CS711: Introduction to Game Theory and Mechanism Design

Endsem – Semester 1, 2018-19, Computer Science and Engineering,
Indian Institute of Technology Kanpur
Total Points: 40, Time: 2 hours, ATTEMPT ALL QUESTIONS

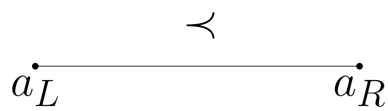
1. Let A be a finite set of alternatives and $f : \mathcal{P}^n \to A$ be a social choice function that is unanimous and strategy-proof, $n \geqslant 3$. Suppose $|A| \geqslant 3$, and $\mathcal{P}$ denotes the set of all possible strict preferences over A.

   Now, consider another social choice function $g : \mathcal{P}^2 \to A$ defined as follows. The SCF g only considers profiles of two agents, denote these two agents as 1 and 2. For every $(P_1, P_2) \in \mathcal{P}^2$, let
   $$g(P_1, P_2) = f(P_1, P_2, P_1, P_1, \ldots, P_1),$$
   i.e., the outcome of g at (P_1, P_2) coincides with the outcome of f at the profile where agents 1 and 2 have types P_1 and P_2 respectively, and all other agents have type P_1.

   Show that g is a dictatorship SCF.

   [Hint: you may use the Gibbard-Satterthwaite characterization result.] **10 points.**

2. Let A be a finite set of alternatives and $\prec$ be an intrinsic linear order over A – see the figure below for an illustration.

   Suppose $a_L, a_R \in A$ be two alternatives such that $a_L \prec a$ for all $a \in A \setminus \{a_L\}$ and $a \prec a_R$ for all $a \in A \setminus \{a_R\}$ - in other words, a_L is the "left-most" alternative and a_R is the "right-most" alternative with respect to $\prec$.

   Let $\mathcal{S}$ be the set of all possible single-peaked strict orderings over A with respect to $\prec$. An SCF $f : \mathcal{S}^n \to A$ maps the set of preference profiles of n agents to A.

   Let $P_i(1)$ denote the peak of agent i in P_i. Suppose f satisfies the following property (call it property Π). There is an alternative $a^* \in A$ such that for any preference profile $(P_1, \ldots, P_n) \in \mathcal{S}^n$, where $P_i(1) \in \{a_L, a_R\}$ for all $i \in N$ with at least one agent's peak at a_L and at least one agent's peak at a_R, $f(P_1, \ldots, P_n) = a^*$.

   Suppose f is strategy-proof, Pareto efficient, anonymous, and satisfies property Π. Then, give a precise (simplified) description of f (using a^*), i.e., for every preference profile P, what is $f(P)$?

   [Hint: you may use the Moulin characterization result and argue where the phantom peaks must be located.] **10 points.**

3. Three friends, numbered 1, 2, and 3, are planning to choose a movie to watch together. The candidate movies are A, B, and C (exactly one of them can be chosen). The values (derived from their film interests) that the friends get from watching these movies are given in the table below.

	A	B	C
1	7	5	1
2	7	5	2
3	2	5	10

They agree that the decision should be 'efficient' as much as possible, i.e., it should maximize the sum value of all the agents, but they are also concerned whether (a) this can be done truthfully, and (b) using only internal transfers, i.e., keeping it budget balanced.

(i) If they decide to ask the agents their valuations and decide the efficient choice of movie without any monetary transfers, will the properties of (a) and (b) be satisfied? Explain why or why not. **2 points.**

(ii) If monetary transfer is allowed and that results in a quasi-linear utility to the agents (the friends), can efficient choice of movie be made in a truthful manner? Provide a payment rule that implements the efficient choice. What is the movie decided and how much should each agent pay? What is the net utility of each of the players? Is this mechanism budget balanced? **1 + 2 + 3 + 3 + 1 points.**

(iii) Suppose they decide on a different mechanism instead. Agent 2's values are not considered in the movie decision process. The mechanism chosen in item (ii) above is used for agents 1 and 3, the monetary surplus generated is transferred to agent 2. Does this mechanism satisfy properties (a) and (b)? Clearly explain why. What are the net utilities of the players? Is this mechanism efficient?

3 + 3 + 2 points.

Good Luck!