CS 687 2018-19 Homework 2

Due: April 17, 2019

1. Let $f : \Re \to \Re$ be a function, and let $X : \Omega \to \Re$ be a random variable having finitely many values.
   a. Show that $H(f(X)) \leq H(X)$.
   b. State and prove a necessary and sufficient condition for $H(f(X)) = H(X)$.
2. Let $X : \Omega \to \Re$ be a random variable taking finitely many values. Show that $H(Y|X) = 0$ iff Y is a function of X.
3. Let X, Y, Z be random variables which can assume finitely many values.
   a. Prove that $I(X;Y) = 0$ if and only if X and Y are pairwise independent.
   b. Suppose someone proposes a definition for a ternary mutual information as follows.

$$I(X;Y;Z) = I(X;Y) - I(X;Y|Z) = H(X) - H(X|Y) - [H(X|Z) - H(X|Y,Z)].$$

   Construct three random variables X, Y and Z where this quantity is negative.
4. Suppose d is a positive real-valued lower semicomputable function. If

$$\sum \frac{2^{d(x)}}{2^{|x|}} \leq 1,$$

   then show that d is a Martin-Löf test.