CS711: Introduction to Game Theory and Mechanism Design

Teacher: Swaprava Nath

Imperfect Information Extensive Form Games

Mixed strategy equivalent to behavioral strategy

Theorem

Let $\langle N, A, \mathcal{H}, \mathcal{X}, P, (u_i)_{i \in N}, (I_i)_{i \in N} \rangle$ *be an IIEFG such that at every vertex of the game tree there are at least two actions. Every behavioral strategy has an equivalent mixed strategy if and only if each information set of player* i *intersects every path emanating from the root at most once.*

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- this will supersede the condition in the previous theorem

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- we are now ready to define **game with perfect recall**

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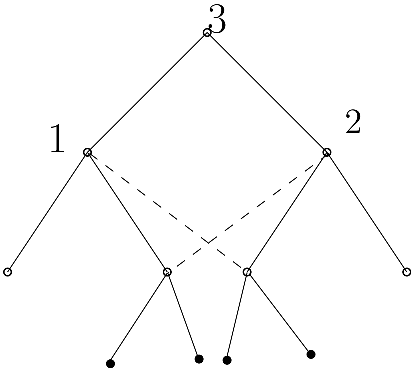
A game is called a game with perfect recall if every player has perfect recall.

Examples



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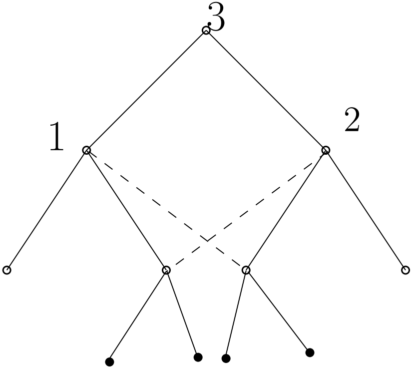

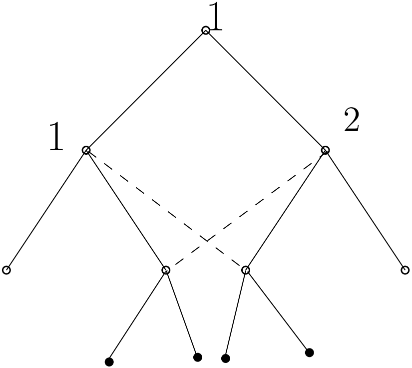

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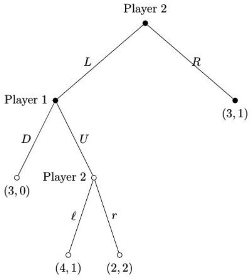
- note that perfect recall subsumes the requirement of the theorem where for every behavioral strategy there is a mixed strategy – hence this is a result that makes these two strategies equivalent

Proof outline

- construction from a given mixed strategy



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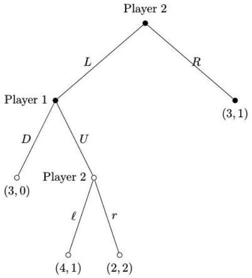

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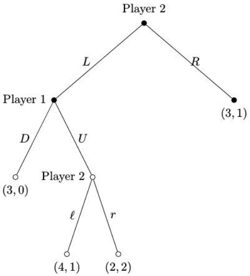

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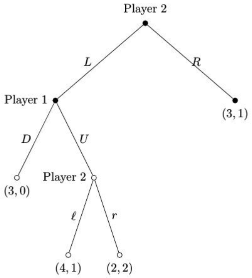

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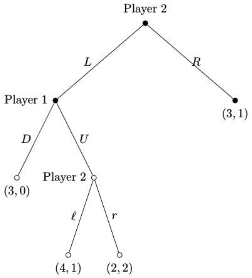

