is the centrality of retailer i in the network. For the experiments, we consider *three* widely used centrality measures: *closeness*, *betweenness* (Freeman, 1977), and *eigenvector* (Newman, 2008). The mechanisms we consider in the `WVT` class will use the weights corresponding to these centralities in Equation (8).

To capture the fairness introduced by a mechanism, we first define the *fairness factor* (FF) of a `WVT` mechanism for an input instance $\mathcal{Z}$ as follows.

$$\text{FF}(\texttt{WVT}, \mathcal{Z}) = 1 - \frac{\mathcal{U}(\texttt{WVT}, \mathcal{Z})}{\mathcal{U}(\texttt{VCG}, \mathcal{Z})} \tag{9}$$

Thus, a mechanism with a higher value of the FF is fairer.

Since the weights of the `WVT` mechanisms can be different, it may not always yield an efficient outcome like the `VCG`. The following metric captures the *inefficiency factor* (IF) of a `WVT` mechanism for an input instance $\mathcal{Z}$.

$$\text{IF}(\texttt{WVT}, \mathcal{Z}) = \frac{\sum_{i \in \mathcal{R}} v_i(\texttt{VCG}(\mathcal{Z}), \mathcal{Z}_i) - \sum_{i \in \mathcal{R}} v_i(\texttt{WVT}(\mathcal{Z}), \mathcal{Z}_i)}{\sum_{i \in \mathcal{R}} v_i(\texttt{VCG}(\mathcal{Z}), \mathcal{Z}_i)} \tag{10}$$

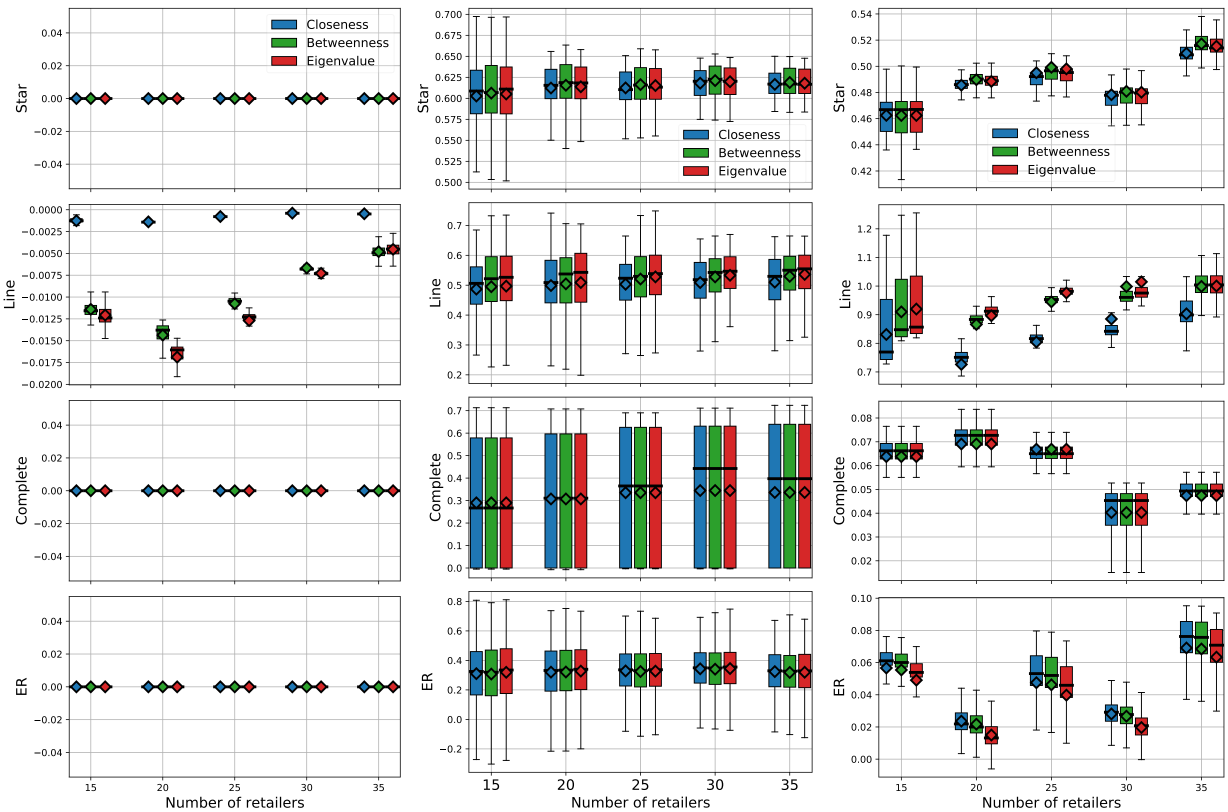
`VCG` provides an efficient outcome; therefore IF can never be greater than zero. A larger negative value will imply that the mechanism is more inefficient than the `VCG`. For brevity, we will omit the arguments of the above factors wherever they are clear from the context.

