Self Organisation in Random Geometric Graph models of Wireless Sensor Networks

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ME Final Presentation

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Outline of Talk

1. Theory
   - Review of Geometric Graph (GG)
   - Assumptions in Literature
   - Motivation for Random GG
   - Paradigms of HD-ED Relationship in RGG
2. Simulations illustrating Point-Node Theorem
3. Application in Localisation
   - Theorem Used
   - Algorithm: Hop Count-derived Distance-based Localisation (HCDL)
4. Performance Comparison
5. Conclusion and Future Work

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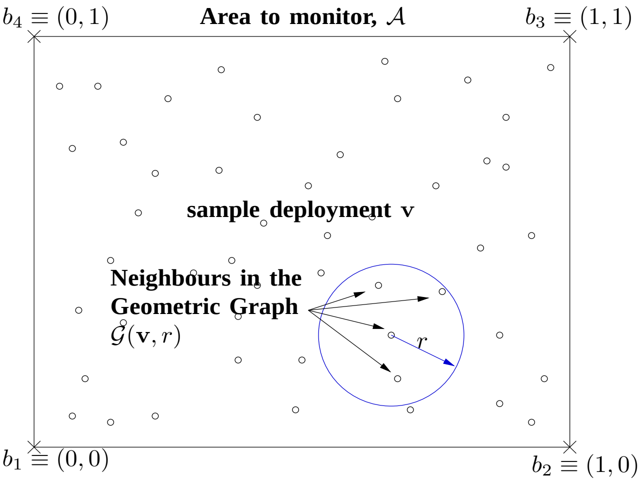
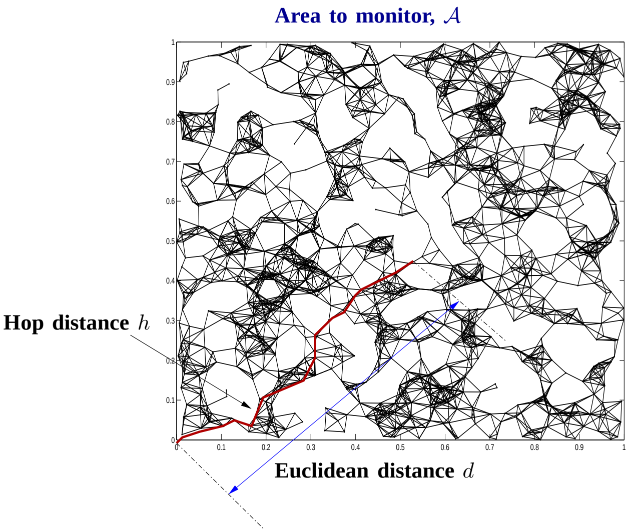
Geometric Graph

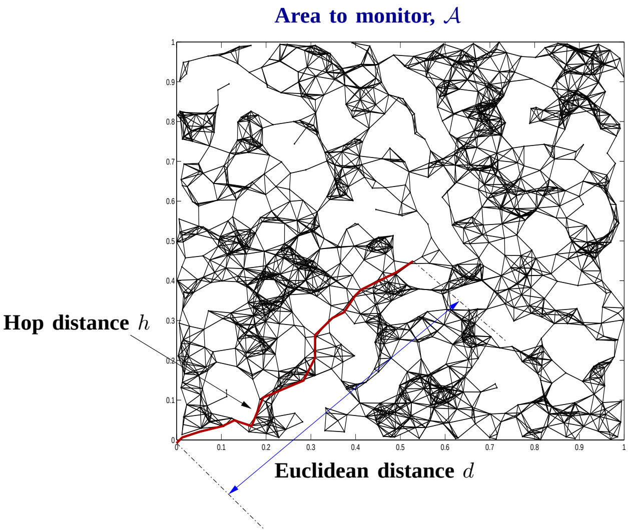


Node locations can be arbitrary or random

Definition of Hop Distance (HD)



Definition of Hop Distance (HD)



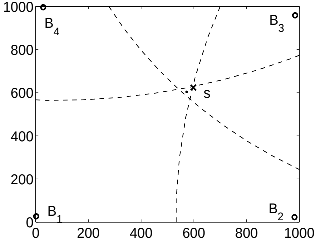
Question: Relation between hop-distance (HD) and Euclidean distance (ED)?