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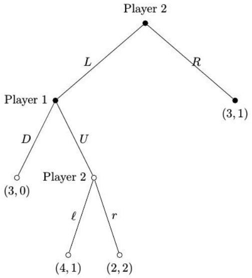
- rest of the argument shows that such a construction is possible due to the facts that the information sets cut every path at most once, and the number of times and actions with which two different paths reach two nodes in the same information set are same



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- example – assume games with perfect recall

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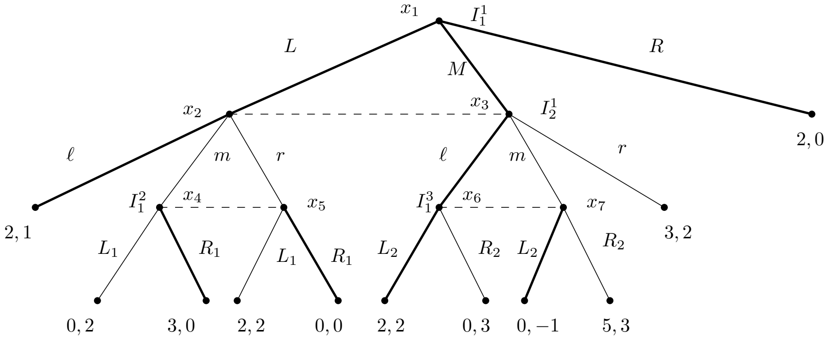

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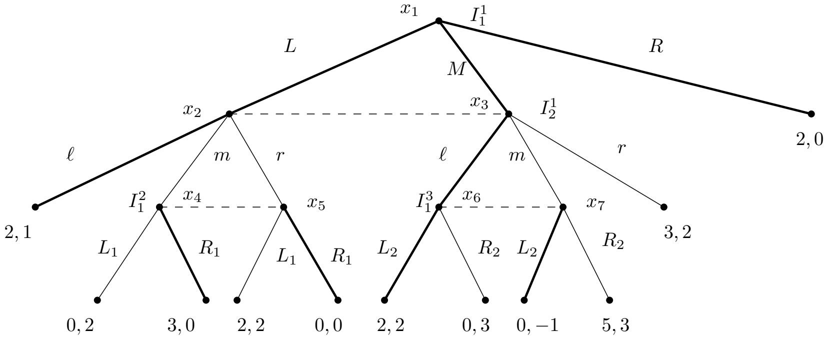

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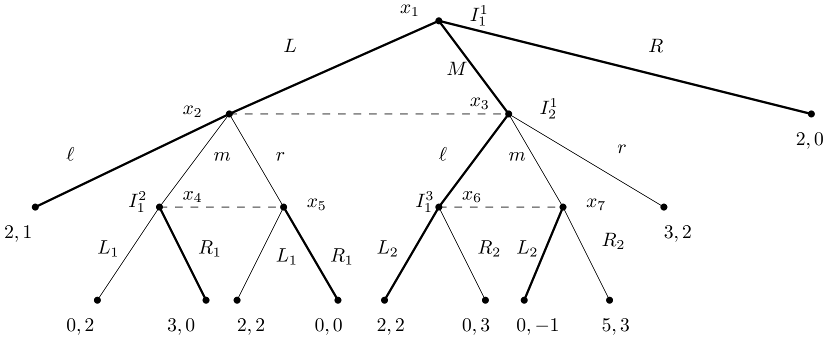

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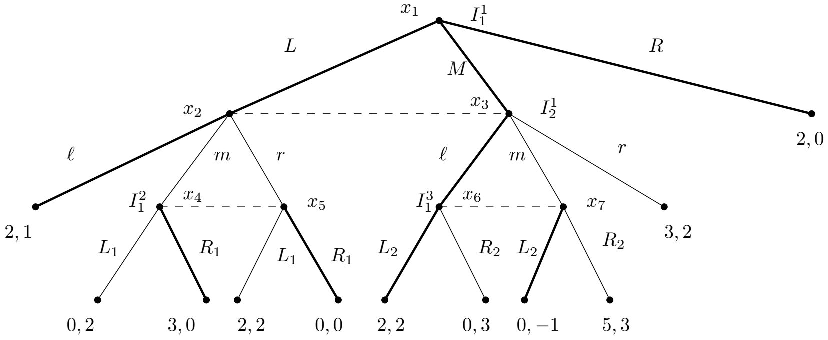