We compare the results given by the above two metrics for *three* `WVT` mechanisms, given by the weights corresponding to three centrality measures on *four* standard network structures, viz., star, line, complete, and Erdős–Rényi networks (with edge forming probability to be 0.5). Figures 2a and 2b show the IF and FF results respectively for an increasing number of retailers.

The parameters chosen are identical for every retailer i and are given by: $\rho_i = 10$, $b_i = 20$, $p_i = 50$, $s_i = 5$. The transportation cost for every edge (i, k) in the network is $\tau_{ik} = 10$. We generate D_i and Q_i from $\mathcal{N}(500, 50)$. The line network has the highest diameter in any connected graph. For a sufficiently large number of retailers on a line network with high edge costs of $\tau_{ik} = 10$ results in an insignificant quantity of transshipment. This is because every retailer has the least number of neighbors with whom the transshipment is beneficial. To reserve the possibility of sufficient transshipment for analysis in a large line network, we choose a low transportation cost per edge ($\tau_{ik} = 1$). The weights are computed via Equation (8) (with $a = 0.5, b = 1$) for the three chosen centrality measures as depicted in Figure 2.

The experiments are repeated 500 times for star, line, and complete networks, by generating those many random instances of D_i and Q_i for every retailer i. For Erdős–Rényi network, we repeat the experiments for 2500 times, by generating 50 demand-supply pairs for every retailer, and for each such instance, by generating 50 Erdős–Rényi random networks.

Figure 2 shows the box plots where the horizontal black lines inside the boxes denote the median and the diamond markers show the mean. From the results of the different number of retailers shown in Figure 2b, we find that the WVT reduces the unfairness (from VCG) by about 60% in star, 50% in line, and 30% in both complete and Erdős–Rényi networks in the case of closeness centrality. The results are similar for betweenness and eigenvector centralities as well. It is interesting to note that this does not come at a big sacrifice in efficiency. For the chosen parameters, only in line networks, WVT compromises up to 2% of the efficiency for eigenvector and betweenness centrality and no significant efficiency loss in case of closeness centrality. For star, complete and Erdős–Rényi networks, WVT makes no compromise in efficiency. From



(a) Inefficiency Factor (IF)

(b) Fairness Factor (FF)

(c) Fractional Budget Surplus (FBS)

Figure 2: Performance of WVT under different metrics.

these results, we conclude that it is possible to transship among the retailers reducing the inequality due to network positions in a truthful, self-participatory manner without a significant compromise in the efficiency.

