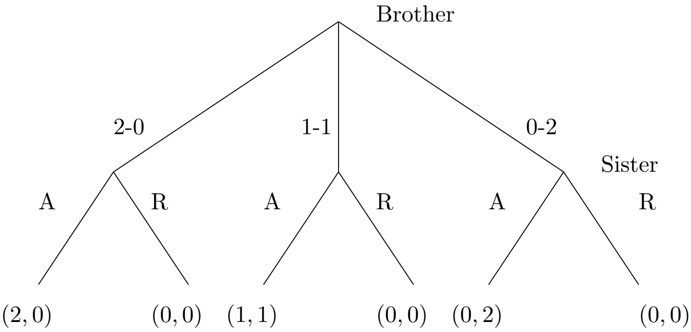
CS711: Introduction to Game Theory and Mechanism Design

Teacher: Swaprava Nath

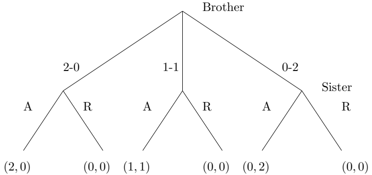
Extensive Form Games

Extensive Form Games

- **Chocolate Division Game:** Suppose a mother gives his elder son two (indivisible) chocolates to share between him and his younger sister. She also warns that if there is any dispute in the sharing, she will take the chocolates back and nobody will get anything. The brother can propose the following sharing options: (2-0): brother gets two, sister gets nothing, or (1-1): both gets one each, or (0-2): both chocolates to the sister. After the brother proposes the sharing, his sister may "Accept" the division or "Reject" it.



Representing the Chocolate Division Game



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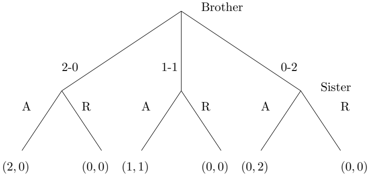

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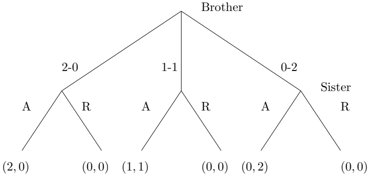

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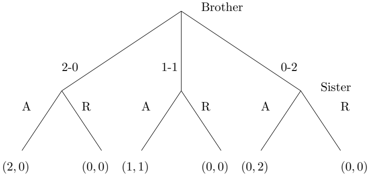

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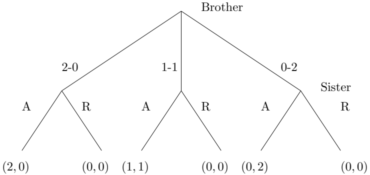

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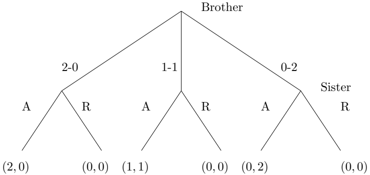

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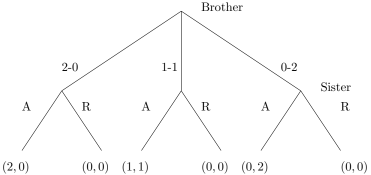

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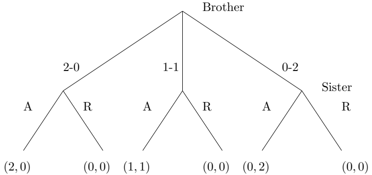

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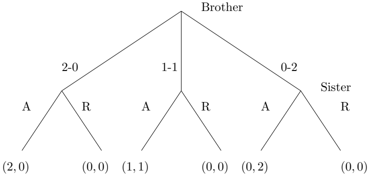

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$$S_1 = \{2-0, 1-1, 0-2\}$$
$$S_2 = \{A, R\} \times \{A, R\} \times \{A, R\} = \{AAA, AAR, ARA, ARR, RAA, RAR, RRA, RRR\}$$

Representing PIEFG as NFG

- Given S_1 and S_2, we can represent the game as an NFG, which can be written in the form of matrix.

