# Perceptual Account of Symbolic Reasoning

Rishav Raj Agarwal

Arpit Agarwal

Guide: Dr. Amitabha Mukerjee

## Abstract

Symbolic reasoning has been thought of as the ability to internally represent numbers, logical and mathematical rules in an abstract and amodal way. The focus has been on the **"inner"** i.e. notations are "translated" into corresponding mental representations. We believe that symbols may act as targets for powerful perceptual and sensorimotor systems as *Landy et al* propose in their: "Perceptual Manipulation Theory"[1]. To test the hypothesis, 2 experiments were designed.

**Keywords**: symbolic reasoning, mathematical cognition, embodied cognition, numerical reasoning, perceptual grouping, unwind strategy.

## Introduction

How does the humans do basic algebra, arithmetic and logical reasoning? The traditional understanding has been centred on the thought that the mathematical symbols are internally represented into abstract concepts. These amodal representation are then acted upon by the all-powerful mind centres dedicated for arithmetic and logical reasoning. Mathematical and especially algebraic reasoning is often taken to be the paradigmatic case of pure symbolic reasoning, and to rely for its successful execution on the use of internally available formal operations (Inhelder & Piaget, 1958).

On this traditional view "inner" is considered to be the ultimate authority taking precedence over any "outer" elements like background, notations used and computer screen. However, if one has to find that simple grouping pressures and background changes could affect the response time and even change the responses for certain subjects, the alternative way of reasoning takes ground. Symbolic reasoning involves the application of peripheral processes to notational structures themselves. Such reasoning requires notations on which to operate, and depends crucially on their physical instantiation and the processes that act on those instantiations (spatial perception, imagined motion, detection of action affordances, and so on). In that sense, such reasoning is modal.

This issue has special importance for understanding mathematical reasoning and learning. Although arithmetic notation may be the best-known example of a purely formal symbol system, arithmetic itself contains a variety of non-formal conventions that relate visual aspects of expressions to their formal structure.

## Perceptual Manipulations Theory

External symbolic notations need not be translated into internal representational structures, but neither does all mathematical reasoning occur by manipulating perceived notations on paper. Rather, complex visual and auditory processes such as affordance learning, perceptual pattern-matching and perceptual grouping of notational structures produce simplified representations of the mathematical problem, simplifying the task faced by the rest of the symbolic reasoning system. Perceptual processes exploit the typically well-designed features of physical notations to automatically reduce and simplify difficult, routine formal chores, and so are themselves constitutively involved in the capacity for symbolic reasoning. Moreover, if a particular symbolic reasoning problem cannot be solved by perceptual processing and active manipulation of physical notations alone, subjects often invoke detail-rich sensorimotor representations that closely resemble the physical notations in which that problem was originally encountered.

## Experiments 1

**Objective**: The experiment tests the effect of non-mathematical grouping pressures on algebraic and logical reasoning of basic algebraic equations.

### Important terms (Refer to Table 1 for Examples)

**Validity**: The equations is said to be valid if the left hand side(LHS) is equal to the right hand side(RHS) for all values possible real values of the variables used. For ex: a + b = b + a.

**Consistent**: The equations in which the grouping pressures correlates with the grouping according to the formal mathematical rules.

**Sensitivity**: When the (in) validity of the equations changes by exchanging the operators, the equations are said to be sensitive. E.g.

$$a + b * c + d = c + d * a + d$$

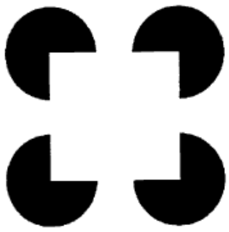

Figure 1 Grouping pressure Used

### Methodology

**Participants**. Thirty undergraduates participated in the experiment which were from the first year and third year. Having passed JEE, we assume that these students have the basic mathematical aptitude required for the experiment.

