CS711: Introduction to Game Theory and Mechanism Design

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Introduction

Engineering Approach to Economic Theory

- Complementary studies: **analysis** and **synthesis**
- E.g., algorithms – analyze to find complexities, then design algorithm of given complexity for a new problem using the pattern
- **Scenario**: multiple **agents** with possibly conflicting objectives: a **game**
- Given game – find a probable outcome, best responses of the agents (players): **game theory**
  - **analysis**
  - **predictive**
- Given a *reasonable* outcome – find a game which gives that as a probable outcome: **mechanism design**
  - **synthesis**
  - **prescriptive**

Example 1: Neighboring Kingdoms' Dilemma

- Kingdoms **A** and **B** have limited options to invest
- Option one: save people of the kingdom from starvation – invest in agriculture
- Option two: save the kingdom from the attack of the other kingdom – invest in defense
- Outcome
  - *not* dependent only on the choice of individual **players**
  - dependent on the choice of **both** the players
- for two players the setup is illustrated by a **game matrix**

A**B**	Agriculture	Defense
Agriculture	5,5 $\mathbf{5, 5}$	0,60,**6**
Defense	6,0**6**,0	1,1**1**,1**1, 1** $\mathbf{1, 1}$

Assumption: preferences are representable using real numbers
Question: what should be a "reasonable" outcome of the above game?

Example 2: Prisoners' Dilemma

- Prisoners 1 and 2 are caught for a crime and are interrogated in separate chambers
- Interrogating officer explains the rules
  - if both confesses the crime – both get 5 years of jail
  - if both denies, some part of the charges still apply – each get 2 years of jail
  - if one confesses but the other denies, the crime will be proved – confessor goes free, denier gets 10 years of jail
- Available choices for the players: **cooperate** [plead not guilty] or **defect** [accept the charge]

Pris 1\Pris 2	Cooperate	Defect
Cooperate	-2,-2 $\mathbf{-2,-2}$	-10,0
Defect	0,-10	-5,-5 $\mathbf{-5,-5}$

"Personal greedy outcome may be far from what is socially optimal"

Formal definitions

Definition (Game)

A **game** is a formal representation of the strategic interaction between the multiple agents that are called **players**.
The choices that are available to the players are called **actions**.
The mapping of the state of the game to the actions is called a **strategy**.

Depending on the context, games can be represented in various ways

- Normal form games
- Extensive form games
- Repeated games
- Stochastic games
- ...

Definition (Game Theory)

Game theory is the formal study of strategic interaction between decision making entities that are **rational** and **intelligent**.

Rationality and Intelligence

Definition (Rationality)

A player is *rational* if she picks actions to **maximize** her utility

Definition (Intelligence)

A player is *intelligent* if she knows the rules of the game perfectly and pick an action considering that there are other rational and intelligent players in the game.

- Intelligence implies that the players have enough computational ability to find the **optimal** action

Objectives of game theory:

- provide **predictions** on the outcome
- find an **equilibrium** (stable point) of the game

Example 3: Fair Division

- One cake: two kids
- Mother decides how to divide the cake
- **Objective:** to ensure that each kid is happy with his/her portion

Fair Division (Contd.)

- Kid 1 thinks he got at least half in his view
- Kid 2 thinks she got at least half in her view
- The division is “fair” – envy-free
- Notions of ‘at least half’ is subjective
- If the mother knows that the kids see the division the same way as she does, the solution is simple – She can divide it and give to the children

Fair Division (Contd.)

- What if Kid 1 has a different notion of equality than that of the mother
- Mother thinks she has divided it equally
- Kid 1 thinks his piece is smaller than Kid 2's
- **Difficulty:**
  - Mother wants to achieve a fair division
  - But does not have enough information to do this on her own
  - Does not know which division is fair
- **Question:**
  - Can she design a mechanism under the incomplete knowledge that achieves fair division?

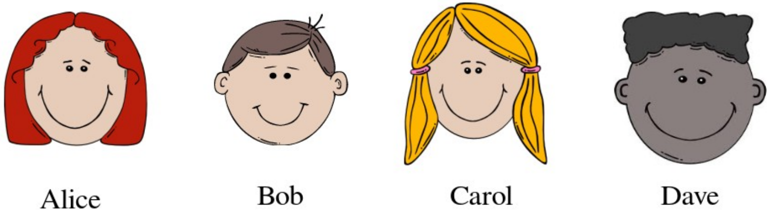
Fair Division: Solution

- Ask Kid 1 to divide the cake into two pieces
- Ask Kid 2 to pick her favorite piece

Why does this work?

- Kid 1 will divide it into two pieces which are equal in *his* eyes
  - Because if he does not, Kid 2 will pick the bigger piece
  - So, he is indifferent among the pieces
  - 
- Kid 2 will pick the piece that is bigger in *her* eyes
  - 

Example 4: Voting



7 voters

3 voters	2 voters	2 voters
$A \succ D \succ B \succ C$	$B \succ A \succ C \succ D$	$C \succ D \succ B \succ A$

And the winner is: A (plurality)

Voting (contd.)

3 voters: $A \succ D \succ B \succ C$
2 voters: $B \succ A \succ C \succ D$
2 voters: $C \succ D \succ BB \succ AAB \succ C \succ D \succ A$

- Give each of the voters a ballot
- Ask to pick one candidate
- Run the *plurality rule*
- A wins!
- Voters could be strategic
- Notice the preferences of the last 2 voters
- They prefer B over A
- Can manipulate to make B the winner

Perhaps the voting rule is flawed? 

Voting (contd.)

