Naïve Set Theory

union, intersection, relative complement, absolute complement

$A \setminus B = \{a \mid a \in A,\ a \notin B\}$ ↓

$A^c = \{u \mid u \in \mathcal{U},\ u \notin A\}$

$\mathcal{U}$ is the universal set.

De Morgan's Laws:

$$(A \cap B)^c = A^c \cup B^c$$

& $$(A \cup B)^c = A^c \cap B^c$$

$\cup, \cap$ are "duals" of each other.

Power Set:

If A is a set, then its power set $\mathcal{P}(A) = \{B \mid B \subseteq A\}$.

Ordered Pair → Cartesian Prod → Relations → Functions → Cardinality

Wrong representation

(a,b) (a,a)

$(a, b) = \{\{a\}, \{\{a\}, b\}\}$. → $\{\{a\}, \{a,b\}\}$

$(a,a) = \{\{a\}, \{\{a\}, a\}\}$. $\{\{a\}, \{a,a\}\}$

$\{\{\{a\}\}, \{\{a\}, b\}\}$ $= \{\{a\}, \{a\}\}$

$= \{\{a\}\}$

$(a_1, a_2, \ldots, a_n) = (a_1, (a_2, \ldots, a_n))$

$A \times B = \{ (a, b) \mid a \in A, \ b \in B \}$

a relation R between A & B is a subset of $A \times B$.

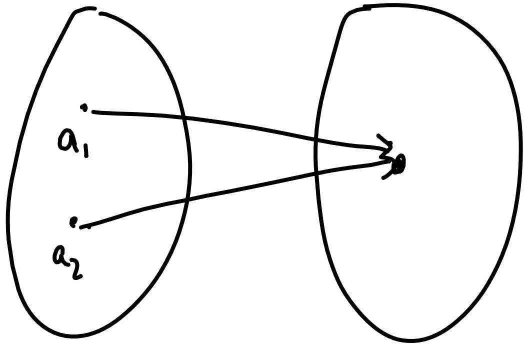
a function $f : A \to B$ is a relation R b/w A & B such that

(1) Every element $a \in A$ appears as the 1st coordinate in some element of R
[total]

(2) $\forall a \in A$ $\forall b, c \in B$ such that $(a,b), (a,c) \in f$ then $b = c$.
[unique image] We can then write $f(a) = b$.

Inverse

$$f^{-1}(b) = \{a \in A \mid f(a) = b\}.$$



Injective (One-to-one) functions

$f: A \to B$ is injective, written $f: A \xrightarrow{1-1} B$ if $\forall a_1, a_2 \in A$

$f(a_1) = f(a_2)$ implies $a_1 = a_2$.

Surjective (onto) functions

$f: A \to B$ is surjective, written $f: A \xrightarrow[\text{onto}]{} B$ if

$\forall b \in B$, $\exists a \in A$ such that $f(a) = b$ $\quad [f^{-1}(b) \neq \emptyset]$.

$f: A \to B$ is a bijection, written $f: A \xrightarrow[\text{onto}]{1\text{-}1} B$, if f is injective & surjective.

Note: For finite sets A & B, if $\exists f: A \xrightarrow[\text{onto}]{1\text{-}1} B$, then A & B have the same number of elements.

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$

A set A is infinite if (and only if) $\exists f: \mathbb{N} \xrightarrow{1\text{-}1} A$

$$\boxed{\sum_{i=1}^{\infty} 1/n^2}$$

A set A is said to have smaller cardinality than a set B if

$\exists f: A \xrightarrow{1-1} B$; written $|A| \leq B$.

- The sets A & B have the same cardinality if

$\exists f_1 : A \xrightarrow{1-1} B$ and $\exists f_2 : B \xrightarrow{1-1} A$.

Theorem [Cantor, Schröder, Bernstein]

A & B have equal cardinality if $\exists g: A \xrightarrow[\text{onto}]{1-1} B$