**Apparatus**. All expressions were presented in black text on a white background, using the Consolas font on Laptop. Monitor resolution was 1,336 * 768, and the monitor size was 15.6 inches. Participants sat approximately 50 cm from the monitors. The symbols had a font size of 70px and spacing of 1pt.

Participants used the keyboard to report validity judgments. The P and Q keys signified valid and invalid judgments, respectively.

**Design**. Our experiment was designed to orthogonally manipulate three factors: validity, consistency and sensitivity. We expected consistent equations to facilitate application of the correct multiplication-before-addition operator rule and inconsistent equations to promote application of an erroneous addition before- multiplication rule. Each participant viewed 20 test stimuli of which 5 were distractors. An individual stimulus consisted of a single symbolic equation. A response was a judgment of that equation's validity. Each stimulus equation consisted of two expressions (a left-hand side and a right hand side) separated by an equals sign. Each expression contained four symbols, connected by three operators. Although operators appeared in the same order on both sides of the equation, the operand order could differ on the left- and right-hand sides. These constraints held for all test equations. Each equation contained four unique symbols; due to their similarity to other symbols, the letters i, l, and o were omitted from the set of available letters. Distractors included different symbols on each side of the equation, division and subtraction, parentheses, and other complicated structures. The purpose of distractor equations was to discourage participants from solving problems using ad hoc shorthands or tricks based on the particular permutations, operator structures, and symbol constraints used in test equations.

Each question had a time limit of 6 seconds after which the stimuli would change automatically. There was a gap of 2 seconds between questions during which a blank screen is shown to the participants.

**Procedure**. Participants were asked to proceed quickly, without sacrificing accuracy; instructions also reminded participants of the order of operations and stepped through a sample arithmetic computation. There was a time restriction; equations remained visible for 600 ms. each stimuli was followed by a 200-ms delay, during which the screen was blank, and after which the next equation was displayed. Every participant received the equation set in the same order.

### Result

- After analysis of the first experiment the consistency was found to have impact on performance of UG students **as the students were able to score more on consistent than inconsistent equations. (Graph 1)**
- **Sensitivity** was found to have **varying** results depending on the values of the other parameters i.e. validity and consistency. (Graph 2)
- The consistent + insensitive equations were found to have lower score than consistent + sensitive equation. This is in contrast with the mechanisms thought for expert UG students according to the theory. (Graph 3)

However the results of the three parts can be substantially different in the case of high school students. This has to be tested.

## Experiment 2

**Objective**: The experiment was designed to test whether a moving background can be used to reveal the kinds of situations (if any) in which people utilize resources dedicated to processing motion, to manipulate mathematical expressions.

### Important concepts

Conceptually, there are (at least) two good strategies for solving such problems:

$$y \times 3 + 2 = 8$$

#### Algebraic strategy

In an algebraic solution, a reasoned constructs the solved equation shown in the expression, and then uses straightforward arithmetic to generate the answer.

$$y = \frac{8 - 2}{3}$$

**Unwind strategy**

In the unwind strategy, one starts by finding the isolated constant, identifies the next available operation on the variable side (+2 in this case), inverts the operation, and solves the resulting problem (8-2). One then uses this number as the starting point, identifies the next available operations on the left, and repeats.