Outline of Talk

Assumption in HCRL

- Hop Count Ratio-based Localisation (HCRL), proposed by Yang et al. [IEEE SECON 2007]
- Assumption: $d \propto h$, hence $\frac{d_1}{d_2} = \frac{h_1}{h_2}$
- Suppose, node location (x, y), Anchors (x_{01}, y_{01}) and (x_{02}, y_{02})

$$\frac{\sqrt{(x - x_{01})^2 + (y - y_{01})^2}}{\sqrt{(x - x_{02})^2 + (y - y_{02})^2}} \approx \frac{h_1}{h_2} \quad \Leftarrow \text{ Equation of circle}$$



Assumption in PDM

- Proximity Distance Map (PDM), proposed by Lim and Hou [IEEE Infocom 2005]
- L anchors, node i has HD vector $\mathbf{h_i} \in \mathbb{N}^L$
- Assumption: ED vector $\mathbf{d_i} = \mathbf{Th_i}$
- ED matrix between anchors, $\mathbf{D} = [\mathbf{d_1}, \cdots, \mathbf{d_L}]$, is known
- HD matrix between anchors, $\mathbf{H} = [\mathbf{h_1}, \cdots, \mathbf{h_L}]$, is computed
- $\mathbf{D} = \mathbf{TH} \Rightarrow \mathbf{T} = \mathbf{DH^T}(\mathbf{HH^T})^{-1}$
- This $\mathbf{T}$ is used for all non-anchor nodes
- Node location estimated from the ED vector $\mathbf{d_i}$

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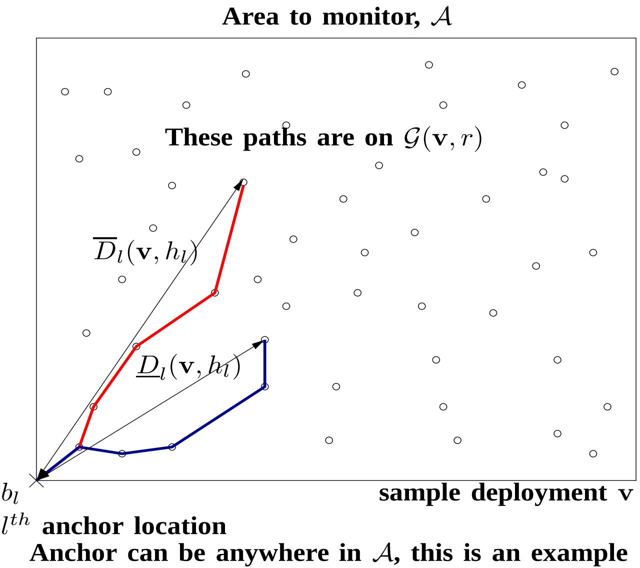
HD-ED Relationship in Arbitrary GG

- Setting:
  - n nodes placed on unit area $\mathcal{A}$ *arbitrarily*
  - $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
  - Geometric graph $\mathcal{G}(\mathbf{v}, r)$ is formed
- Notation:
  - $\mathcal{N} = [n] = \{1, 2, \cdots, n\}$, the index set of the nodes
  - $H_{l,i}(\mathbf{v})$ = HD of node i from l^{th} anchor on $\mathcal{G}(\mathbf{v}, r)$, for the deployment $\mathbf{v}$
  - $D_{l,i}(\mathbf{v})$ = Euclidean distance of node i from anchor b_l for the deployment $\mathbf{v}$.

$$\overline{D}_l(\mathbf{v}, h_l) = \max_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$

$$\underline{D}_l(\mathbf{v}, h_l) = \min_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$

Graphical Illustration



Lemma

For arbitrary **v** *and* $h_l \geq 2$, $r < \underline{D}_l(\mathbf{v}, h_l) \leq \overline{D}_l(\mathbf{v}, h_l) \leq h_l r$ *and both bounds are sharp.*

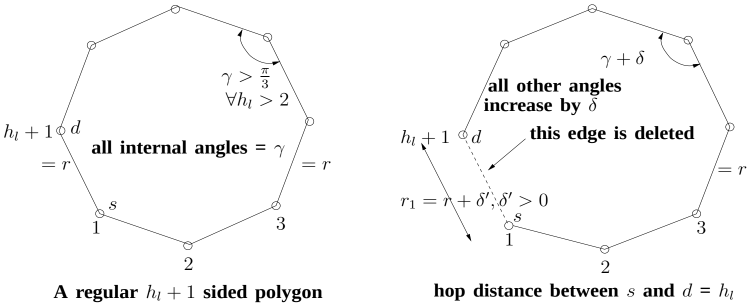


Figure: Node placement on the right achieves the lower bound of ED

- HD does not give useful information about ED in Arbitrary GG

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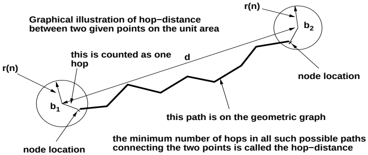
Paradigms of HD-ED Relationship in RGG