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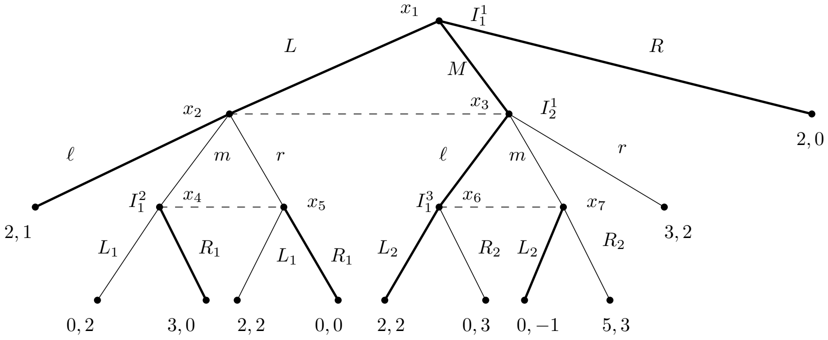

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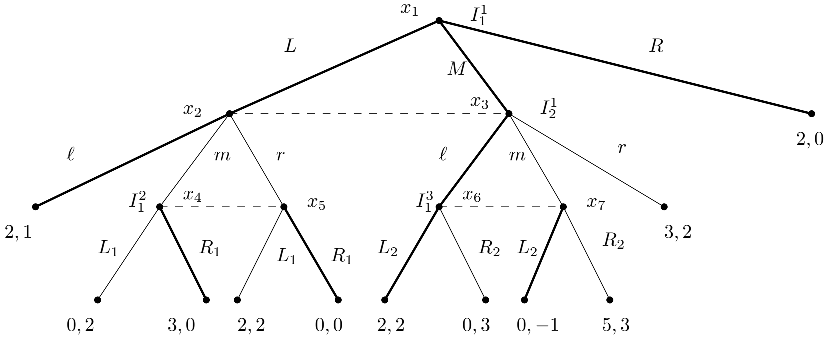

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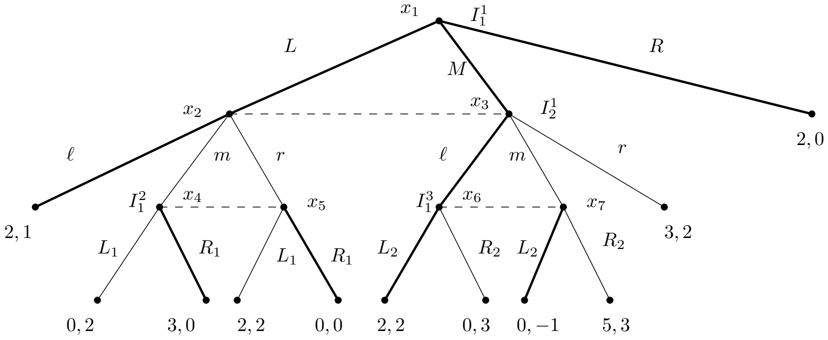

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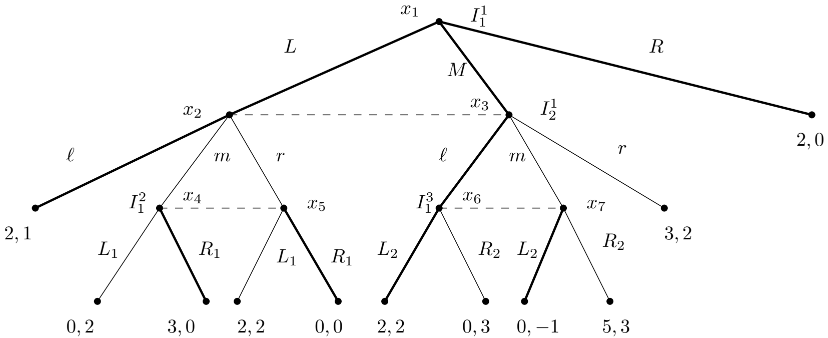