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- For the given example, we can express the utility function as in the following table:

B \ S	AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
2-0	(2,0)	(2,0)	(2,0)	(2,0)	(0,0)	(0,0)	(0,0)	(0,0)
1-1	(1,1)	(1,1)	(0,0)	(0,0)	(1,1)	(1,1)	(0,0)	(0,0)
0-2	(0,2)	(0,0)	(0,2)	(0,0)	(0,2)	(0,0)	(0,2)	(0,0)

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0-2	(0,2)	(0,0)	(0,2)	(0,0)	(0,2)	(0,0)	(0,2)	(0,0)

- Observe that there are many PSNEs in the given game, some of which leads to quite nonintuitive solutions. The PSNEs are marked in **Bold**.

B \ S	AAA	AAR	ARA	ARR	RAA	RAR	RRA	RRR
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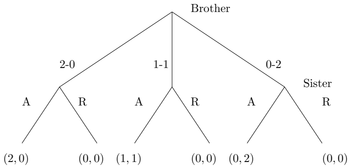

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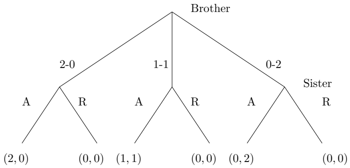

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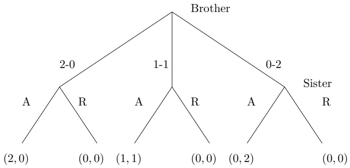

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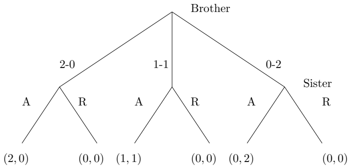

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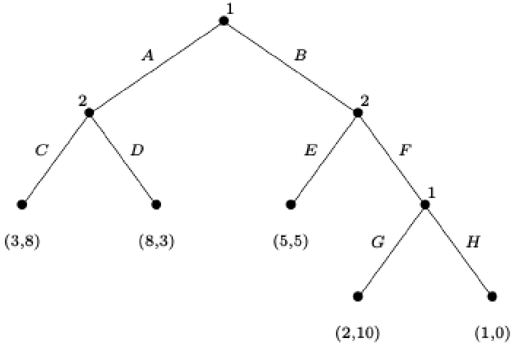
Theorem

Every finite PIEFG has a PSNE.

But PSNE is a Weak Guarantee

- Consider the PIEFG



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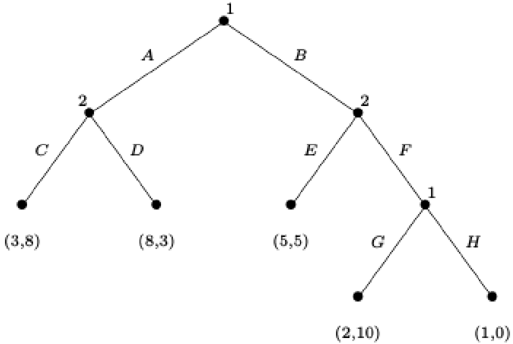

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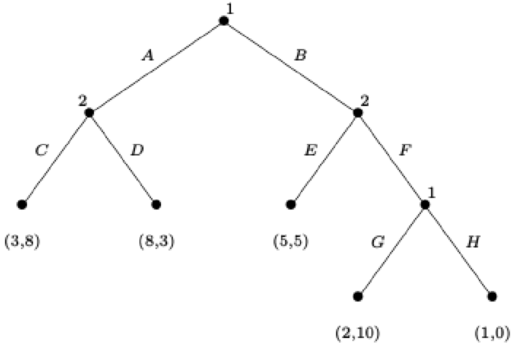

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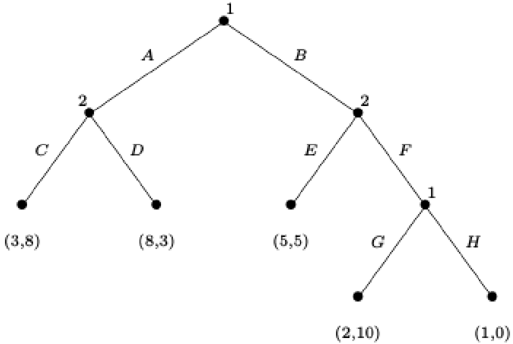