- HD-ED Relationship between Fixed Points
- HD-ED Relationship between Random Nodes
- HD-ED Relationship between Fixed Point and Random Node

HD-ED Relationship between Fixed Points

- Setting:
  - n nodes placed on unit area $\mathcal{A}$ *Uniform i.i.d.*
  - Node location vector $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
  - $\mathbb{P}^n(.)$ is the probability measure
  - Geometric graph $\mathcal{G}(\mathbf{v}, r(n))$ is formed
- Notation:
  - $H_{b_1 b_2}(\mathbf{v})$ is the hop distance between any two points b_1 and b_2 on $\mathcal{A}$, for the sample deployment $\mathbf{v}$
  - We will take $r(n) = c\sqrt{\frac{\log n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$, a constant, to guarantee asymptotic connectivity (Gupta and Kumar, 1998)



HD-ED Relationship between Fixed Points (Contd.)

Theorem

For all ϵ, $1 > \epsilon > 0$, *if* $c^2(\epsilon) \geq \frac{2}{q\sqrt{1-p^2}}$, *where* p *and* q *are any two constants satisfying* $1-\epsilon < p < 1$ *and* $0 < q < p - (1-\epsilon)$ *and* $p \geq 2q$, *on a unit square* $\mathcal{A}$,

$$\lim_{n\to\infty} \mathbb{P}^n \left\{ \mathbf{v} : \forall z_1, z_2 \in \mathcal{A}, \frac{\overline{z_1 z_2}}{r(n,\epsilon)} \leq H_{z_1 z_2}(\mathbf{v}) < \frac{\overline{z_1 z_2}}{(1-\epsilon) r(n,\epsilon)} \right\} = 1$$

where $r(n,\epsilon) = c(\epsilon)\sqrt{\frac{\log n}{n}}$.

- i.e., with high probability, ED between any two points is roughly equal to HD $\times$ radius of the RGG, where the radius is larger than the critical radius by a constant factor

HD-ED Relationship between Random Nodes

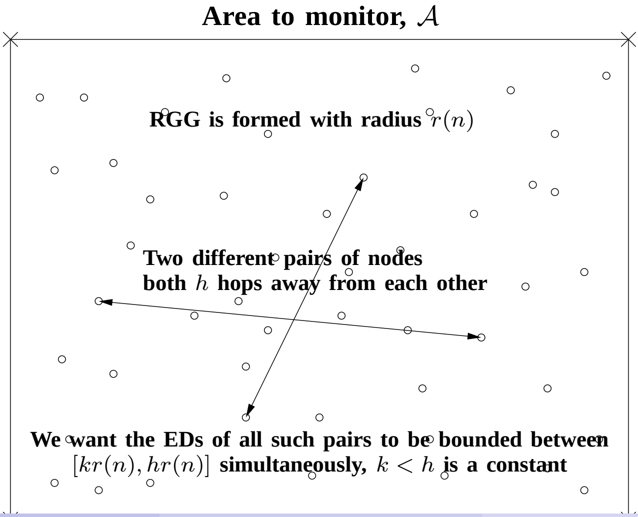
- Setting:
  - n nodes placed on unit area $\mathcal{A}$ *Uniform i.i.d.*
  - Node location vector $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
  - $\mathbb{P}^n(.)$ is the probability measure
  - Geometric graph $\mathcal{G}(\mathbf{v}, r(n))$ is formed
- Notation:
  - $\mathcal{N} = [n] = \{1, 2, \cdots, n\}$, the index set of the nodes, i.e., node $i \in \mathcal{N}$ has a location v_i on $\mathcal{A}$.
  - $D_{a,b}(\mathbf{v})$: The Euclidean distance on $\mathcal{A}$ between two nodes a and b, $a, b \in \mathcal{N}$, for the sample deployment $\mathbf{v}$.
  - $H_{a,b}(\mathbf{v})$: The hop distance on $\mathcal{G}(\mathbf{v}, r(n))$ between two nodes a and b, $a, b \in \mathcal{N}$, for the sample deployment $\mathbf{v}$.

$$\overline{D}(\mathbf{v}, h) = \max_{\{(a,b)\in\mathcal{N}^2 : H_{a,b}(\mathbf{v})=h\}} D_{a,b}(\mathbf{v})$$

$$\underline{D}(\mathbf{v}, h) = \min_{\{(a,b)\in\mathcal{N}^2 : H_{a,b}(\mathbf{v})=h\}} D_{a,b}(\mathbf{v})$$

HD-ED Relationship between Random Nodes (Contd.)