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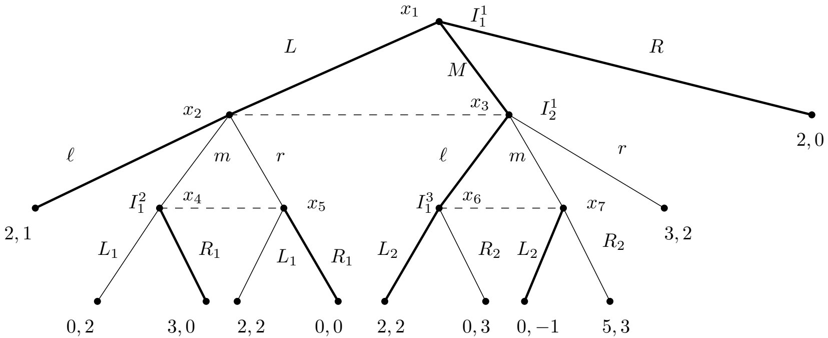

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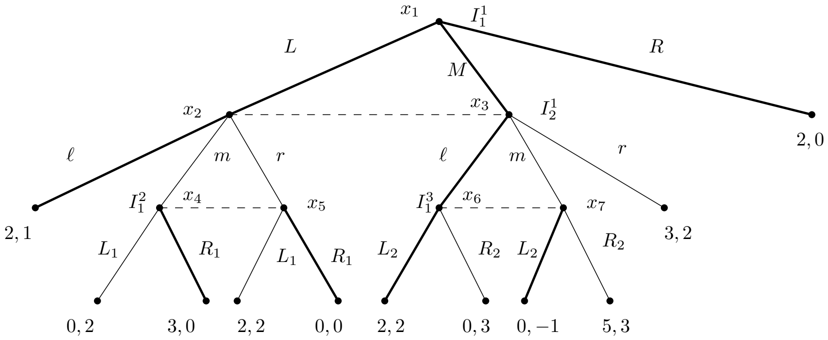

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- connection between **action** and **belief** at an information set

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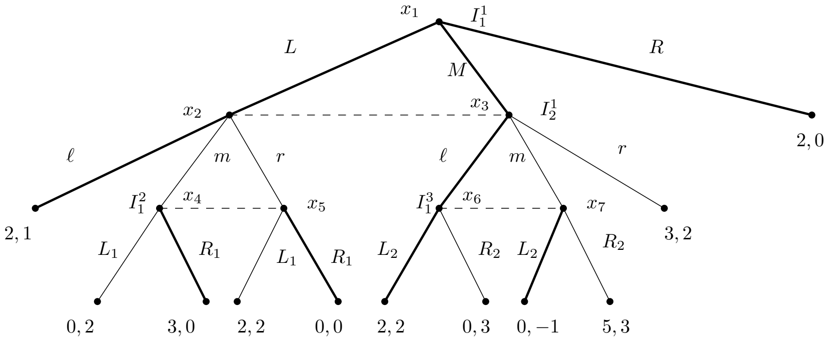

