CS-XXX: Game Theory and Collective Choice	Jul-Nov 2017
Lecture 31: October 25, 2017	
Lecturer: Swaprava Nath	*Scribe(s): Garima Shakya*

Disclaimer: *These notes aggregate content from several texts and have not been subjected to the usual scrutiny deserved by formal publications. If you find errors, please bring to the notice of the Instructor at swaprava@cse.iitk.ac.in.*

31.1 Recap

In the previous lecture, we discussed the advantages of VCG mechanism such as it is DSIC, never runs into deficit, never charges a losing agent and is individually rational for agents to partipate. It is the most-used mechanism with money, but it needs to be carefully applied and its good to know the limitations of VCG.

31.2 Criticism of VCG

31.2.1 Privacy:

As in VCG mechanism the agents have to report their valuations truthfully. If the interaction had to happen beyond one round, it may be preferable to use a mechanism that uses the minimal information needed for the current round and not have spillover effect. Because, if the agents know the valuations of other agents they can change their valuation in the next round such that they can get more utility. Another possibility is that, the auctioneer can introduce fake bidders to extract more from a rich bidder.

31.2.2 Susceptibility to collusion:

Let us take an example. As shown in the figure 31.1, by the VCG mechanism the player 1 and 2 should pay 150 and 50 respectively.

	A	B	Payment
1	200	0	150
2	100	0	50
3	0	250	0

Figure 31.1: P^{VCG} before manipulation

But, the player 1 and 2 can change the amount of payment they have to make by manipulating their valuation together, as shown in the figure 31.2.

	A	B	Payment
1	250	0	100
2	150	0	0
3	0	250	0

Figure 31.2: P^{VCG} after manipulation

Therefore, VCG is not group strategy proof.

31.2.3 Not frugal

VCG mechanism does not charge an amount close to the seller's valuation/cost. It guarateed to bring revenue or no deficit, but the payment could be very large. Let us take an example of shortest path, routing a packet or delivery of an item as shown in the figure 31.3.

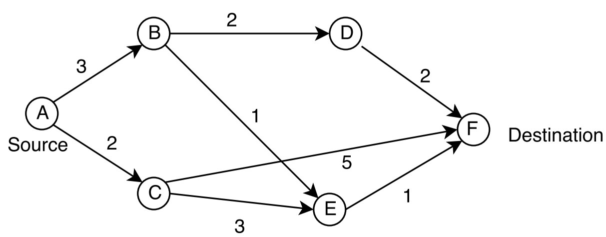

Figure 31.3: A network to deliver an item from A to F

Consider each edge as an agent. The efficient allocation for the shortest path is $A \to B \to E \to F$ with cost 4. Notice the effect of payment to $A \to B$ due to the cost of $A \to C$. If the cost of $A \to C$ is 2 then the payment of $A \to B$ is:

$$P^{AB} = \sum_{j \neq i} v_j(x^*_{-j}) - \sum_{i \in N} v_i(x^*) = 0 - (-1) + (-2) - 0 + (-3) - 0 = -4$$

Now, if cost of $A \to C$ becomes some $x > 2$, then $P^{AB} = -(x+2)$ and $A \to B$ will get more money than before but, the cost of it is still same as 3.

31.2.4 Revenue Monotonicity Violated

The system maintains the revenue monotonicity, if the revenue increases with the increase in number of agents. But, the VCG mechanism violates the revenue monotonicity. Let us take an example given in the figure 31.4.

	X	Y	P^{VCG}
1	0	90	0
2	100	0	90

Figure 31.4: The P^{VCG} before merging of agents

But, if a new agent participates then the P^{VCG} for all the agent changes and the result is shown in figure 31.5.

	X	Y	P^{VCG}
1	0	90	0
2	100	0	0
3	100	0	0

Figure 31.5: The P^{VCG} after merging of agents

the split or merging of agents is problematic.