- We want bounds on the ED between any pair of nodes which are at a hop-distance h from each other



HD-ED Relationship between Random Nodes (Contd.)

Define,

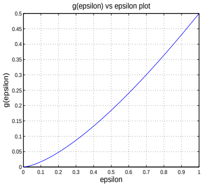
$$E_h(n, \epsilon) = \left\{ \mathbf{v} : \left((1-\epsilon)(h-1) - \frac{1}{2} \right) r(n, \epsilon) \leq \underline{D}(\mathbf{v}, h) \leq \overline{D}(\mathbf{v}, h) \leq hr(n, \epsilon) \right\}$$

Theorem

For $1 > \epsilon > 0$, *if* $c^2(\epsilon) \geq \frac{1}{g(\epsilon)}$, *where*
$g(\epsilon) = q(\epsilon)\sqrt{1 - p^2(\epsilon)}$, *and*
$p(\epsilon) = \frac{1-\epsilon+\sqrt{(1-\epsilon)^2+8}}{4}$,
$q(\epsilon) = \frac{-3(1-\epsilon)+\sqrt{(1-\epsilon)^2+8}}{4}$,

$$\mathbb{P}^n(E_h(n, \epsilon)) = 1 - \mathcal{O}\left(\frac{n^{1-c^2(\epsilon)g(\epsilon)}}{\ln n} \right)$$

Thus, $\lim \mathbb{P}^n(E_h(n, \epsilon)) = 1$



HD-ED Relationship between Fixed Point and Random Node

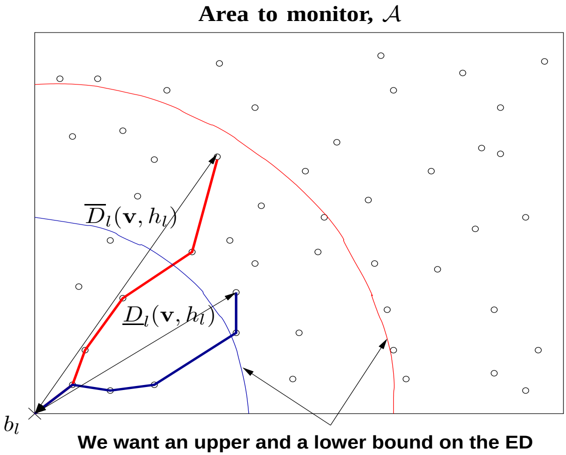
- Setting:
  - n nodes placed on unit area $\mathcal{A}$ *Uniform i.i.d.*
  - Node location vector $\mathbf{v} = [v_1, v_2, \cdots, v_n] \in \mathcal{A}^n$
  - $\mathbb{P}^n(.)$ is the probability measure
  - Geometric graph $\mathcal{G}(\mathbf{v}, r(n))$ is formed
- Notation:
  - $\mathcal{N} = [n] = \{1, 2, \cdots, n\}$, the index set of the nodes
  - $H_{l,i}(\mathbf{v})$ = HD of node i from l^{th} anchor on $\mathcal{G}(\mathbf{v}, r(n))$, for the deployment $\mathbf{v}$
  - $D_{l,i}(\mathbf{v})$ = Euclidean distance of node i from anchor b_l for the deployment $\mathbf{v}$.

$$\overline{D}_l(\mathbf{v}, h_l) = \max_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$

$$\underline{D}_l(\mathbf{v}, h_l) = \min_{\{i \in \mathcal{N} : H_{l,i}(\mathbf{v}) = h_l\}} D_{l,i}(\mathbf{v})$$

HD-ED Relationship between Fixed Point and Random Node (Contd.)

- We want bounds on the ED between a fixed point and all nodes at a hop-distance h from the point



HD-ED Relationship between Fixed Point and Random Node (Contd.)