$$y \times 3 + 2 = 8$$

$$y \times 3 = 8 - 2$$

$$y = \frac{8-2}{3}$$

Figure 2 Illustration of motion Strategy

## Methodology

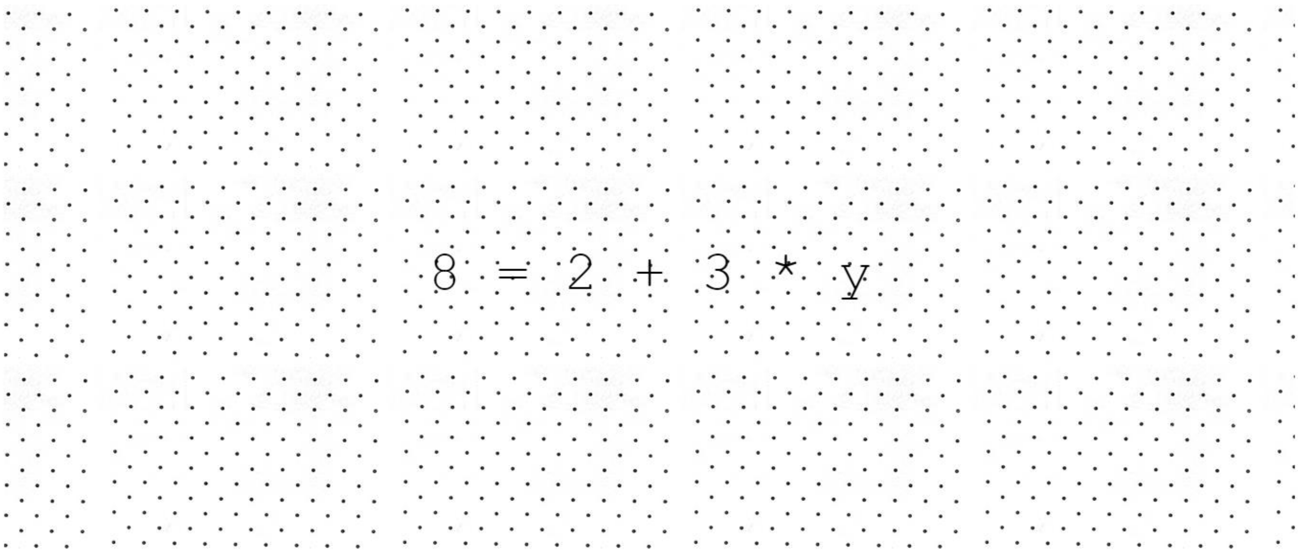


Figure 3 Screenshot of Expt. 2

**Participants**. Thirty undergraduates participated in the experiment which were from the first year and third year. Having passed JEE, we assume that these students have the basic mathematical aptitude required for the experiment.

**Apparatus**. All expressions were presented in black text on a white background, using the Consolas font on Laptop. Monitor resolution was 1,336 * 768, and the monitor size was 15.6 inches. Participants sat approximately 50 cm from the monitors. The symbols had a font size of 70px and spacing of 1pt. The background has is dotted and can move left to right or right to left.

Participants used the keyboard to report answers using the numpad.

**Design**.

The equations are of the "solve for y" type. We aim to test if background motion acts as stimulus to test whether unwind strategies are actually used during computation.

The first 10 questions move left to right and the last ten move from right to left. Although, the participants are not informed of this fact.

Each question had a time limit of 6 seconds after which the stimuli would change automatically. There was a gap of 2 seconds between questions during which a blank screen is shown to the participants.

**Procedure**. Participants were asked to proceed quickly, without sacrificing accuracy; instructions also reminded participants of the order of operations and stepped through a sample arithmetic computation. There was a time restriction; equations remained visible for 600 ms. each stimuli was followed by a 200-ms delay, during which the screen was blank, and after which the next equation was displayed. Every participant received the equation set in the same order.

**Results**

- The results are concurrent with the hypothesis that the expected score is in accordance with the movement. Thus, unwind strategies are used to complete algebraic tasks. (Graph 4)

However the results are supposed to be more prominent for the high school or $8^{th}$ grade students to whom the concepts are taught are recently taught.

Discussion

Perceptual and motor processing is central to symbolic reasoning. The problem as represented perceptually already differs substantially from the problem as it is presented notationally. Perceptual processes re-organize and simplify the symbolic problems we are faced with. On this view, the relevant perceptual processes are taken to be central components of the properly mathematical reasoning.

## Future work

- Experiment 2 will be conducted on novice and expert subjects to find out whether **perceptual stimuli** affects symbolic reasoning equally for both levels of mathematical expertise.
- The experiment 2 can also be conducted with different velocity and size level of the background dots.

