First Course Handout for CS687 2025-26 II

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1 Instructor details

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All announcements for the course will be made over email.

2 Overview and Learning outcomes

The course is a part of mathematical logic, especially the theory of computation. The content matter of this course can be viewed as an *application* of the theory of computation to the mathematical areas of information theory, and probability theory. This is unusual because in computer science, typically we apply mathematical tools to computer science, rather than the other way around. The content will emphasize proofs and rigor while conveying the intuition behind the subject.

The initial part of the course will focus on randomness of finite strings.

The initial lectures [8 lectures] will cover some of the basic techniques from set theory and computability theory needed for the course.

The course covers the motivations and basics of algorithmic information theory, namely, the notions of

1. Plain Kolmogorov Complexity [3 lectures]
2. Self-delimiting Kolmogorov Complexity [7 lectures]

Further, we will show that incompressible objects contain many properties which we expect from random objects. [4 lectures]

We will introduce some basic inequalities from classical information theory. [3 lectures]

The second half of the course will focus on the randomness of infinite objects. We will introduce the notion of Martin-Lof randomness using constructive measure [5 lectures] and cover the important properties of symmetry of relative randomness, i.e. van Lambalgen Theorem [5 lectures] and the Kucera-Gacs theorem [5 lectures].

3 Grading Policy

1. Homeworks - 2 (weightage 2x10 = 20 percent)
2. Quizzes - 2 (weightage 2x10 = 20 percent)
3. Midsem - 30 percent
4. Endsem - 30 percent.

The homeworks will be individual effort. Quizzes, Midsem and Endsem will be in-class (i.e. the exams will be held during the institute's midsem and endsem slots, in the lecture halls, and the quizzes will be held in separate 1 hour slots during the semester).

4 Attendance policy

Attendance is recommended.

5 Academic Integrity

Please refer to the CSE department guidelines on academic ethics (`https://www.cse.iitk.ac.in/pages/AntiCheatingPolicy.html`).

6 Makeup and late submission policy

Late submissions will incur a penalty of 10 percent up to 1 week after the deadline. After that, submissions will no longer be accepted.

Makeup exams will be provided only according to the institute norms.

7 Lecture Notes

1. Set theoretic and Computability Theoretic Preliminaries
2. Plain Kolmogorov Complexity
3. Self-delimiting Kolmogorov Complexity
4. Applications to probability theory
5. Martin-Löf randomness and properties of ML-randoms: van Lambalgen theorem and the Kučera-Gàcs theorem

8 Textbooks

Lecture notes will be provided. The following textbooks may be referred.

1. Li, Vitanyi. An Introduction to Kolmogorov Complexity and its Applications, 4th Edition, Springer, 2019.

1. Downey, Hirschfeldt. Algorithmic Randomness and Complexity, Springer, 2010.
2. Nies, Computability and Randomness, Oxford, 2009.
3. Shen, Uspensky, Vereschagin, "Kolmogorov Complexity and Algorithmic Randomness", AMS, 2017.