Define, $E_{h_l}(n) = \{\mathbf{v} : (1-\epsilon)(h_l - 1)r(n) \leq \underline{D}_l(\mathbf{v}, h_l) \leq \overline{D}_l(\mathbf{v}, h_l) \leq h_l r(n)\}$

Theorem

For a given $1 > \epsilon > 0$, *and* $r(n) = c\sqrt{\frac{\ln n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$,

$$\mathbb{P}^n(E_{h_l}(n)) = 1 - \mathcal{O}\left(\frac{1}{n^{g(\epsilon)c^2}}\right)$$

where $g(\epsilon) = q(\epsilon)\sqrt{1 - p^2(\epsilon)}$, $p(\epsilon) = \frac{1-\epsilon+\sqrt{(1-\epsilon)^2+8}}{4}$, $q(\epsilon) = \frac{-3(1-\epsilon)+\sqrt{(1-\epsilon)^2+8}}{4}$.

$$\text{Hence, } \lim_{n\to\infty} \mathbb{P}^n(E_{h_l}(n)) = 1$$

Since $g(\epsilon) \downarrow$ as $\epsilon \downarrow$, the rate of convergence slows down

Proof Techniques (1/4)

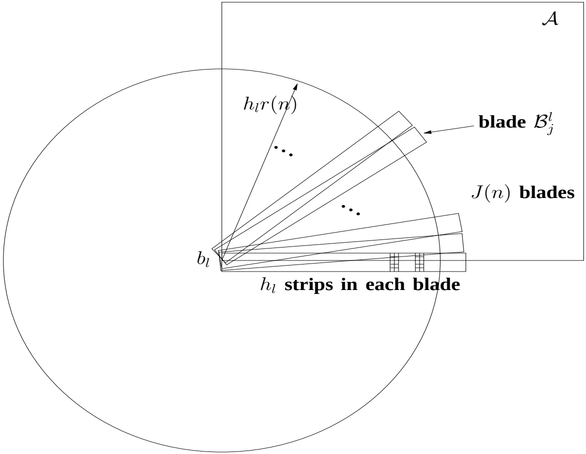


Figure: Construction using the blades cutting the circumference of the circle of radius $h_l r(n)$.

Proof Techniques (2/4)

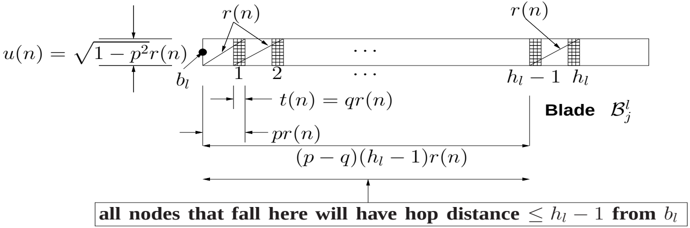


Figure: The construction with h_l hops.

$$A_{i,j}^l = \{\mathbf{v} : \exists \text{ at least one node in the } i^{th} \text{ strip of } \mathcal{B}_j^l\}$$

$$\{\cap_{j=1}^{J(n)} \cap_{i=1}^{h_l-1} A_{i,j}^l\}$$
$$\subseteq \quad \{\mathbf{v} : (p-q)(h_l-1)r(n) \leq \underline{D}_l(\mathbf{v}, h_l) \leq \overline{D}_l(\mathbf{v}, h_l) \leq h_l r(n)\}$$

Proof Techniques (3/4)

$$
\begin{aligned}
\mathbb{P}^n\left(\cap_{j=1}^{J(n)}\cap_{i=1}^{h_l-1} A_{i,j}^l\right) &= 1-\mathbb{P}^n\left(\cup_{j=1}^{J(n)}\cup_{i=1}^{h_l-1} {A_{i,j}^l}^c\right)\\
&\geq 1-\sum_{j=1}^{J(n)}\sum_{i=1}^{h_l-1}\mathbb{P}^n\left({A_{i,j}^l}^c\right)\\
&\geq 1-(h_l-1)\left\lceil\frac{\pi h_l}{2\sqrt{1-p^2}}\right\rceil(1-u(n)t(n))^n\\
&\geq 1-(h_l-1)\left\lceil\frac{\pi h_l}{2\sqrt{1-p^2}}\right\rceil e^{-nu(n)t(n)}\\
&= 1-(h_l-1)\left\lceil\frac{\pi h_l}{2\sqrt{1-p^2}}\right\rceil e^{-nq\sqrt{1-p^2}r^2(n)}\\
&= 1-(h_l-1)\left\lceil\frac{\pi h_l}{2\sqrt{1-p^2}}\right\rceil n^{-q\sqrt{1-p^2}c^2}\overset{n\to\infty}{\longrightarrow}1
\end{aligned}
$$

