Application of Kolmogorov Complexity to Probability Theory.

A proof of Weak Law of Large Numbers using KC.

One version of WLLN is:

If $X_1, X_2, \dots,$ is an infinite sequence of independent, identically distributed random variables such that $\mathbb{E}X_1 < \infty$ (hence $\mathbb{E}X_2, \mathbb{E}X_3 \dots < \infty$)

then

$$\lim_{n\to\infty} \underbrace{\frac{X_1 + X_2 + \cdots + X_n}{n}}_{\text{"Sample mean"}} = \mathbb{E}X_1 .$$

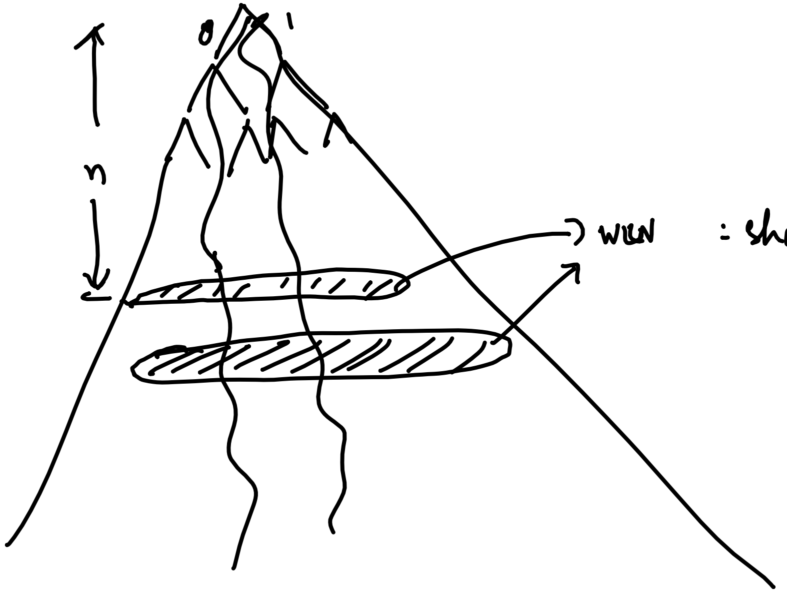
(Jakob Bernoulli, 1713)

A simple proof using the assumption that $\mathbb{E}X^2 < \infty$ is essentially Chebyshev's inequality.

We now show a proof using KC. (Gács, Ann. Prob, 1987).

(Borel, 1909, Cantelli, 1913)

Difference b/w WLLN & SLLN.



→ WLLN : shaded paths have ½ 0s nearly

SLLN : $P\left\{X \in \{0,1\}^{\infty} \;\middle|\; \lim_{n\to\infty} \frac{X_1+\dots+X_n}{2} = \mathbb{E}X_1\right\} = 1.$