## Acknowledgement

We would like to thank Dr. Amitabha Mukherjee for guiding us through this project. Dr. David Landy valuable suggestions helped us in designing experiments and result analysis. The campus community of IIT Kanpur is to be thanked for participating in our experiments.

## References

1. Landy, D., and Goldstone, R. L. (2007a). How abstract is symbolic thought? J. Exp. Psychol. Learn. Mem. Cogn.33, 720–733. doi: 10.1037/0278-7393.33.4.720
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3. Landy, D., and Goldstone, R. L. (2010). How much symbolic manipulation is just symbolic pushing?
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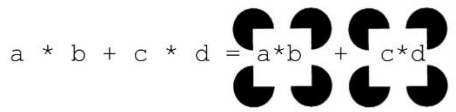
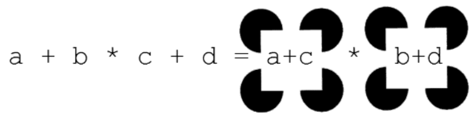
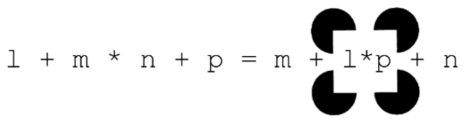
# Appendices

## Tables

Table 1: Permutations and Mathematical Properties of Right-Hand Side Orderings, for the Operator Structure Plus-Times-Plus

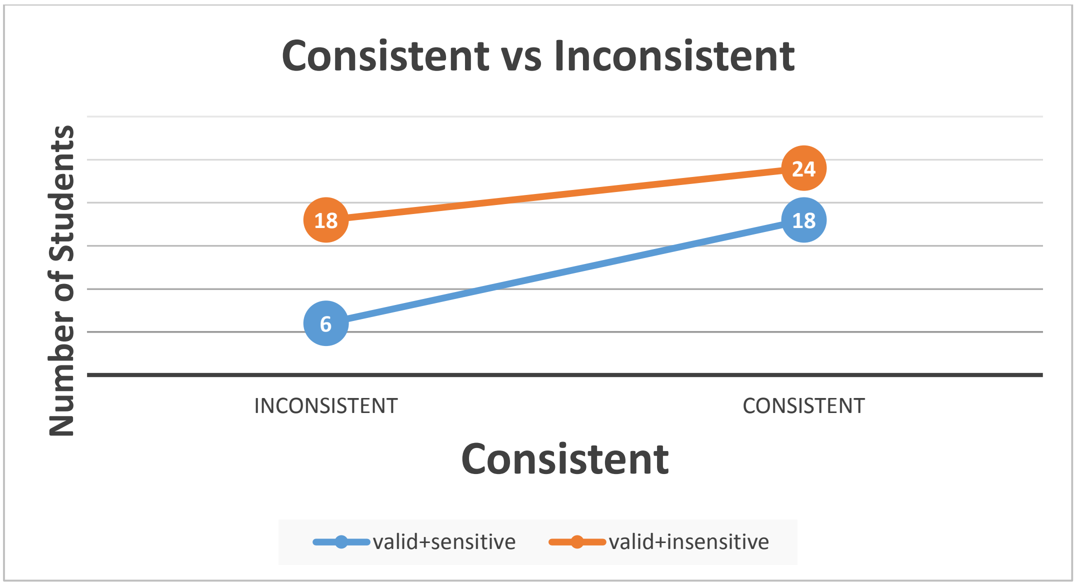
| **Permutation** | **Left-hand side** | **Right-hand side** | **Valid** | **Valid if + precedes *?** | **Sensitivity** |
|---|---|---|---|---|---|
| a b c d | a+b*c+d | =a+b*c+d | True | True | Insensitive |
| d c b a | a+b*c+d | =d+c*b+a | True | True | Insensitive |
| b c a d | a+b*c+d | =b+c*a+d | False | False | Insensitive |
| c a d b | a+b*c+d | =c+a*d+b | False | False | Insensitive |
| a c b d | a+b*c+d | =a+c*b+d | True | Flase | Sensitive |
| d b c a | a+b*c+d | =d+b*c+a | True | Flase | Sensitive |
| c d a b | a+b*c+d | =c+d*a+b | False | True | Sensitive |

Table 2: Examples of stimuli used. Implied oval-shaped regions embedded in the equations were used to create perceptual grouping.