Proof Techniques (4/4)

- We take $p - q = 1 - \epsilon$, and maximise $q\sqrt{1-p^2}$
- Gives $p(\epsilon) = \frac{1-\epsilon+\sqrt{(1-\epsilon)^2+8}}{4}$, $q(\epsilon) = \frac{-3(1-\epsilon)+\sqrt{(1-\epsilon)^2+8}}{4}$
- Define $g(\epsilon) = q(\epsilon)\sqrt{1-p^2(\epsilon)}$, Hence,

$$\mathbb{P}^n\{\mathbf{v} : (1-\epsilon)(h_l - 1)r(n) \leq \underline{D}_l(\mathbf{v}, h_l) \leq \overline{D}_l(\mathbf{v}, h_l) \leq h_l r(n)\}$$

$$= 1 - \mathcal{O}\left(\frac{1}{n^{g(\epsilon)c^2}}\right)$$

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1000 nodes : 5 hops

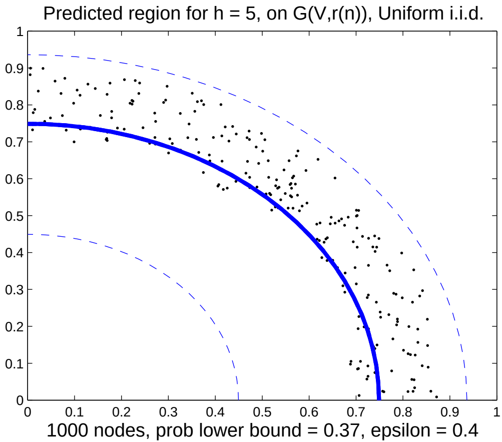

Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows ED $(h_1 - 1)r(n)$ from b_1 for 1000 nodes, 5 hops, $\epsilon = 0.4$, $\mathbb{P}^n(E_1(n)) \geq 0.37$. $r(n) = \frac{4}{\sqrt{\pi}}\sqrt{\frac{\ln n}{n}}$.

5000 nodes : 5 hops

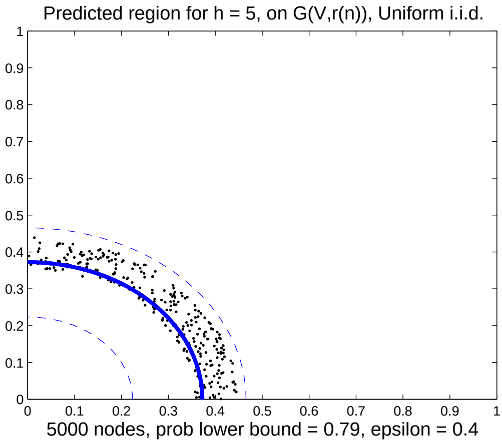

Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows ED $(h_1 - 1)r(n)$ from b_1 for 5000 nodes, 5 hops, $\epsilon = 0.4$, $\mathbb{P}^n(E_1(n)) \geq 0.79$. $r(n) = \frac{4}{\sqrt{\pi}}\sqrt{\frac{\ln n}{n}}$.

5000 nodes : 10 hops

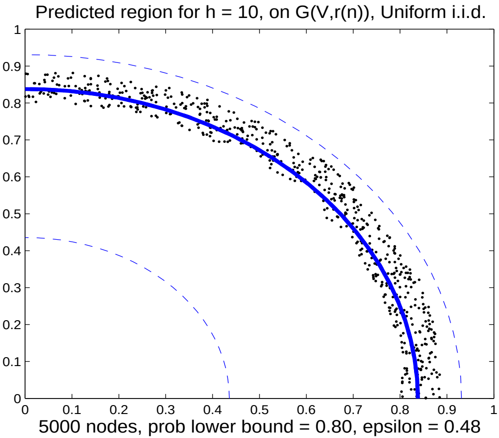

Figure: The dashed line shows the ED bounds given by the point-node theorem, the solid line shows ED $(h_1 - 1)r(n)$ from b_1 for 5000 nodes, 10 hops, $\mathbb{P}^n(E_1(n)) \geq 0.80$. $r(n) = \frac{4}{\sqrt{\pi}}\sqrt{\frac{\ln n}{n}}$.