Remarks on the IF plots (Figure 2a): It is interesting to note that the IF is very close to zero, which is an effect of the event that the social welfare, i.e., the sum of the valuations, was almost same in different graphs for most of the random instances. This is even though the allocations in WVT and VCG were not the same always. The different allocations by WVT and VCG ($\mathcal{A}^{\texttt{WVT}}$ and $\mathcal{A}^{\texttt{VCG}}$ respectively) do change the individual utilities of the retailers. However, in the experiment with the chosen parameters, we found that the total quantity of the transshipment, i.e., $\sum_{i,k\in\mathcal{R}} a_{ik}$ is almost same in $\mathcal{A}^{\texttt{WVT}}$ and $\mathcal{A}^{\texttt{VCG}}$. As all the individual parameters (except the transportation costs, which are identical over the edges) are identical for every retailer in both the allocations, the difference in the social welfare is only due to the difference in total transportation cost $\sum_{i,k\in\mathcal{R}} \tau_{ik} a_{ik}^{\texttt{WVT}}$ and $\sum_{i,k\in\mathcal{R}} \tau_{ik} a_{ik}^{\texttt{VCG}}$. Since these values are insignificant in comparison to the optimal social welfare ($\sum_{i\in\mathcal{R}} v_i(\texttt{VCG}(\mathcal{Z}), \mathcal{Z}_i)$), the IF looks arbitrarily close to zero in the figure. We could have chosen a larger value of τ_{ik}, which needs to be large enough to be comparable to the optimal social welfare. But such a large value of τ_{ik} reduces the quantity of transshipment significantly, making the need of the WVT mechanism insignificant. Hence, even if the allocation by WVT and VCG are very distinct, the change in the social welfare is insignificant.

5.3 Budget surplus

The monetary transfers in mechanisms serve as an instrument to ensure truthfulness and certain other properties. However, it is desirable that the mechanism designer do not earn a significant surplus of these payments or run into a large deficit to run the mechanism. Ideally, one would like to have the sum of all these payments to be zero (which means the money is only redistributed) and we call such mechanisms to be *budget balanced*. However, in mechanisms with monetary transfers, ensuring both efficiency and budget balance are not generically possible (Green and Laffont, 1979). In the WVT mechanism, there are two components of the monetary transfer: (a) the transshipment prices computed by the WVT, which are one-to-one transactions between the retailers, and hence, the transshipment prices are budget balanced ($\sum_{i,k \in \mathcal{R}} \mathfrak{p}_{ik} = 0$) by design, and (b) the side-payments, $\mathcal{P}_i$s, which exist to ensure certain desirable properties of the mechanism, e.g., truthfulness. However, in this setup, the positive surplus of $\sum_{i \in \mathcal{R}} \mathcal{P}_i$ has an advantage since it can be easily distributed to the customers on the platform (who are not the players in this mechanism) as gift coupons or monetary discounts and the mechanism can be budget balanced. However, if the surplus is negative, i.e., resulting in a deficit, we need a larger value of $\sum_{i \in \mathcal{R}} \mathcal{P}_i$ so that the deficit can be minimized, for any mechanism f and input instance $\mathcal{Z}$.

Therefore, a larger value of $\sum_{i \in \mathcal{R}} \mathcal{P}_i$ will be more preferred. We capture how much WVT increases the surplus over VCG using the *fractional budget surplus* (FBS) factor defined as follows.

$$\text{FBS}(\texttt{WVT}, \mathcal{Z}) = \frac{\sum_{i \in \mathcal{R}} \mathcal{P}_i^{\texttt{WVT}}(\mathcal{Z}) - \sum_{i \in \mathcal{R}} \mathcal{P}_i^{\texttt{VCG}}(\mathcal{Z})}{\sum_{i \in \mathcal{R}} v_i(\texttt{VCG}(\mathcal{Z}), \mathcal{Z}_i)} \tag{11}$$

An FBS factor of 0.05 therefore implies that the surplus under WVT increases by 5% of the optimal welfare than that of VCG under the same instance $\mathcal{Z}$. The optimal welfare in all experiments were always positive.

Figure 2c shows that for star and line networks the budget surplus increases by $45-52\%$ and $70-100\%$ respectively. For complete and Erdős–Rényi networks the increase is of $4-7\%$ and $2-7\%$ respectively. The results for the three centrality measures are similar.

6 Conclusions and future work

In this paper, we study the excess inventory or unmet demands faced in the supply chain by the retailers who interact with the consumers on a shopping platform. The retailers are connected by a geographical network and have multi-dimensional *private* information. The objective was to ensure truthful revelation of the private information with incentives to the retailers to participate in the mechanism. We provided a mechanism called WVT that satisfies both. In addition, it does not compromise on the efficiency too much, has a better fairness factor and budget surplus compared to the classic VCG mechanism.