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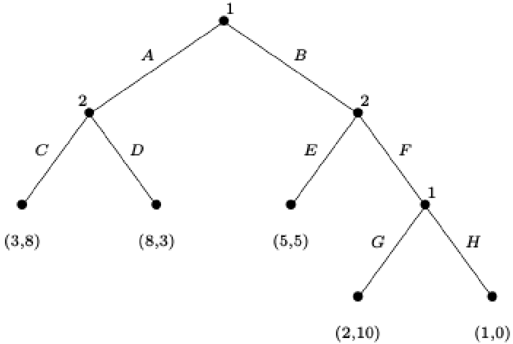

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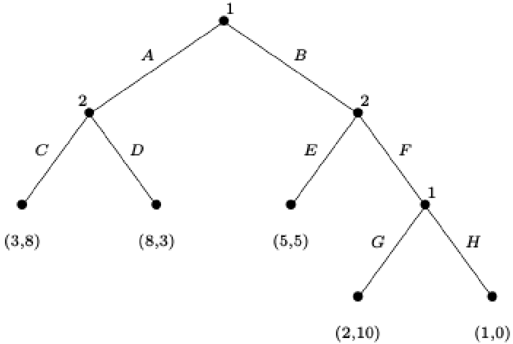

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- PSNEs: (AG,CF), (AH,CF), (BH,CE)
- **non-credible threat** again!
- better notion of rational outcome will be that considers a history and ensures utility maximization for the agent

Subgame and Subgame Perfection

- Subgame is a game rooted at a vertex

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Given a perfect-information extensive-form game G, the subgame of G rooted at node h is the restriction of G to the descendants of h. The set of subgames of G consists of all of subgames of G rooted at some node in G.

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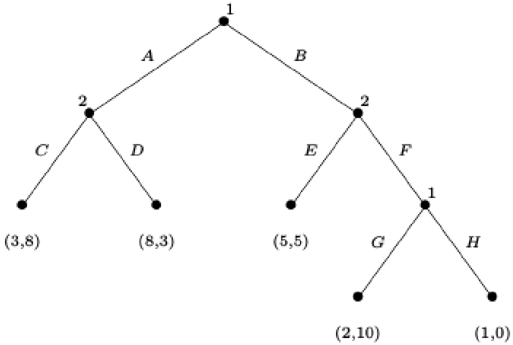
Definition (Subgame Perfect Nash Equilibrium)

The *subgame perfect Nash equilibrium* (SPNE) of a game G are all strategy profiles $s \in S := \bigtimes_{i \in N} S_i$ such that for any subgame G' of G the restriction of s to G' is a Nash equilibrium of G'.

Example

- Consider the PIEFG



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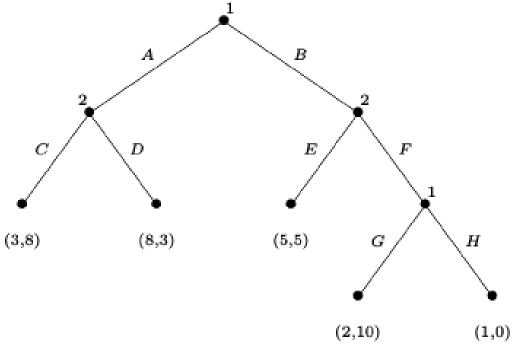

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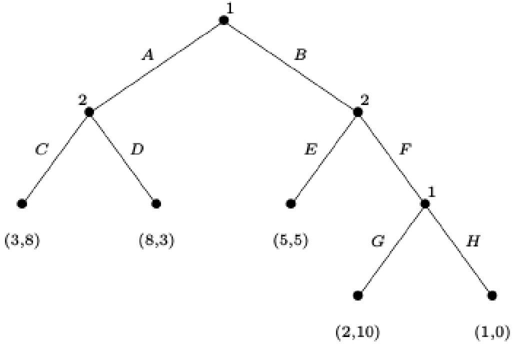

