CS711: Introduction to Game Theory and Mechanism Design

Teacher: Swaprava Nath

Introduction

Engineering Approach to Economic Theory

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  - **synthesis**
  - **prescriptive**

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Defect	0,-10	-5,-5

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"Personal greedy outcome may be far from what is socially optimal"

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Definition (Game Theory)

Game theory is the formal study of strategic interaction between decision making entities that are **rational** and **intelligent**.

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Objectives of game theory:

- provide **predictions** on the outcome
- find an **equilibrium** (stable point) of the game

Example 3: Fair Division

- One cake: two kids
- Mother decides how to divide the cake
- **Objective:** to ensure that each kid is happy with his/her portion

Fair Division (Contd.)

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- Kid 2 thinks she got at least half in her view
- The division is "fair" – envy-free
- Notions of 'at least half' is subjective
- If the mother knows that the kids see the division the same way as she does, the solution is simple – She can divide it and give to the children

Fair Division (Contd.)

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- Kid 1 thinks his piece is smaller than Kid 2's

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- What if Kid 1 has a different notion of equality than that of the mother
- Mother thinks she has divided it equally
- Kid 1 thinks his piece is smaller than Kid 2's
- **Difficulty:**
  - Mother wants to achieve a fair division
  - But does not have enough information to do this on her own
  - Does not know which division is fair

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- Mother thinks she has divided it equally
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- **Difficulty:**
  - Mother wants to achieve a fair division
  - But does not have enough information to do this on her own
  - Does not know which division is fair
- **Question:**
  - Can she design a mechanism under the incomplete knowledge that achieves fair division?

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- Kid 2 will pick the piece that is bigger in *her* eyes
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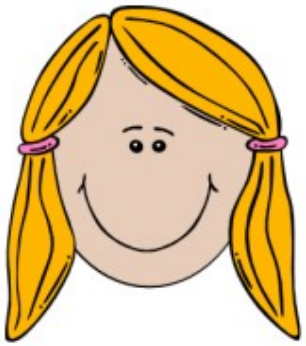
Example 4: Voting

Alice

Bob



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Example 4: Voting

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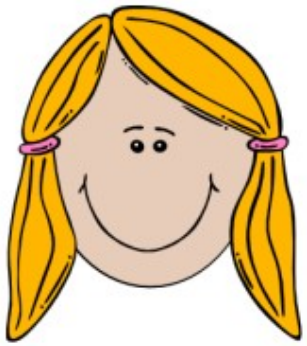

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7 voters

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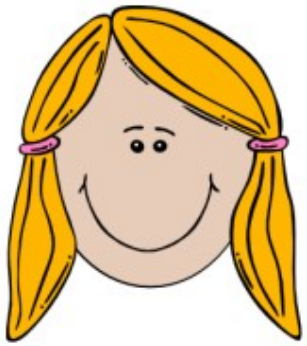

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7 voters

3 voters	2 voters	2 voters
$A \succ D \succ B \succ C$	$B \succ A \succ C \succ D$	$C \succ D \succ B \succ A$

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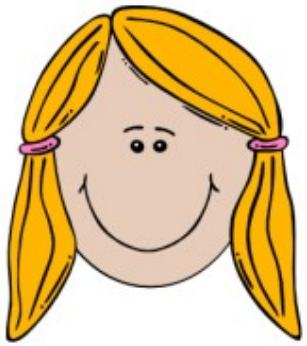

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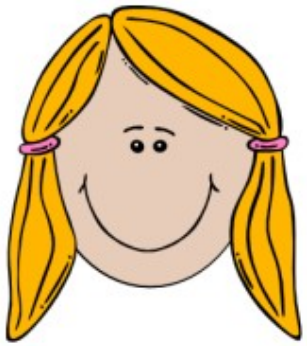

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And the winner is: A (plurality)

Voting (contd.)

3 voters:	$A \succ D \succ B \succ C$
2 voters:	$B \succ A \succ C \succ D$
2 voters:	$C \succ D \succ B \succ A$

- Give each of the voters a ballot
- Ask to pick one candidate
- Run the *plurality rule*

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- Notice the preferences of the last 2 voters
- They prefer B over A

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Perhaps the voting rule is flawed? 

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- How about a different voting rule
- Ask the voters to submit the whole preference profile
- Give scores to the each candidate $=$ number of pairwise elections won

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- **Copeland voting rule**

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Coincidence?

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Theorem (Gibbard 73, Satterthwaite 75): With unrestricted preferences and three or more distinct alternatives, no rank order voting system can be unanimous, truthful, and non-dictatorial

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- Other applications:
  - Sponsored search advertisements [Google, Facebook etc.]

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- Teams are put in groups – every team plays each other in the group, top 2 teams advance to knock-out stages
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- Similar incident: Olympic 2012, London, women's doubles badminton

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- **Mechanism design**
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  - **Social choice settings**
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Take aways from this class

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  - Understand the interplay between incentives and computation in the design of socio-economic systems
  - Develop applicable models of complex Internet systems
  - Analyze the behavior of systems that include people, computational agents, and firms, and involve strategic behavior
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- Make a deployable AI system that does this automatically
  - As a product or a deliverable for industrial applications – building systems that are guaranteed to perform
  - Research front: push the frontiers of research with the knowledge of current state-of-the-art

Expectations

- What you can expect from us
  - We will work hard to make this course useful for you (but we cannot do the work and learn the material for you)
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  - Attend classes regularly, come to class on time, and ask questions if something is unclear
  - Return the assigned tasks, e.g., assignments, scribe notes etc., on time
  - Adopt academic integrity (see: https://www.cse.iitk.ac.in/pages/AntiCheatingPolicy.html)
  - Have a positive attitude towards learning topics of this course

Logistics

Information:

- **Class times and venue:** Mon Thu 14.00 – 15.15, RM 101
- Instructor: Swaprava Nath, swaprava@cse.iitk.ac.in, send mail with [CS711] in the subject, or post on Piazza
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Reference text: No specific one. The following books could be helpful.

1. **Game Theory** – Michael Maschler, Eilon Solan, Shmuel Zamir (few copies of this book are available in the library)
2. **Multiagent Systems** – Y. Shoham and K. Leyton Brown, Cambridge University Press, online copy available
3. **Game Theory and Mechanism Design** – Y. Narahari, World Scientific and IISc Press – Indian edition available

Logistics (Contd.)

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Virtual classroom:

- Piazza: register yourself and post questions/clarifications there – check the course homepage for details

Thank you! Questions?