CS711: Introduction to Game Theory and Mechanism Design

Assignment 2 – Semester 1, 2018-19
Computer Science and Engineering, Indian Institute of Technology Kanpur
Total Points: 20, Time: 2 days, ATTEMPT ALL QUESTIONS
Please email (with subject 'CS 711 Assignment 2') your solutions to the instructor (swaprava@cse.iitk.ac.in) with cc to the TAs as a single PDF file generated through LaTeX. See the course webpage for LaTeX tutorials. Please submit the solution PDF named as {roll number}.pdf, e.g., 1234567.pdf.
There is no need to write the question again in the solution PDF.

1. Consider a two agent model with three alternatives $\{a, b, c\}$. Table 1 shows two preference profiles of the agents. Suppose f is an *onto* SCF with $f(P_1, P_2) = a$.

P_1	P_2	P_1'	P_2'
a	c	b	a
b	b	a	b
c	a	c	c

Table 1: Two Preference Profiles

(a) Suppose the domain of preferences is of unrestricted strict preferences. Show that if f is **strategyproof** then $f(P_1', P_2') = b$. You are allowed to use the result that for any preference profile $(\bar{P}_1, \bar{P}_2)$, $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$ and the fact that strategyproofness implies monotonicity (but do not use any other result from the lectures, e.g., Gibbard-Satterthwaite theorem).

5 points.

Solution:[sketch] This is case 3 of lemma 20.5 in the lecture notes.[1] The alternatives are renamed – player 1 and 2's favorite alternatives are a and c respectively in profile P, while it is b and a respectively in P'. Hence to show this result, one needs to prove the cases 1 and 2 of the same lemma before proving case 3. ■

(b) Now, suppose that these preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$. Does the earlier conclusion hold in this case? Explain clearly why or why not the earlier proof go through? If the conclusion is false, provide a mechanism that can have $f(P_1', P_2') = a$.

1 + 2 + 2 points.

Solution:[sketch]

(b).1 No.

(b).2 The proofs in the earlier case constructs certain preference profile which is disallowed in the single peaked domain. E.g., the proof of the fact that "for any preference profile $(\bar{P}_1, \bar{P}_2)$, $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$" – uses the construction of a preference

[1] https://www.cse.iitk.ac.in/users/swaprava/courses/cs698w/scribe/lecnotes.pdf

that places a and c next to each other. This is not possible in the single peaked domain when the intrinsic ordering is $a < b < c$. Similar construction is used in case 1 as well. Therefore the same proof technique does not hold in the single peaked domain.

(b).3 'Pick the left-most peak' mechanism is strategyproof in the single-peaked domain, and that yields the result of $f(P'_1, P'_2) = a$ in this setting. Note that the failure of the earlier proof technique does not imply that the statement "if f is **strategyproof** then $f(P'_1, P'_2) = b$" is false. Rather this counterexample shows that indeed there are more strategyproof rules in this domain.

■

2. Let X be a set of projects. A social choice function chooses a non-empty subset of projects. Agent i has a linear ordering P_i over the set of projects X. Agent i evaluates subsets of projects by extending P_i in the following manner: for any pair of subsets of projects $S, T \subseteq X$, S is preferred to T if the highest ranked project in S (according to P_i) is better than the highest ranked project in T - if these two projects are the same, then S and T are indifferent.

   Suppose $|X| \geq 2$. Will the Gibbard-Satterthwaite result apply here? Discuss your answer.

   5 points.

   Solution:[sketch] The set of alternatives is the set of all subsets of objects: $\{S : S \subseteq X\}$. If there are at least 2 projects then, the set of alternatives is at least 3. Now, consider two alternatives S and T such that $S \subsetneq T$. By definition of the preference ordering, any agent is either indifferent between S and T or likes T to S. Hence, the preference ordering where S is ranked higher than T can never arise. This is a restriction of the domain and we cannot apply the Gibbard-Satterthwaite result here. ■

3. Consider the single-peaked domain model. A social choice function f is manipulable by a group of agents $K \subseteq N$ if for some preference profile (P_K, P_{-K}) there exists some preference profile P'_K of agents in K such that $f(P'_K, P_{-K}) P_i f(P_K, P_{-K})$ for all $i \in K$. A social choice function f is **group strategy-proof** if cannot be manipulated by any group of agents. Is the median voter SCF group strategy-proof? **5 points.**

   Solution:[sketch] Yes, the median voter SCF is group strategy-proof. The proof is similar to the proof that shows that the median voter SCF is strategy-proof. A group of agents can change the outcome if they shift the median – which can happen if at least one agent shifts her peak to the other side of the median, but this agent will not like that outcome over the current outcome. ■

Good Luck!