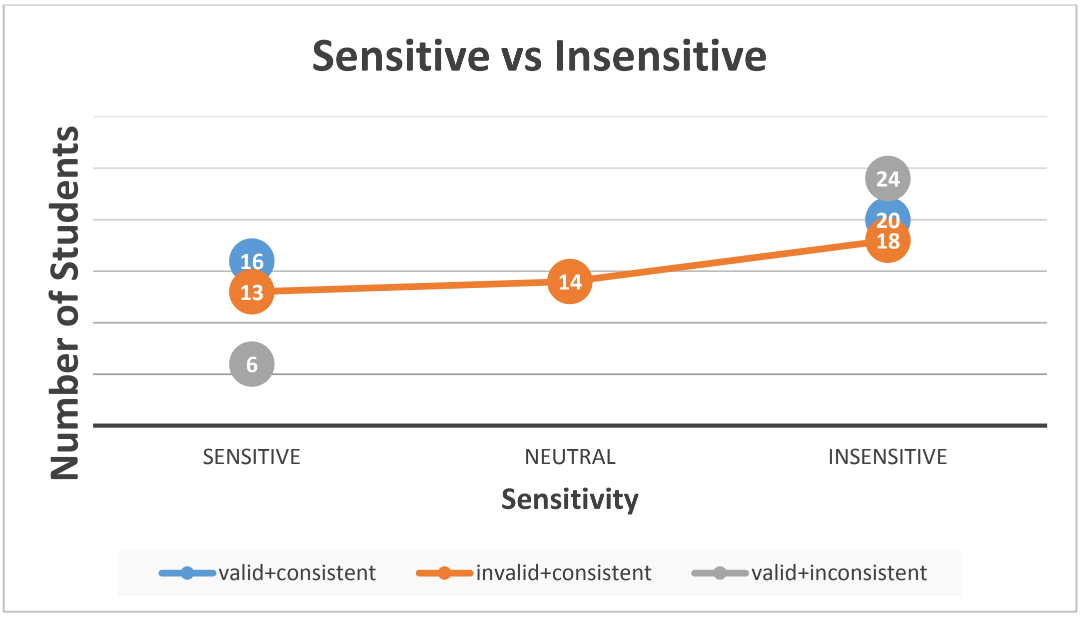
| **Permutation** | **Structure** | **Consistency** | **Validity** | **Example** |
|---|---|---|---|---|
| a b c d | * + * | Consistent | Valid |  a * b + c * d = a*b + c*d |
| a b c d | + * + | Inconsistent | Valid |  a + b * c + d = a+c * b+d |
| l m n p | + * + | Consistent | Invalid |  l + m * n + p = m + l*p + n |
| a b c d | + * + | Neutral | Invalid | a + b * c + d = e + b * c + d |
| h k u s | + * + | Neutral | Valid | h + k * u + s = h + k * u + s |

