CS698A: Selected Areas of Mechanism Design **Jan-Apr 2018**

Lecture 2: Cooperative Game Theory and Correlated Equilibrium

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2.1 Cooperative Game Theory

Cooperative game theory is a game paradigm where players can communicate with each other and "decide" on a joint strategy to be played. However, each player can break this strategy and unilaterally deviate while playing. Thus, the "decisions" are only binding if they are self-enforcing.

2.1.1 The traffic control game

		Player 2	
		Go	Stop
Player 1	Go	-10,-10	5,0
	Stop	0,5	0,0

Figure 2.1: The traffic control game

In the above example, both (Stop, Go) and (Go, Stop) are Nash equilibria. However, players do not know which particular equilibrium will be played, leading to the need of a trusted mediator. The mediator assigns a probability to each of the joint strategy profiles. It then randomly samples a joint strategy profile and suggests a strategy to the players in accordance with this joint profile. We formalize this notion below.

Definition 2.1 *Correlated strategy: A correlated strategy is a mapping* $\Pi : S \to [0,1]$ *where* S *is the set of joint strategy profiles* $S = S_1 \times S_2 \times \ldots S_n$ *where* S_i *represents the strategy space of the* i^{th} *player and* $\sum_{s \in S} \Pi(s) = 1$

While the players are free to accept/disregard the strategy suggested by the mediator, the mediator's role only makes sense if the players are not incentivized to deviate from the strategy suggested by the mediator, leading us to our next solution concept of **correlated equilibria**.

2.2 Correlated equilibria

Definition 2.2 *Correlated Equilibria: A correlated equilibria is a correlated strategy* Π *satisfying*

$$\sum_{s_{-i} \in S_{-i}} \Pi(s_i, s_{-i}) u_i(s_i, s_{-i}) \geq \sum_{s_{-i} \in S_{-i}} \Pi(s_i, s_{-i}) u_i(s_j, s_{-i}) \; \forall s_j \in S_i \; \forall s_{-i} \in S_{-i}$$

where s_{-i} is a joint strategy profile of all players apart from the i^{th}, S_{-i} is the set of all joint strategy profiles of all players apart from the i^{th} and u_i is the utility function of the i^{th} player.

2.2.1 Entertainment selection - An illustration of correlated equilibria

		Player 2	
		Cricket(C)	Football(F)
Player 1	Cricket(C)	2,1	0,0
	Football(F)	0,0	1,2

Figure 2.2: The entertainment selection game.

Consider the entertainment selection game mentioned above. Let us take a correlated strategy $\Pi(C, C) = 1/2$, $\Pi(F, F) = 1/2$, $\Pi(C, F) = 0$ and $\Pi(F, C) = 0$. Suppose, player 1 is suggested to play F. Let us look at the payoff of player 1 if it accepts the suggestion and if it rejects the suggestion.
$P_1(\text{Accept suggestion}) = \Pi(F, F)u_i(F, F) + \Pi(F, C)u_i(F, C) = (1/2)(1) + (0)(0) = 1/2$
$P_1(\text{Reject suggestion}) = \Pi(F, F)u_i(C, F) + \Pi(F, C)u_i(C, C) = (1/2)(0) + (0)(2) = 0$

The remaining cases can be analyzed similarly. The complete analysis gives us the conclusion that it is never profitable for a player to deviate from the suggested strategy, and hence, the correlated strategy is a correlated equilibrium.

An interesting point is that the expected payoff for the correlated equilibrium mentioned above for each of the players is 3/2 which is greater than the expected payoff of the mixed strategy nash equilibrium, which is 2/3. This clearly illustrates the advantage of a mediator in a cooperative game.

2.2.2 Best response and correlated equilibrium - a close relationship

Definition 2.3 *The best response set of i for a strategy s_i in a correlated strategy Π is defined as:*

$$B_i(\Pi, s_i) = argmax_{s_j \in S_i} \sum_{s_{-i} \in S_{-i}} \Pi(s_i, s_{-i})u_i(s_j, s_{-i})$$

Lemma 2.4 *A correlated strategy Π is a correlated equilibrium if $\forall s_i \in S_i\ \forall i \in N,\ s_i \in B_i(\Pi, s_i)$*

2.2.3 Computation of correlated equilibria

If there are n players in a game, with m strategies for each of the players, the computation of a correlated equilibrium reduces to a linear programming problem with $\mathcal{O}(nm^2)$ constraints, with $\mathcal{O}(m^2)$ constraints for each of the players. This is much faster than the mixed strategy nash equilibrium computation which takes $\mathcal{O}(2^{mn})$ time.

2.3 Axiomatic Bargaining

The axioms in axiomatic bargaining refer to the goals of the designer. Bargaining has the following characteristics:

- Two individuals have the possibility of concluding to a mutually beneficial agreement.
- There must be a conflict of interest on which agreement to choose or reject.
- Each individual should approve the agreement

2.3.1 Two agent bargaining problem

The two agent bargaining problem is denoted by (F,v) where

- F is the feasible set, $F \subseteq R^2$ and is a closed, convex subset of R^2
- v is the disagreement point, $v \in R^2$
- $F \cap \{(x_1, x_2) \in R^2 | x_1 \geq v_1, x_2 \geq v_2\} \neq \emptyset$