CS 687 2026: Homework 1

Due Date: February 13, 2026

1 Part 1: Preliminaries

1. Show that L and L^c are acceptable if and only if there is a Turing machine M which, for every string x, halts and accepts x if $x \in L$ and halts and rejects x if $x \notin L$. [10 points]

2. Show that every infinite computably enumerable language contains an infinite decidable subset. [10 points]

3. Call a set $A \subset \mathbb{N}$ *sparse* if for every $k \in \mathbb{N}$, among the first 2^k natural numbers, the set A contains at most k numbers.

   Consider the set of all sparse subsets of $\mathbb{N}$. Is this set countable or uncountable? Prove your claim. [10 points]