

CS 350 2022 Homework 2

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October 14, 2022

1 Instructions

Due date: October 25, 2022

2 Questions

1. Derive the list of free variables in the following λ term. Outline your derivation according to the rules given in the notes.

$(\lambda x.y(xx))(\lambda y.x(yy))(\lambda z.y)$

2. Evaluate the following λ expressions using α and β reduction rules to obtain the normal form. Please stop the reduction when you first obtain the normal form.

(a) $(\lambda ab \cdot ba)ab$

(b) $(\lambda x \cdot xx)(\lambda a \cdot a)$.

(c) $(\lambda x \cdot xx)(\lambda x \cdot xx)$.

3. Construct a λ term that does not have a normal form - *i.e.* construct a term which can always be β reduced further. Explain why this term has this property in one or two sentences.
4. Based on the Church representation of Boolean values given in the notes, define the λ term which computes the "or" of Boolean values - *i.e.* a term which takes two arguments, and evaluates to the Boolean representation of True if either of them is True, and to False if both of them are False.
5. What is the set of fixed points of the λ term $(\lambda x \cdot x)$?

6. Consider an enriched λ calculus which has natural numbers available, has a normal if-then-else construct, and has the operators $+$, $-$ and $==$. Using the Y-combinator, define the following *recursive* function to sum the first n numbers.

$\text{sum} = \lambda n. \text{if } n == 0 \text{ then } 0 \text{ else } n + (\text{sum } n - 1).$