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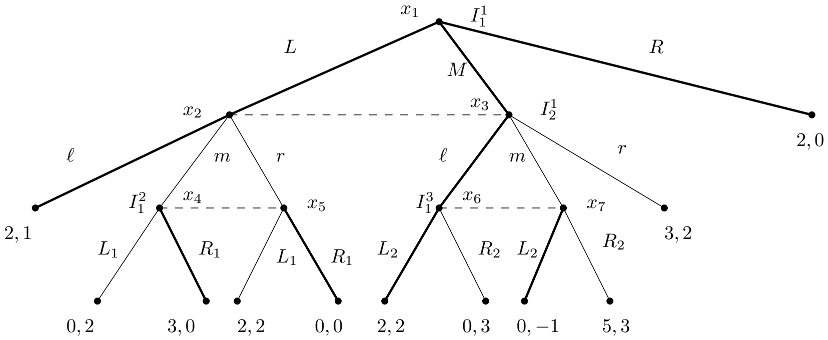

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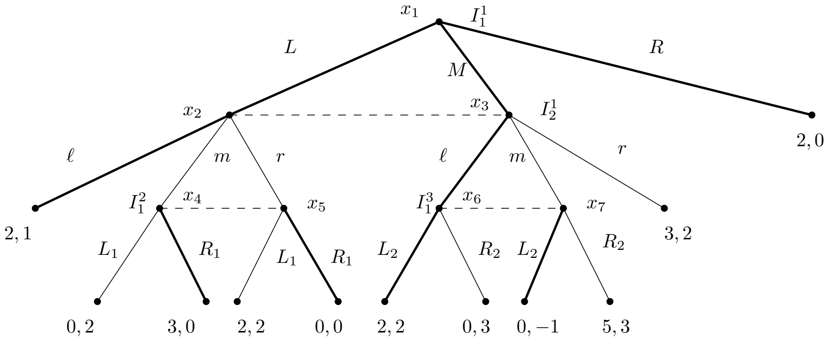

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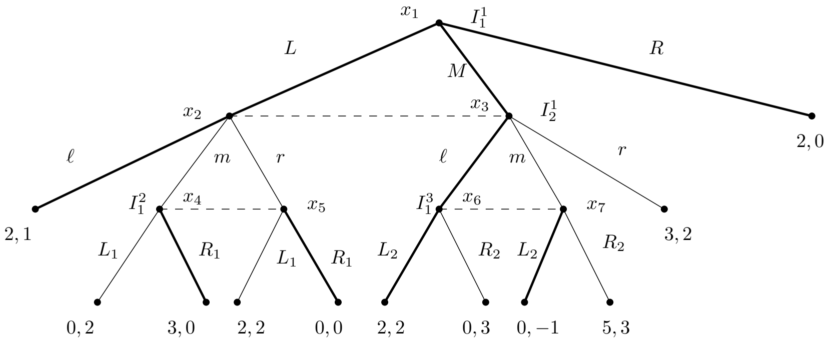
- *choose an action maximizing expected payoff at each information set* – **sequential rationality**

Example (contd.)



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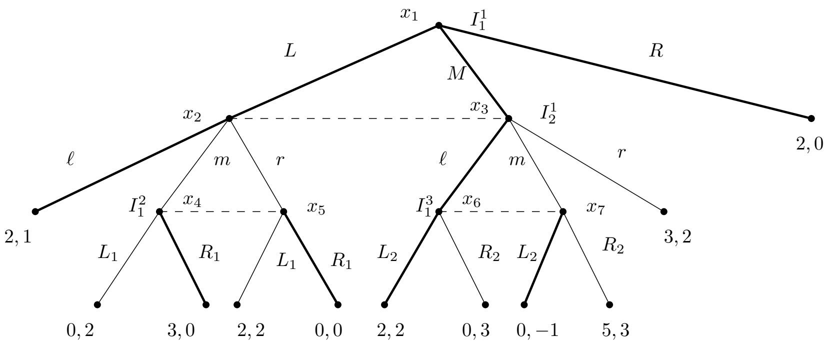

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- is the action of player 2 sequentially rational with her belief?