There are multiple directions to explore in the future. For example, to study other aspects of fairness for lateral transshipment, such as reward or compensation, to make the allocation more equitable and maintain the desirable properties for a multi-agent system such as truthfulness and others. Another possible extension is to find group-strategyproof mechanisms. The payment method in our mechanism does not ensure group strategyproofness and may meet coalitional deviations if the retailers can communicate outside the platform and trust each other.

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Appendix

Proof of Lemma 4

Proof. Let us assume for the contradiction that, there exist an agent i for having true private information as, $\mathcal{Z}_i = (Q_i, b_i, s_i)$, but misreports it as $\mathcal{Z}_i^{'} = (Q_i, b_i, s_i^{'})$, and gets better utility[6]. Suppose $\mathcal{X}(\mathcal{Z'}_i, \mathcal{Z}_{-i}) = \mathcal{X}'$ and $\mathcal{X}(\mathcal{Z}_i, \mathcal{Z}_{-i}) = \mathcal{X}^*$. The utility of i for $\mathcal{X}'$ is:

$$
\begin{aligned}
u_i(\mathcal{X}^{'}, \mathcal{Z}_i) &= v_i(\mathcal{X}', \mathcal{Z}_i) - \mathcal{P}_i(\mathcal{Z}_i', \mathcal{Z}_{-i}) \\
&= v_i(\mathcal{X}', \mathcal{Z}_i) - \frac{1}{w_i}\left(\sum_{\ell \in \mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}_{-i}^*, \mathcal{Z}_\ell) - \sum_{\ell \in \mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}^{'}, \mathcal{Z}_\ell)\right) \\
&= \frac{1}{w_i}\left(\sum_{\ell \in \mathcal{R}} w_\ell \; v_\ell(\mathcal{X}^{'}, \mathcal{Z}_\ell) - \sum_{\ell \in \mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}_{-i}^*, \mathcal{Z}_\ell)\right)
\end{aligned}
$$

Similarly, the utility of i for $\mathcal{X}^*$ is:

$$
= \frac{1}{w_i}\left(\sum_{\ell \in \mathcal{R}} w_\ell \; v_\ell(\mathcal{X}^*, \mathcal{Z}_\ell) - \sum_{\ell \in \mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}_{-i}^*, \mathcal{Z}_\ell)\right)
$$

If i gets better utility by misreporting her private information as $\mathcal{Z}_i'$, then

$$
\sum_{\ell \in \mathcal{R}} w_\ell \; v_\ell(\mathcal{X}^{'}, \mathcal{Z}_\ell) > \sum_{\ell \in \mathcal{R}} w_\ell \; v_\ell(\mathcal{X}^*, \mathcal{Z}_\ell)
$$

The above inequality leads to the contradiction that $\mathcal{X}^*$ is optimal for the true private information. Therefore, the mechanism is dominant strategy truthful in every period and no retailer can get better utility by misreporting the salvage value. □

Proof of Theorem 2

Proof. Consider agent i. The utility of i under `WVT` is $v_i(\mathcal{X}) - \mathcal{P}_i$

$$
= \frac{1}{w_i}\left(\sum_{\ell \in \mathcal{R}} w_\ell \; v_\ell(\mathcal{X}^*, \mathcal{Z}_\ell) - \sum_{\ell \in \mathcal{R}\setminus\{i\}} w_\ell \; v_\ell(\mathcal{X}_{-i}^*, \mathcal{Z}_\ell)\right) \geqslant 0
$$

Note that the different term in the parentheses is always non-negative since $\mathcal{X}^*$ is the optimal allocation for all allocations. In particular, $\mathcal{X}_{-i}^*$ is also a feasible allocation when agent i is present. Hence the inequality follows. □

[6] In this part of the section, we do not consider the direct revenue received by individual retailers before the transshipment, in the utility; as that has no affect in the decisions made by the mechanism.