Fractions and Division: As a predicate of Mathematical Achievement

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Whole Number Properties: A recollection

- Can be represented by a single symbol
- Have unique successors
- Are countable
- Never decrease on multiplication
- Never increase on division

Mathematical Understanding of Fractions

- During mathematical understanding of fractions, the central structure of the whole numbers, **the number line**, is extended to the rational numbers
- Coming to understanding that all numbers have magnitudes that can be assigned specific locations on the number line

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  - 4/3 no meaning as we can't have four parts of an object that is divided into three parts

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- Probably because of growing percentage of well paying jobs requiring mathematical proficiency

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- Learning about fractions requires children to recognize many properties of whole numbers that are not true of numbers in general and also recognize that they possess magnitudes which can be ordered on number lines

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- Fifth grade fraction knowledge predicts the mastery of algebra and overall mathematics achievement in high school even after controlling IQ, reading achievement, working memory, family income and education and whole number knowledge

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- Symbolic - competence with conventional representations
- Non-symbolic - competence with concrete stimuli, dots, proportion, areas etc

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  independently add numerator and denominator, shows lack of understanding that addition produces number greater than both the addends
- $1/3 * 2/3 = 2/3$
  lack of understanding that multiplication by a number less than 1 give a smaller number

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- Linguistic - one third v/s of three parts one

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- IPS is sensitive to distances between fractions and not to distances between numerator and denominator
- Recent studies have confirm to this point of view, showing that fractions can indeed be encoded by numerical values

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- *"fractions and early knowledge of division accounted for independent variance in later algebra knowledge and overall mathematics achievement indicated that neither relation explained the other"*
- *"the unique predictive value of early fractions and division knowledge seems to be due to many students not mastering fractions and division and to those operations being essential for more advanced mathematics, rather than simply to fractions and division being relatively difficult to master"*

Improving fraction education

Instruction should focus on magnitude of fractions integrating the conceptual and procedural understanding because the study of magnitudes is essential to understanding of fractions like wholes.

Questions

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References:

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