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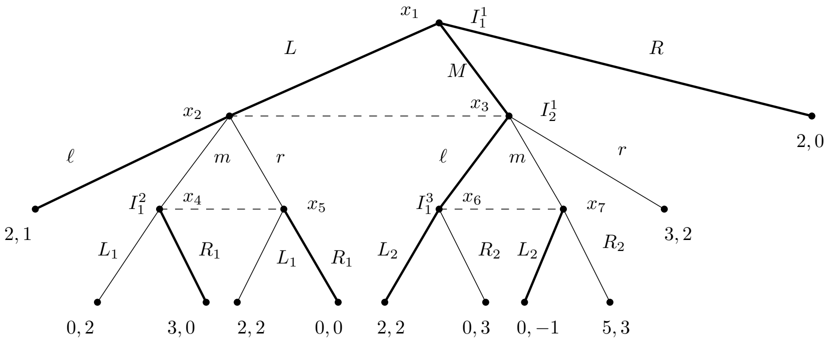

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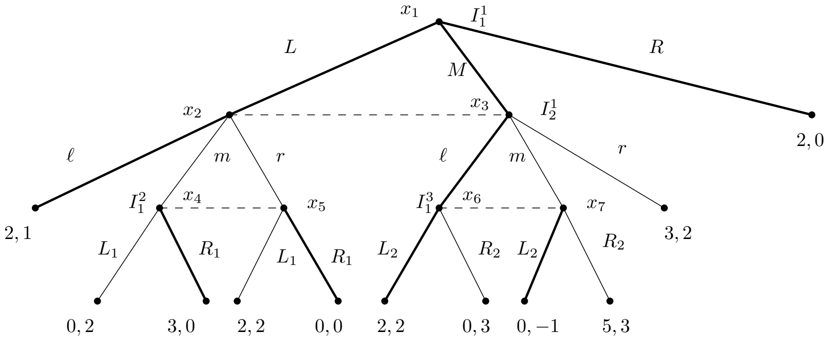
- $\ell \to \frac{5}{9}1 + \frac{4}{9}2 = \frac{13}{9}$

Example (contd.)



- given these, player 1, at I_1^1, what will be the best response?

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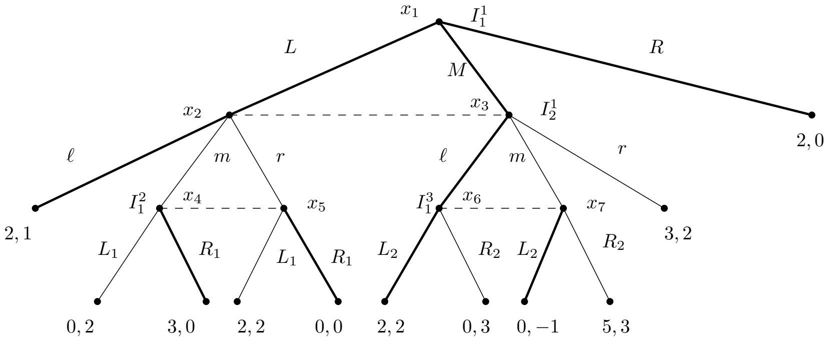

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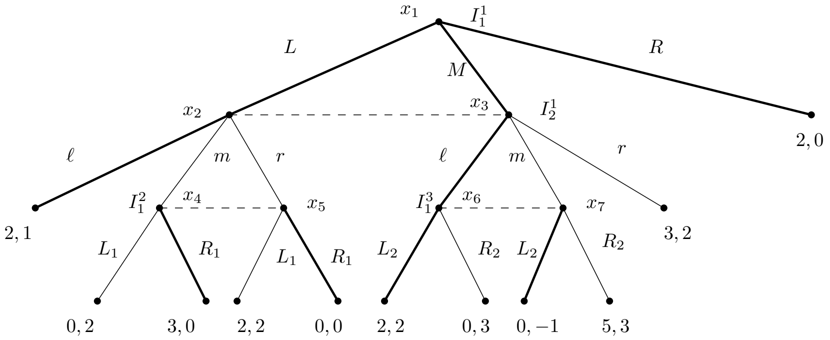

