How many strings are incompressible in Σ^n?

<u>Lemma</u> The number of strings of length n with complexity at most $n-c$, is at most $2^{n-c+1}-1$.

2^{n-c}

<u>Proof</u>

The number of programs of length $\leq n-c$ is at most $2^{n-c+1}-1$. Each such program outputs at most one string. The total number of distinct outputs is $\leq 2^{n-c+1}-1$.

$\square$

Lemma

For every large n, $\exists$ a string of length n with complexity $> n$.

$\exists N_0 \ \forall n > N_0$: for all large enough n

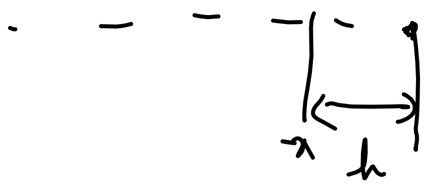
$\forall N_0 \ \exists n > N_0$: for infinitely many n.

$x+h$ for strings x & integers h.

Lemma (Landmark lemma)

There is a constant c such that $\forall x\in\Sigma^* \ \forall h\in\mathbb{N}$

$$\left| C(x+h) - C(x) \right| \le 2|h| + c$$



Proof idea

Let p_x be the shortest program for x. So $\Psi(p_x, \lambda) = x$.

$x+h$ can be produced by applying to the tuple $\langle h, p_x \rangle$, the function

$$F(\langle a, b\rangle) = a + \Psi(b).$$