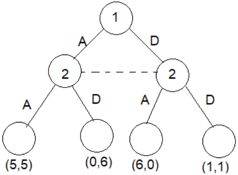
CS711: Introduction to Game Theory and Mechanism Design

Teacher: Swaprava Nath

Imperfect Information Extensive Form Games

Notation



Imperfect-information extensive form game Representation of Neighboring Kingdom Dilemma

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A	5,5	0,6
D	6,0	1,1

Normal Form Representation

Definition (Imperfect Information Extensive Form Game)

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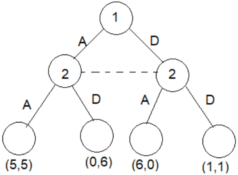

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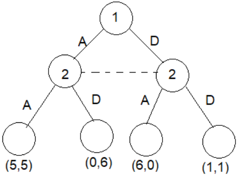

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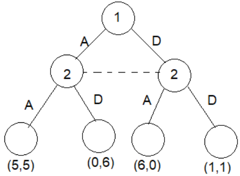

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IIEFG (contd.)

- set I_i for every player i, is a collection of information sets $I_i^j, j = 1, \ldots, k(i)$. Information sets are collection of histories where the player at that history is uncertain about which history has been reached.

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Definition (Strategy Set)

Strategy set of player $i, i \in N$ is defined as the Cartesian product of the actions available to player i at his information sets, i.e.,

$$S_i = \bigtimes_{\tilde{I} \in I_i} \mathcal{X}(\tilde{I}) = \bigtimes_{j=1}^{k(i)} \mathcal{X}(I_i^j).$$

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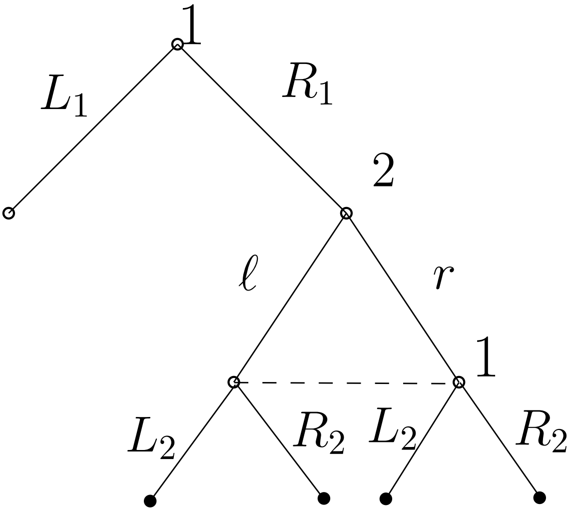
    - player can randomly pick the action at *an information set* – **behavioral strategies**

Example 1



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- mixed strategies look a "richer" or "larger" concept
- can a player attain higher payoff in one strategy than the other? – contrast the relation between Nash and correrated equilibria

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- equivalent strategies induce same probability of reaching a vertex

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Theorem (Utility Equivalence)

If a mixed strategy σ_i and a behavioral strategy b_i of a player i are equivalent, then for every mixed/behavioral strategy vector σ_{-i} of the other players and for every player $j \in N$

$$u_j(\sigma_i, \sigma_{-i}) = u_j(b_i, \sigma_{-i}).$$

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Corollary

Let $\sigma = (\sigma_i)_{i \in N}$ *be a mixed strategy profile. For each player* i, *let* b_i *be a behavioral strategy that is equivalent to* σ_i. *Let* $b = (b_i)_{i \in N}$. *Then for every* $i \in N$

$$u_i(\sigma) = u_i(b).$$

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- does equivalence always hold?