- given these, player 1, at I_1^1, what will be the best response?
- in all actions, the utility is 2, but a different strategy will change the numbers for other information sets
- **strategy profile σ is sequentially rational for all the players**

Formal definitions

Definition (Belief)

Let the *information sets* of player i be $I_i = \{I_i^1, I_i^2, ..., I_i^{k(i)}\}$. In an IIEFG, the belief of player i is a map $\mu_i^j : I_i^j \to [0, 1]$, such that,

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Definition (Bayesian belief)

A belief $\mu_i := (\mu_i^j, j = 1, \ldots, k(i))$ of player i is Bayesian with respect to the behavioral strategy σ, if it is derived from the strategy profile σ using Bayes' rule, i.e.,

$$\mu_i^j(x) = \frac{P_\sigma(x)}{\sum_{y \in I_i^j} P_\sigma(y)}, \forall x \in I_i^j, \ \forall j = 1, \ldots, k(i).$$

Definitions (contd.)

Definition (Sequential rationality)

A strategy σ_i of player i at an information set I_i^j is **sequentially rational** given σ_{-i} and partial beliefs μ_i if $\forall \sigma_i'$

$$\sum_{x \epsilon I_i^j} \mu_i(x) u_i(\sigma_i, \sigma_{-i} | x) \geqslant \sum_{x \epsilon I_i^j} \mu_i(x) u_i(\sigma_i', \sigma_{-i} | x).$$

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The pair (σ, μ) is called an **assessment**

Relationship with Nash

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Theorem

In a PIEFG, a behavioral strategy profile σ is an SPNE if and only if the pair $(\sigma, \widehat{\mu})$ is sequentially rational. [In PIEFG, every information set is singleton, hence $\widehat{\mu}$ is the degenerate distribution at that singleton.]

Equilibrium with sequential rationality

Definition (Perfect Bayesian Equilibrium)

An assessment (σ, μ) is a *perfect Bayesian equilibrium* (PBE) if *for every player* i

1. μ_i is Bayesian with respect to σ,
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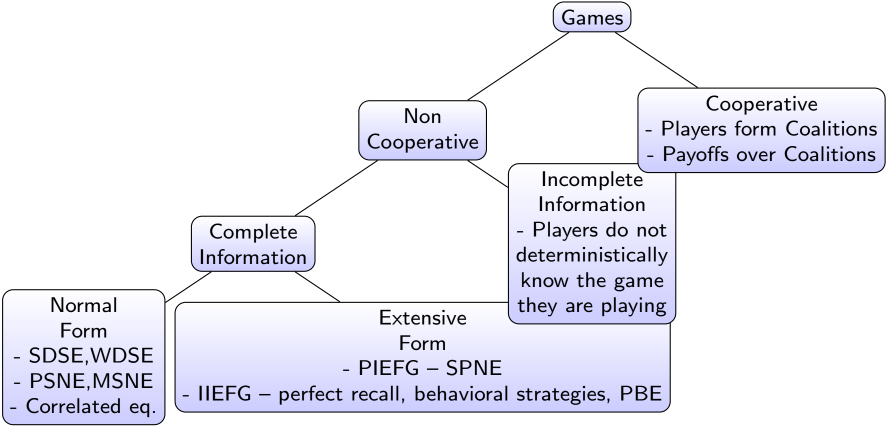
Theorem

For every Perfect Bayesian Equilibrium (PBE) (σ, μ), σ *is a Mixed Strategy Nash Equilibrium (MSNE).*

Ecosystem


Games
Non Cooperative
Cooperative
- Players form Coalitions
- Payoffs over Coalitions
Complete Information
Incomplete Information
- Players do not deterministically know the game they are playing
Normal Form
- SDSE,WDSE
- PSNE,MSNE
- Correlated eq.
Extensive Form
- PIEFG – SPNE
- IIEFG – perfect recall, behavioral strategies, PBE