

CS711: Introduction to Game Theory and Mechanism Design

Assignment 2 – Semester 1, 2018-19

Computer Science and Engineering, Indian Institute of Technology Kanpur

Total Points: 20, Time: 2 days, ATTEMPT ALL QUESTIONS

Please email (with subject ‘CS 711 Assignment 2’) your solutions to the instructor (swaprava@cse.iitk.ac.in) with cc to the TAs as a single PDF file generated through L^AT_EX. See the course webpage for L^AT_EX tutorials. Please submit the solution PDF named as {roll number}.pdf, e.g., 1234567.pdf.

There is no need to write the question again in the solution PDF.

1. Consider a two agent model with three alternatives $\{a, b, c\}$. Table 1 shows two preference profiles of the agents. Suppose f is an *onto* SCF with $f(P_1, P_2) = a$.

P_1	P_2	P'_1	P'_2
a	c	b	a
b	b	a	b
c	a	c	c

Table 1: Two Preference Profiles

- (a) Suppose the domain of preferences is of unrestricted strict preferences. Show that if f is **strategyproof** then $f(P'_1, P'_2) = b$. You are allowed to use the result that for any preference profile (\bar{P}_1, \bar{P}_2) , $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$ and the fact that strategyproofness implies monotonicity (but do not use any other result from the lectures, e.g., Gibbard-Satterthwaite theorem).

5 points.

- (b) Now, suppose that these preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$. Does the earlier conclusion hold in this case? Explain clearly why or why not the earlier proof go through? If the conclusion is false, provide a mechanism that can have $f(P'_1, P'_2) = a$.

1 + 2 + 2 points.

2. Let X be a set of projects. A social choice function chooses a non-empty subset of projects. Agent i has a linear ordering P_i over the set of projects X . Agent i evaluates subsets of projects by extending P_i in the following manner: for any pair of subsets of projects $S, T \subseteq X$, S is preferred to T if the highest ranked project in S (according to P_i) is better than the highest ranked project in T - if these two projects are the same, then S and T are indifferent.

Suppose $|X| \geq 2$. Will the Gibbard-Satterthwaite result apply here? Discuss your answer.

5 points.

3. Consider the single-peaked domain model. A social choice function f is manipulable by a group of agents $K \subseteq N$ if for some preference profile (P_K, P_{-K}) there exists some preference profile P'_K of agents in K such that $f(P'_K, P_{-K}) P_i f(P_K, P_{-K})$ for all $i \in K$. A social choice function f is **group strategy-proof** if cannot be manipulated by any group of agents. Is the median voter SCF group strategy-proof? **5 points.**

Good Luck!