3 voters: $A \succ D \succ B \succ C$
2 voters: $B \succ A \succ C \succ D$$B \succ$ A $\succ C \succ D$$B \succ C \succ D \succ$ A
2 voters: $C \succ D \succ B \succ A$

- How about a different voting rule
- Ask the voters to submit the whole preference profile
- Give scores to the each candidate = number of pairwise elections won
- **Copeland voting rule**
- Assume a fixed tie-breaking rule $A \rightarrow B \rightarrow C \rightarrow D$
- Scores: A=2, B=2, C=1, D=1 – A wins!
- But the second group of voters prefer B over A
- Scores: A=0, B=2, C=2, D=2 – B wins!

Is it manipulable?

Coincidence?

3 voters: $A \succ D \succ B \succ C$
2 voters: $B \succ A \succ C \succ D$
2 voters: $C \succ D \succ B \succ A$

- **Question:** can we design any *truthful* voting scheme that *aggregates* all voters' opinions?
- **Answer:** No!


Allan Gibbard


Mark Satterthwaite

Theorem (Gibbard 73, Satterthwaite 75): With unrestricted preferences and three or more distinct alternatives, no rank order voting system can be unanimous, truthful, and non-dictatorial

Inverse Game Theory: Mechanism Design

- Objectives are to start with
- **Goal:** to design the game
- Such that the objectives are satisfied in **an** equilibrium of the game

Definition (Mechanism Design)

- A *mechanism* is a protocol of interaction between multiple agents.
- *Mechanism design* is a formal way to designing the protocol such that the desirable properties are satisfied in an equilibrium of the game induced by the protocol.
- The set of properties may be unsatisfiable simultaneously. In such a case, mechanism design formally argues the impossibility of the properties.

- The guarantees are **prescriptive**.
- Other applications:
  - Sponsored search advertisements [Google, Facebook etc.]

Why Design a Game?

- In sports: world cup football, cricket, and many more has round robin tournament, not in lawn tennis
- Teams are put in groups – every team plays each other in the group, top 2 teams advance to knock-out stages
- Is this a good tournament design? **No!**
- World Cup Football 1982, Group II
- Teams: Austria, Algeria, West Germany, Chile
- Game 1: Algeria beat West Germany 2-1 – a shock
- Game 2: Austria beat Algeria 2-0
- Game 3 Algeria beat Chile 3-2
- Algeria was going to be the first African team to qualify to the knockout stages
- Last match of the group: Austria vs West Germany
- West Germany needed to win to progress to the next round – it was anticipated that their chance was thin against mighty Austria
- After 10 minutes of furious attack, West Germany scored a goal
- Then both the teams stopped playing – **disgrace of Gijon**
- Similar incident: Olympic 2012, London, women's doubles badminton

Course Outline and Goals

- **Non-cooperative game theory**
  - **Complete information simultaneous move games**
  - **Complete information sequential move games**
  - **Incomplete information games**
- **Mechanism design**
  - **Social welfare settings**
  - **Social choice settings**
  - **Domain restrictions**
- **Applications of mechanism design**

Take aways from this class

- Apply principles of economics and computation to
  - Understand the interplay between incentives and computation in the design of socio-economic systems
  - Develop applicable models of complex Internet systems
  - Analyze the behavior of systems that include people, computational agents, and firms, and involve strategic behavior
  - Solve both mathematical and conceptual problems involving such systems, including problems you have not seen before
  - Write programs that implement strategic agents and mechanisms
- Build a taste for mathematical description of a social problem
  - The model and axioms of desirable properties and their interactions
  - Theorems and their proofs
  - Recognizing how the concepts and ideas in the course form a coherent framework for economics and computation
- Make a deployable AI system that does this automatically
  - As a product or a deliverable for industrial applications – building systems that are guaranteed to perform
  - Research front: push the frontiers of research with the knowledge of current state-of-the-art

Expectations

- What you can expect from us
  - We will work hard to make this course useful for you (but we cannot do the work and learn the material for you)
  - We will be available for assistance throughout the semester and look forward to meeting you in person
  - We will do our best to promptly answer your questions – via Piazza
  - We will listen to constructive comments and be open to suggestions
- What we expect from you
  - Attend classes regularly, come to class on time, and ask questions if something is unclear
  - Return the assigned tasks, e.g., assignments, scribe notes etc., on time
  - Adopt academic integrity (see: https://www.cse.iitk.ac.in/pages/AntiCheatingPolicy.html)
  - Have a positive attitude towards learning topics of this course

Logistics

Information:

- **Class times and venue:** Mon Thu 14.00 – 15.15, RM 101
- Instructor: Swaprava Nath, swaprava@cse.iitk.ac.in, send mail with [CS711] in the subject, or post on Piazza
- TAs: Garima Shakya, garima@cse.iitk.ac.in, Souradeep Chandra, souradeepc@cse.iitk.ac.in, Piazza will be better
- Course homepage: **https://swaprava.wordpress.com/cs711/**

Reference text: No specific one. The following books could be helpful.

1. **Game Theory** – Michael Maschler, Eilon Solan, Shmuel Zamir (few copies of this book are available in the library)
2. **Multiagent Systems** – Y. Shoham and K. Leyton Brown, Cambridge University Press, online copy available
3. **Game Theory and Mechanism Design** – Y. Narahari, World Scientific and IISc Press – Indian edition available

Logistics (Contd.)

Evaluation:

- Two tests – midterm and endterm (35% on each)
- Two assignments (15% on each) – solutions should be typeset in LaTeX (talk to me after class if you don't have exposure to LaTeX)
  - Collaboration is cool, but copying isn't
  - Mention each collaborators' name in the assignments – this does not affect your score in any way – but the solutions you write must be self-written

Virtual classroom:

- Piazza: register yourself and post questions/clarifications there – check the course homepage for details

Thank you! Questions?