Encoding $e: \Sigma^* \to P$, where P is a prefix-free set. (prefix set)

Then $e(x,y) = e(x) \cdot e(y)$.

We can select $e(x) = bd(|x|)\, 01\, x$ — this is a prefix code.

$$|e(x)| \le 2 \log(|x|) + |x| + O(1).$$

Theorem There is a universal partial computable function $\psi: \Sigma^* \dashrightarrow \Sigma^*$ with prefix free domain such that for any partial computable function $\varphi: \Sigma^* \dashrightarrow \Sigma^*$ with prefix-free domain $\exists\, c_\varphi$ $\forall x \in \Sigma^*$, $y \in \Sigma^*$

$$C_\psi(x|y) \le C_\varphi(x|y) + c_\varphi.$$

Assume that we restrict to partial comp. fns. with prefix free domains. & fix ψ as our universal partial comp fn with prefix domain.

We call

$$C_\psi(x|y) = K(x|y)$$

(self delimiting Kolmogorov complexity)

$$C_\psi(x|\lambda) = K(x).$$

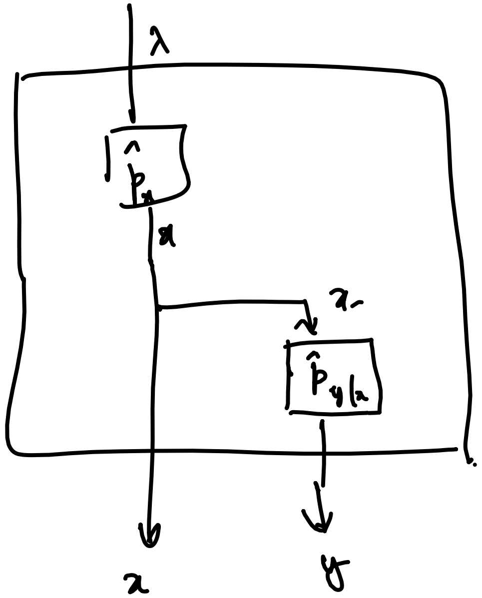
(1) K is subadditive

$$\underset{\hat{p}(x)\hat{p}(y)}{K(x,y)} \leq \underbrace{K(x)}_{\hat{p}_x} + \underbrace{K(y)}_{\hat{p}_y} + O(1)$$

$$K(x,y) \le K(x) + \boxed{K(y|x)} + O(1)$$

by $\hat{p}_x$ $\hat{p}_{y|x}$

concatenation



$$K(y|x) \le K(y)$$

similar to

$$H(X,Y) = H(X) + H(Y|X)$$ for Shannon entropy.

How bad is K wrt C?

K is in general greater than C.

$$C(x|y) \le K(x|y) + O(1).$$

$$K(x|y) \le C(x|y) + 2\log C(x,y) + O(1).$$

Let $p_{x|y}$ be the shortest program producing x from y.

$$|e(p_{x|y})| \le 2\log(|p_{x|y}|) + 2 + |p_{x|y}|$$

$$= 2\log C(x|y) + C(x|y) + O(1).$$

Goal Is the following true, in order to make K behave like entropy:

$\forall x, y \in \Sigma^*$ $$K(x, y) \geq K(x) + K(y|x) + O(1) \;?$$