## Graphs

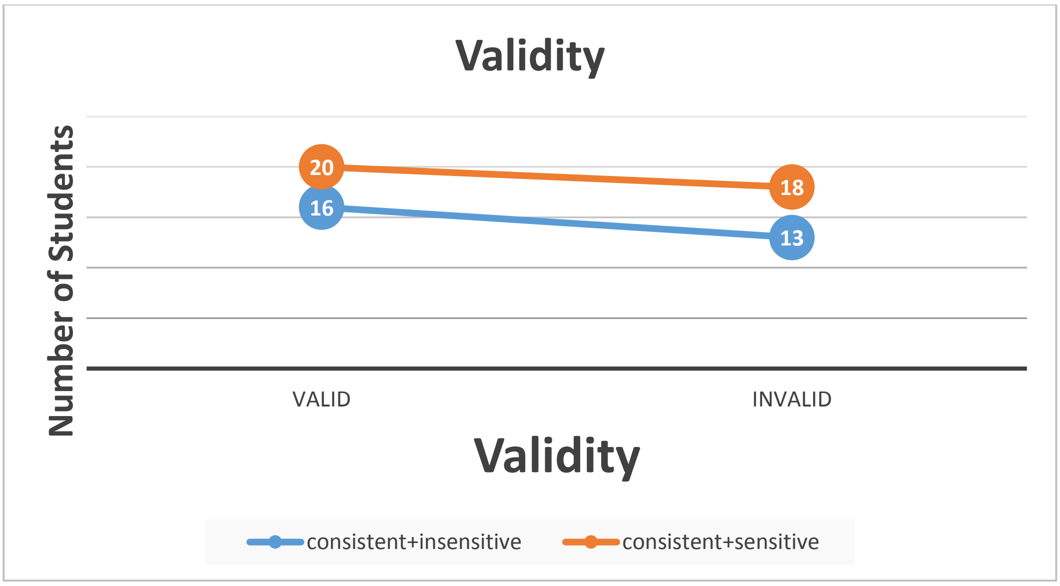
Graph 1



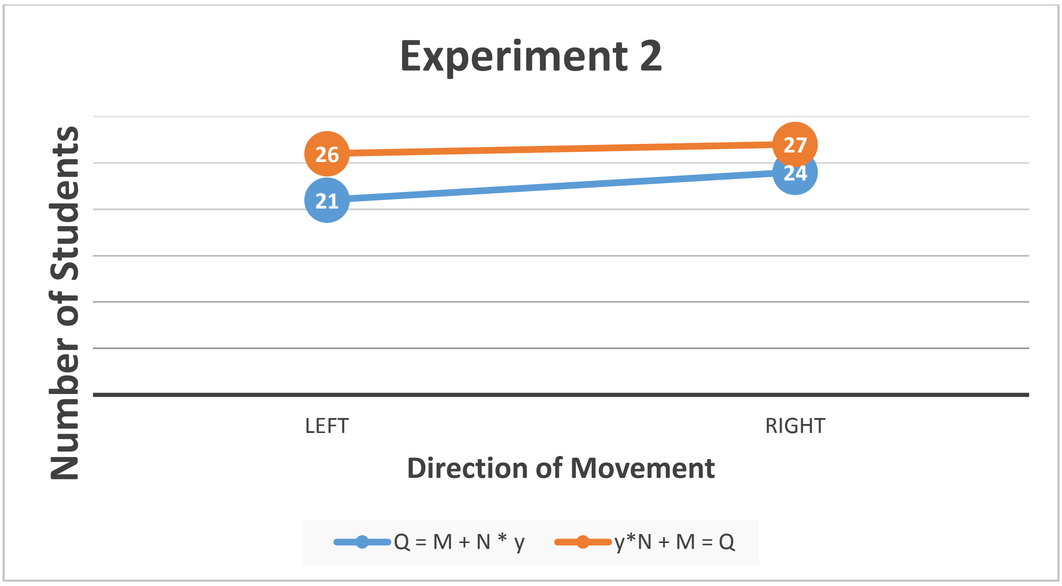

Graph 2




Graph 3


Validity
Number of Students
20
18
16
13
VALID
INVALID
Validity
consistent+insensitive
consistent+sensitive

Graph 4


Experiment 2
Number of Students
26
27
21
24
LEFT
RIGHT
Direction of Movement
Q = M + N * y
y*N + M = Q