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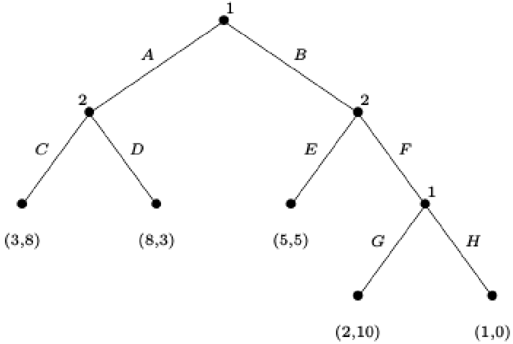

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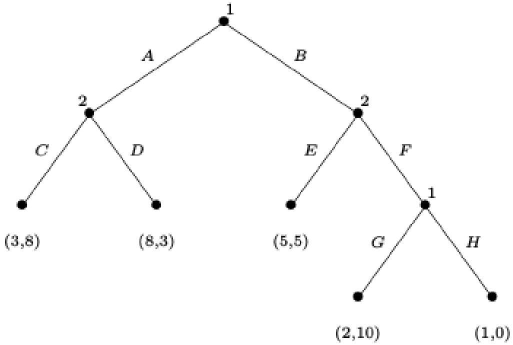

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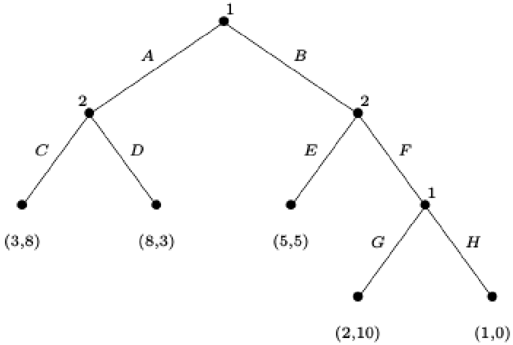

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- are they all SPNE?
- how to find them?