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Theorem Used

- For localisation we need multiple (say L) anchors
- hop-distance vector, $\mathbf{h} = [h_1, \cdots, h_l, \cdots, h_L] \in \mathbb{N}^L$
- For L anchors, all possible $\mathbf{h}$ vectors are not *feasible*
- $\mathcal{H}(n)$: set of all *feasible* $\mathbf{h}$ vectors (it depends on n)

Recall, $E_{h_l}(n) = \{\mathbf{v} : (1-\epsilon)(h_l - 1)r(n) \leq \underline{D}_l(\mathbf{v}, h_l) \leq \overline{D}_l(\mathbf{v}, h_l) \leq h_l r(n)\}$

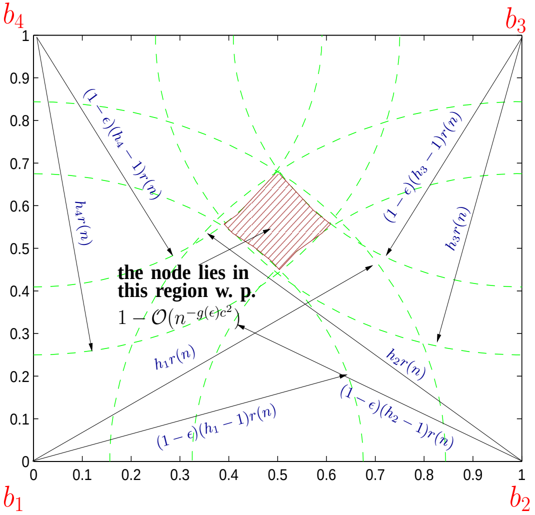
Theorem

For a given $1 > \epsilon > 0$, and $r(n) = c\sqrt{\frac{\ln n}{n}}$, $c > \frac{1}{\sqrt{\pi}}$,
$\forall \mathbf{h} = [h_1, \cdots, h_l, \cdots, h_L] \in \mathcal{H}(n)$,

$$\mathbb{P}^n \left(\cap_{l=1}^{L} E_{h_l}(n) \right) = 1 - \mathcal{O}\left(\frac{1}{n^{g(\epsilon)c^2}} \right)$$

where $g(\epsilon) = q(\epsilon)\sqrt{1 - p^2(\epsilon)}$,
$p(\epsilon) = \frac{1-\epsilon+\sqrt{(1-\epsilon)^2+8}}{4}$, $q(\epsilon) = \frac{-3(1-\epsilon)+\sqrt{(1-\epsilon)^2+8}}{4}$

Illustration



Outline of Talk

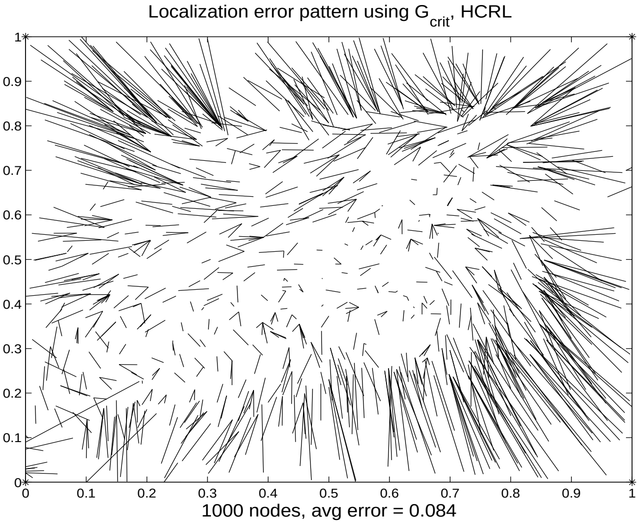
Algorithm: Hop Count-derived Distance-based Localisation (HCDL)[1]

- **STEP 1: (Initialisation)** Each node finds the hop-distance vector $\mathbf{h} = [h_1, \cdots, h_L]$
- **STEP 2: (Region of Intersection)** For a certain node, set an ϵ, small enough, and find the region of intersection formed by the annuli of radii $[(1-\epsilon)(h_l - 1)r(n), h_l r(n)]$ centred at the l^{th} anchor location, $l = 1, \cdots, L$
- **STEP 3: (Terminating Condition)**
  - **IF** there is an intersection, declare the centroid of the region of intersection as the estimate of the node. GO TO **STEP 4**.
  - **ELSE** increase ϵ by an amount k, $0 < k < 1$. GO TO **STEP 2**.
- **STEP 4: (Repetition)** Repeat **STEP 2** to **STEP 3** for all n nodes.
- **STEP 5: STOP**