31.2.5 Not fully Budget Balanced

VCG mechanism does not always satisfies the balanced budget, almost always some surplus will be left. This surplus can not be redistributed as that will change the payoff of the agents and it can not be invested to things that can effect the payoffs. Therefore, it has to be destroyed and known as *money burning.*

Caution: *This does not mean that the VCG mechanism is useless. Rather it is the most-used meachanism with money, but it needs to be carefully applied and it is good to know its limitations.*

31.3 Generalization of VCG

From the previous section we get to know about some of the limitations of VCG mechanism. Therefore, the class of mechanism is needed to be expanded to take care of multiple limitations by considering :
1.) Budget Balance issue.
2.) Equal weightage for every agent may not be required.

31.3.1 Affine maximizer allocation rule

- It is the superclass of VCG mechanism and is an efficient allocation.
- It can ask a Gibbard-Satterthwaite kind of question in the quasi-linear setting with public goods.

The allocation rule is:

$$f^{AM} \in \underset{a \in A}{\text{argmax}} \sum_{i \in N} \big(w_i \theta_i(a)\big) + \kappa(a)$$

where, $w_i \geq 0$, $\forall i \in N$ and not all zero.
and $\kappa : A \to \Re$

Definition 31.1 *An Affine maximizer rule with weights w_i, $i \in N$ and κ satisfies independence of non-influential agents (INA) if for all $i \in N$ with $w_i = 0$ we have:*

$$f(\theta_i, \theta_{-i}) = f(\theta_i^{'}, \theta_{-i}), \forall \theta_i, \theta_i^{'}, \theta_{-i}$$

Tie-breaking requirement: If a dictatorial rule has some value for two different alternatives and the tie is broken by the valuation of a non-dictatorial agent then, it violates INA.

Theorem 31.2 *An INA affine maximizer rule is implementable.*

Proof: Consider the payment

$$p_i^{AM} = \begin{cases} \frac{1}{w_i}\Big[h_i(\theta_{-i}) - \Big(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta))\Big)\Big] & \forall i : w_i > 0 \\ 0 & \forall i : w_i = 0 \end{cases}$$

The payoff for agent i, if $w_i > 0$,

$$= \theta_i(f^{AM}(\theta)) - p_i^{AM}(\theta_i, \theta_{-i})$$

$$= \frac{1}{w_i}\Big[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta)) - h_i(\theta_i)\Big]$$

$$\geq \frac{1}{w_i}\Big[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta_i^{'}, \theta_{-i})) + \kappa(f^{AM}(\theta_i^{'}, \theta_{-i})) - h_i(\theta_i)\Big]$$

$$= \theta_i(f^{AM}(\theta_i^{'}, \theta_{-i})) - \underbrace{\frac{1}{w_i}\Big[h_i(\theta_i) - \Big(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta_i^{'}, \theta_{-i})) + \kappa(f^{AM}(\theta_i^{'}, \theta_{-i}))\Big)\Big]}_{= p_i^{AM}(\theta_i^{'}, \theta_{-i})}$$

therefore,

$$\theta_i(f^{AM}(\theta)) - p_i^{AM}(\theta_i, \theta_{-i}) \geq \theta_i(f^{AM}(\theta_i^{'}, \theta_{-i})) - p_i^{AM}(\theta_i^{'}, \theta_{-i})$$

and for $i : w_i = 0$, due to INA,

$$f(\theta_i, \theta_{-i}) = f(\theta_i^{'}, \theta_{-i}), \forall \theta_i, \theta_i^{'}, \theta_{-i}$$

and payment is zero. So, the agent is weakly truthful. ■

Similar to GS theorem, we ask what if the valuations are unrestricted.

Theorem 31.3 (Roberts's 1979) *Let A be finite with $|A| \geq 3$. If the type space is unrestricted, then every onto and implementable allocation rule must be an affine maximizer.*

As before, restricting the space of valuations types gives us more mechanisms that are DSIC. For example, there are mechanisms other than VCG or Groves mechanisms that are truthful in auction settings.