CS-698W: Game Theory and Collective Choice **Jul-Nov 2017**

Lecture 29: Game Theory – An Introduction

Lecturer: Swaprava Nath *Scribe(s): Anil Kumar Gupta*

29.1 Recap

In the last lecture we studied about the VCG mechanism of allocation and payment. We made a claim that the payment for an agent that is not allocated any object in an auciton is zero and proved it. In this lecture we continue the analysis of VCG and make another claim.

29.2 VCG continued

29.2.1 Claim 2:

In the allocation of goods, VCG mechanism is idividually rational, i.e., the payoffs of every agent is non-negative.

Proof:

The payoff of an agent (using earlier notation of X and Y) $X \in \arg\max_{a\in A} \sum_{i\in N} v_i(a,\theta_i), Y \in \arg\max_{a\in A} \sum_{j\neq i} v_j(a,\theta_j)$

$$v_i(X,\theta_i) - \sum_{j\neq i} v_j(Y,\theta_j) + \sum_{j\neq i} v_j(X,\theta_j) \tag{29.1}$$

$$= \sum_{j\in N} v_j(X,\theta_j) - \sum_{j\neq i} v_j(Y,\theta_j) \text{ [add and subtract } v_i(Y,\theta_i)] \tag{29.2}$$

$$= \underbrace{\frac{(\sum_{j\in N} v_j(X,\theta_j) - \sum_{j\in N} v_j(Y,\theta_j))}{\geq 0}}_{\geq 0} + \underbrace{\frac{v_i(Y,\theta_i)}{\geq 0}}_{\geq 0} \geq 0 \tag{29.3}$$

This concludes the proof. ■

29.3 Application Domain : Internet Advertising

Internet advertising is delivering promotional marketing messages called Ads to consumers when a user searches something on a search engine or visits a website.

29.3.1 Advantages of Internet Advertising

Internet advertising is much better than the conventional advertising in news papers/articles or radio because of the following advantages.

1. **User Data:** Using Internet Ads, the advertiser can gain information about which set of buyers are interested in which target products as the interested users are very likely to click the Ads.
2. **Measurable Actions:** Classification of buyers is possible using Internet advertising. The buyers can be classified into a set of groups and be shown ads according to their interest.
3. **Low Latency:** Internet Advertising allows the auctions to happen on the fly. The Auction takes place just before the Ad is to be shown. For example in search engines like Google, bidding takes place after a keyword has been search and Ad for the company which wins the bid is shown. This also leaves allows any Customer buying Ads to enter any time and also enables automation of the whole process.

29.3.2 Types of Ads on Internet

1. **Sponsored Search Ads**
   In this type of Ads, the advertisers bid on the keywords that are entered by the user while searching on a search engine. The problems with this method and their solutions are.
   - Some words can have multiple meanings based on the context. Like Jaguar can mean a car or an animal. So to prevent the car Ads when the animal Jaguar is searched, stop words are used. Stop words are a set of words which if they are used with a particular word (here, Jaguar), the Ad (for car Jaguar)is not presented.
   - One user can search for the same keyword multiple times. So to prevent activity of a malicious user, to prevent draining of money from the Advertiser and prevent irritating the user with same Ad every time, a cap is set on number of times an Ad can be shown to a user.
2. **Contextual Ads**
   These type of Ads are presented to a user based on the context of the Web Page or the mail. For example, Gmail can present you certain type of Ad after reading your email.
3. **Display Ads**
   It is the classical way of displaying the Ads just like the banner Ads in the newspapers.

29.3.3 Position Auction

This type of Auction is used to sell multiple Ad slots in a webpage. The following set of notations are used.

$$N = \{1, 2, 3, \ldots, n\}, \text{set of bidders and } M = \{1, 2, 3, \ldots, m\} \text{ set of slots and } m \geq n$$

Here among the set of slots, 1 represents the best position and m represents the worst position.

Advertiser Value It is the valuation that is provided to the advertiser by ads om the Internet. The following simplifying assumptions are considered.

1. Clicks generate the value to the bidders

2. All the clicks are valued equally no matter which position it came from. This means that the role of position is only in the probability of being clicked. This assumption helps in decoupling the position effect from the value effect.

Hence the Expected value of Agent i, when her Ad is at position j is

$$v_{ij} = CTR_{ij} * v_i$$

where click through rate or CTR_{ij} is the probability bidder i gets clicked at position $j \in \{0, 1\}$ The CTR_{ij} can be decomposed into two terms, the quality of Ad of the advertiser i, CTR_i which is quality effect and the position of the Ad which is pos_j as

$$CTR_{ij} = CTR_i * pos_j, \quad CTR_i \in [0, 1], pos_j \in [0, 1]$$

29.3.4 Deciding the Auction Mechanism

The problem of deciding the what mechanism to use for allocation of resources(here the position at which an Ad is displayed) and what payment should be made by each advertiser. The following are the methods of auctions

1. **Early position Auction:** In this type of auction, the received bids are sorted in decreasing order of value and the positions are allotted according to the this order of value, the highest bidder receiving the first position and lowest bidder receiving last value. This type of bidding puts all risk on the advertiser
2. **Pay per click model:** In this mechanism, the price charged to the advertiser is proportional to the number of clicks the users made on the Ad. This mechanism puts all the risk on the website because a there could be an Ad that is at one of the top positions but does produce any revenue.
3. **Rank by expected revenue:** In this approach the advertisers are ranked by product of estimated CTR and the bid value of the advertiser. This mechanism shares the risk on advertiser as well as the website.

   We use the following notations :

   - $\boldsymbol{eCTR_i}$ **:** the value of the click through rate of the Ad i estimated by the search engine
   - $\boldsymbol{b_i}$ **:** the amount that agent i is willing to pay if a click occours
   - $\boldsymbol{x_i \in M}$ the position assigned to agent i s.t. $x_i \neq x_k, i, k \in N$
   - $\boldsymbol{x = (x_1, x_2, \ldots, x_n)}$ **:** Allocation of ads to positions

   Here we assume that the Ads are by decreasing order of their expected revenue that is decreasing order of the value of product $eCTR_i.b_i$. Suppose the agent i reports his bid as b_i Then

   $$\hat{v_i}(x) = pos_{x_i}(eCTR_i.b_i)$$

   is the reported value of the agent for the allocation x. This method of allocation is exactly same as the VCG mechanism of allocation.

   Detremining the Allocation:

   We want the allocation x^* to be such that it maximizes the total value recived by the advertizers i.e.

   $$x^* = \max_x \sum_{i \in N} \hat{v_i}(x)$$