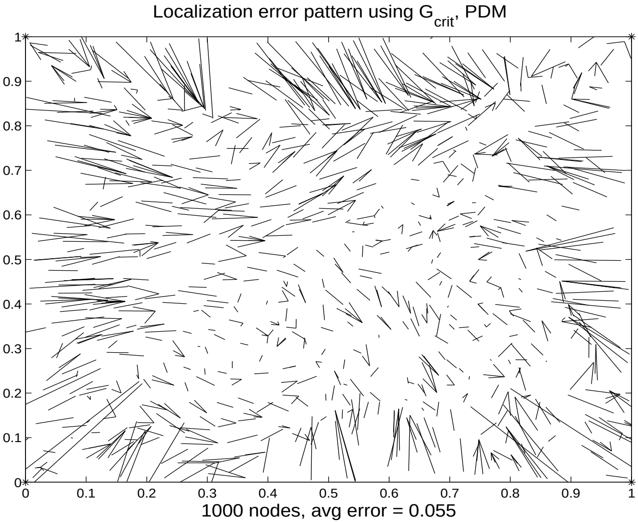
[1] This is a joint work with Venkatesan N.E. and Prof. P. Vijay Kumar

Outline of Talk

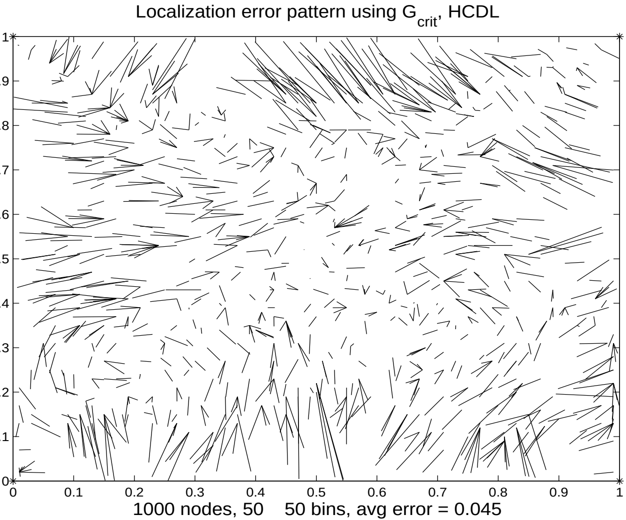
HCRL (Yang et al. 2007): Localisation Error Pattern



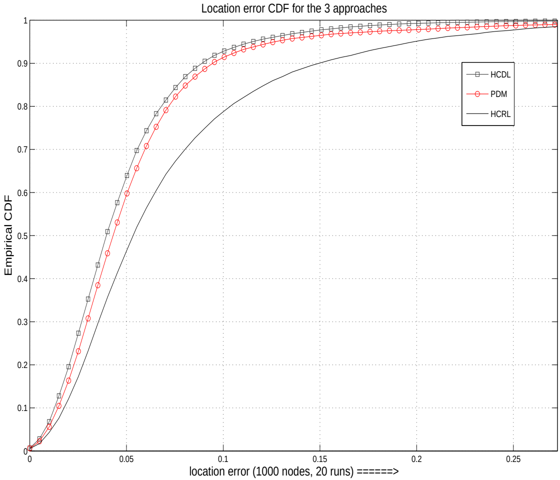
PDM (Lim and Hou, 2005): Localisation Error Pattern



HCDL: Localisation Error Pattern



Cumulative Distribution of Error



Outline of Talk

Conclusion and Future Work

- Assumed a Geometric Graph model of the Wireless Sensor Network
- HD is not a good measure for ED for *arbitrary* node placements
- Three paradigms of HD-ED proportionality for *random* node placements
  - Sufficiency theorems for ED-HD relationships in point-point, node-node and point-node paradigms
  - For point-point and node-node cases, the radius of the GG is larger than the critical radius
  - For point-node case, theorem is valid for critical radius too
- For point-node theory, given HD $= h$, $(1 - \epsilon)(h - 1)r <$ ED $\leq hr$ with high probability
- Proposed algorithm HCDL based on this theory
- Performs better than HCRL and PDM


Thank You