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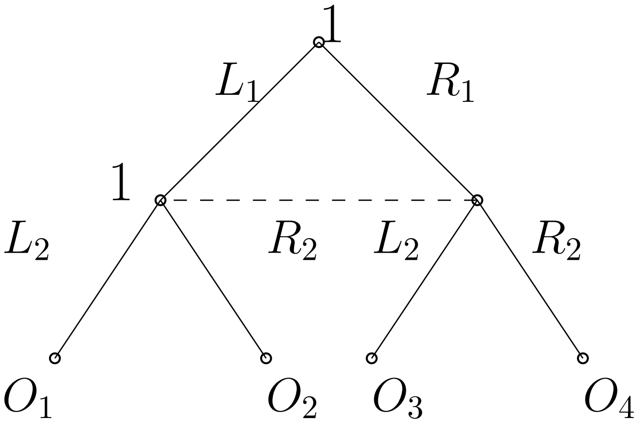

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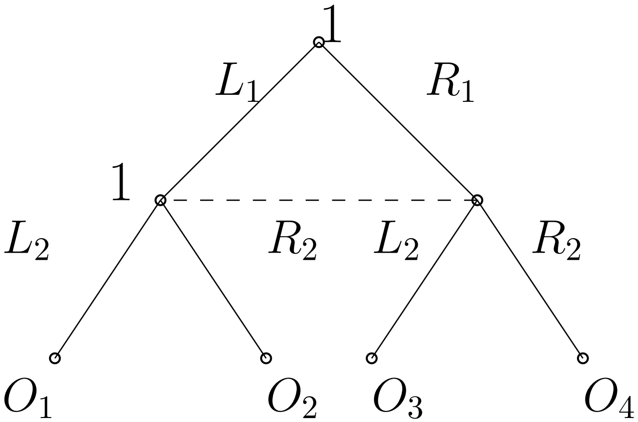
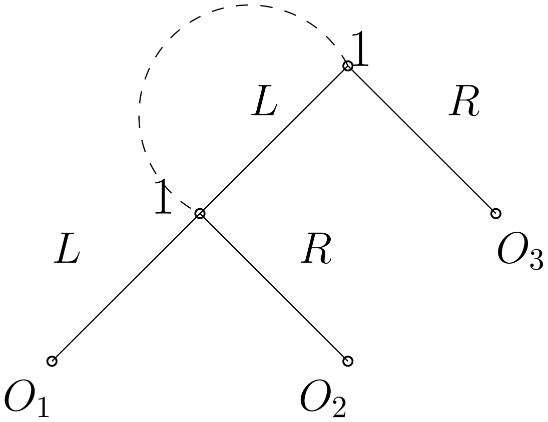


mixed strategy with no equivalent behavioral strategy

Example 3



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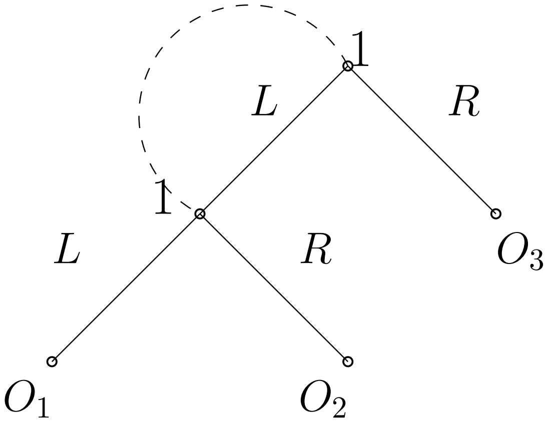

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- but behavioral strategies can randomize on every vertex of an information set independently, then x can be reached in a behavioral strategy with positive probability

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Theorem

Let $\langle N, A, \mathcal{H}, \mathcal{X}, P, (u_i)_{i \in N}, (I_i)_{i \in N} \rangle$ be an IIEFG such that at every vertex of the game tree there are at least two actions. Every behavioral strategy has an equivalent mixed strategy if and only if each information set of player i intersects every path emanating from the root at most once.

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- by the previous lemma, then this game has a behavioral strategy of player i that has no equivalent mixed strategy

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- this sequence of actions with their probabilities can be determined at the beginning of the game, hence it is an **equivalent** mixed strategy

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- **given:** each information set of player i intersects every path emanating from the root at most once
- **to show:** every behavioral strategy of player i has an equivalent mixed strategy
- let b_i be a behavioral strategy of player i, i.e., it gives a probability distribution over the actions at I_i^k as $b_i(I_i^k)$
- since every information set is cut at most once by any path from root to a node x, player i can pick the sequence of actions leading to x with a probability $=$ the product of all the probabilities given by the behavioral strategies at those information sets
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