CS 601 Homework 1
Due Date: Thursday, 16 November 2017

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1. Let Ω be a discrete sample space, and $\mathcal{F}$ be the set of events defined in the class. Show that there is a function $Q : \Omega \to [0, 1]$ such that the following hold.

a. $\forall x \in \Omega$, $0 \leq Q(x) \leq 1$.

b. $\sum_{x\in\Omega} Q(x) = 1$.

c. $Q(x) = P(\{x\})$.

(The converse also holds, but you are not required to show this.) [10 points]

2. Which is greater, the expectation of the square of a random variable, or the square of its expectation? Justify your answer. [5 points]

3. If X and Y are independent random variables, then show that $\text{Var}(X + Y)$=$\text{Var}(X)$+$\text{Var}(Y)$. [5 points]

4. Let X_1, X_2, X_3, X_4 be 3-wise independent random variables. Prove or disprove : they are pairwise independent. [10 points]

5. For the geometric distribution, we did not show in class that it is a probability distribution. Complete that argument by showing that

$$\sum_{i=1}^{\infty}(1-p)^i p = 1.$$

[5 points]

6. What is the probability that you need to have n trials in order to get k heads? (This is a generalization of the geometric distribution.) [10 points]

7. Consider the usual algorithm for finding the maximum number in an unsorted array of n integers using a linear scan.

Initially, the maximum of the elements seen so far is $-\infty$. You store the maximum seen so far in a variable. When you encounter a new element, if it is greater than the maximum seen so far, then you reset the variable to the new value. Otherwise you leave the variable unchanged.

What is the expected number of times you need to reset the variable? Assume that the entries in the array are coming from a range $[1 \ldots N]$. [15 points]

(One more question to be added based on tail bounds.)