Computing SPNE: Backward Induction

- computation

```
function BackwardInduction (node h) returns u(h)
if h ∈ Z then
    return u(h)                                        // h is a terminal node
best_util ← −∞
forall a ∈ χ(h) do
    util_at_child ← BackwardInduction(σ(h, a))
    if util_at_child_ρ(h) > best_util_ρ(h) then
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- **good news:** not only are we guaranteed to find an SPNE, the algorithm is quite simple
- **bad news:** need to enumerate all possible vertices of the game tree – e.g., for chess, this is around 10^{150} vertices

Implications of Backward Induction and Criticisms

- SPNE: guaranteed to exist for finite PIEFGs (exercise)

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- SPNE: guaranteed to exist for finite PIEFGs (exercise)
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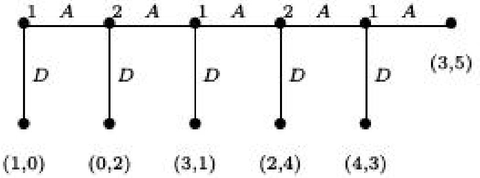
- **Limitations of SPNE:** Centipede game

Centipede Game

- In this game two players makes alternate decisions, at each turn choosing between going "down" and ending the game or going "across" and continuing it except at the last node where going "across" also ends the game. The payoffs are constructed in such a way that the player achieves higher payoffs by choosing "down".



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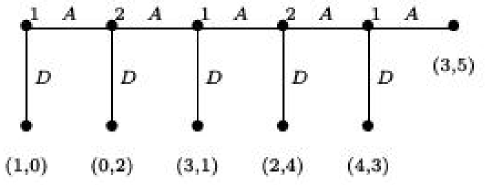

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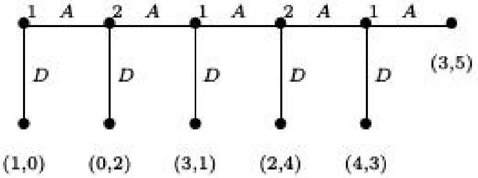
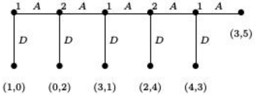


- SPNE of this game?
- what is the problem with that prediction?

Empirical Studies on Centipede



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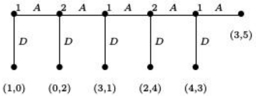

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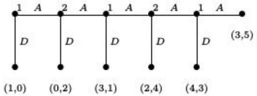

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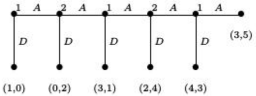

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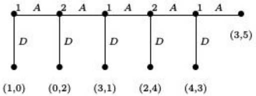

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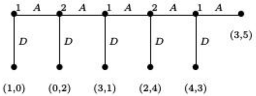

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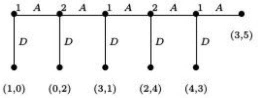

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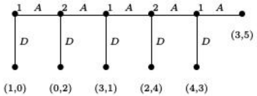

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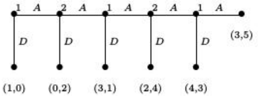

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- with a rising Elo, the probability of continuing the game declines
- all Grandmasters in the experiment stopped at their first chance

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- **Question:** represent neighboring kingdoms' dilemma with an EFG

A\B	Agriculture	Defense
Agriculture	5,5	0,6
Defense	6,0	1,1

Imperfect Information Extensive Form Games

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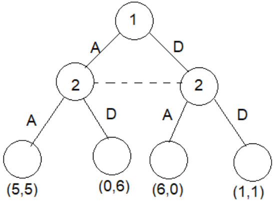
- We need the **Imperfect Information Extensive Form Games**

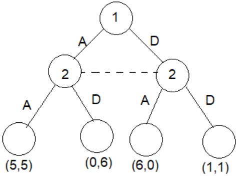


Imperfect-information extensive form game Representation of Neighboring Kingdom Dilemma

1 \ 2	A	D
A	5,5	0,6
D	6,0	1,1

Normal Form Representation

Notation



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Definition (Imperfect Information Extensive Form Game)

An *imperfect information extensive form game* is a tuple

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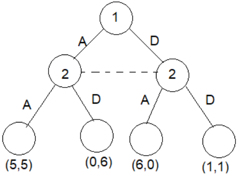

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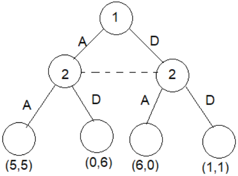

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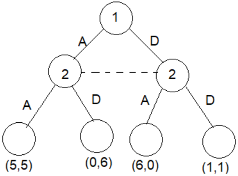

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IIEFG (contd.)

- set I_i for every player i, is a collection of information sets $I_i^j, j = 1, \ldots, k(i)$. Information sets are collection of histories where the player at that history is uncertain about which history has been reached.

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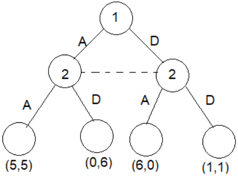
Definition (Strategy Set)

Strategy set of player $i, i \in N$ is defined as the Cartesian product of the actions available to player i at his information sets, i.e.,

$$S_i = \bigtimes_{\tilde{I} \in I_i} \mathcal{X}(\tilde{I}) = \bigtimes_{j=1}^{k(i)} \mathcal{X}(I_i^j).$$

Representations



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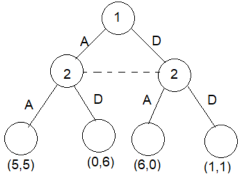

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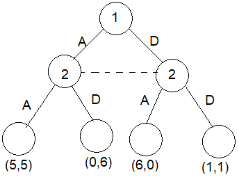

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- however every NFG can be transformed into an IIEFG, and vice versa – this is an equivalent representation, but clearly one representation is more appropriate for one setting than the other