Support Vector Machines and their Applications

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Support Vector Machines

- What is being “supported” ?

Support Vector Machines

- What is being "supported" ?
- How can vectors support anything ?

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- How can vectors support anything ?
- Wait !! Machines ?? - Is this a Mechanical Engineering Lecture ?

The Learning Methodology

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- a palindrome and a non-palindrome ?

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- a palindrome and a non-palindrome ?
- a graph with and without cliques of size bigger than 1000 ?

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Is it possible to write an algorithm to distinguish between ...

- a handwritten 4 and a handwritten 9 ?

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Is it possible to write an algorithm to distinguish between ...

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- a positive movie review and a negative movie review ?

Statistical Machine Learning

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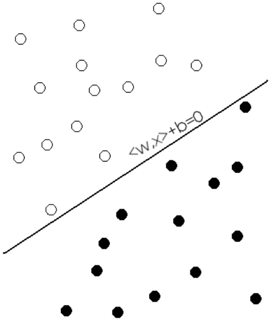
- In other words be probably-approximately-correct
- The motto - Let the data decide the algorithm

Expert Systems

... a computing system capable of representing and reasoning about some knowledge rich domain, such as internal medicine or geology ...

Introduction to Expert Systems, Peter Jackson, Addison Wesley Publishing Company, 1986.

Linear Machines



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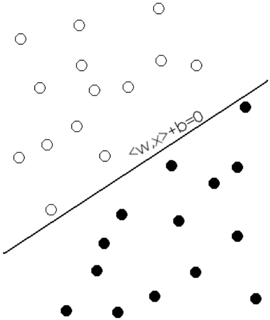

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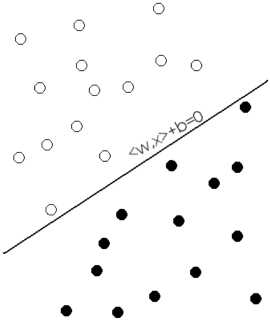
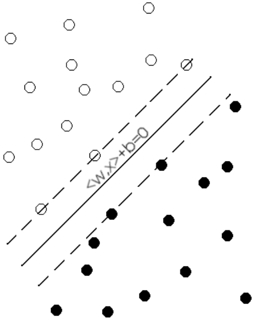


- Arguably the simplest of classifiers acting on vectoral data
- Numerous Learning Algorithms - Perceptron, SVM

Support Vector Machines



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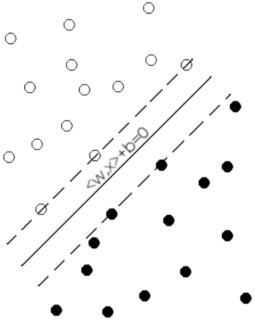

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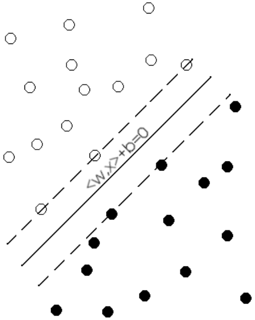

- A "special" hyperplane - with the maximum margin
- Margin of a point measures how far is it from the hyperplane

Learning the Maximum Margin Classifier

$$
\begin{aligned}
\text{minimize}_{\vec{w},b} \quad & \|\vec{w}\|_2 \\
\text{subject to} \quad & y_i(\langle \vec{w} \cdot \vec{x_i} \rangle + b) \geq 1, \\
& i = 1, \ldots, l.
\end{aligned}
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- A *Linearly-constrained Quadratic program*

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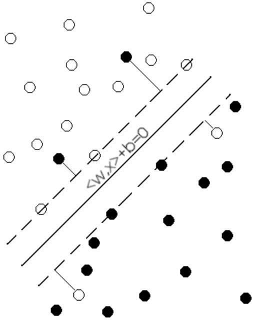
- A *Linearly-constrained Quadratic program*
- Solvable in polynomial time - several algorithms known
- Does not give us much insight into the nature of the hyperplane

Non-linearly Separable Data



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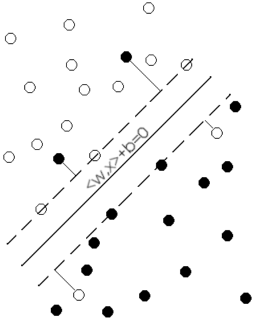

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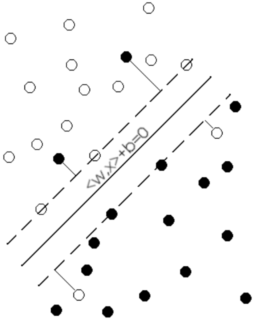

- Use slack variables to allow points to lie on the “wrong” side of the hyperplane
- Can still be solved using a QCQP

Learning the Soft Margin Classifier

$$\begin{aligned}
\text{minimize}_{\vec{w},b} \quad & \|\vec{w}\|_2 + C\sum_{i=1}^{l}\xi_i^2 \\
\text{subject to} \quad & y_i(\langle \vec{w}\cdot\vec{x}_i\rangle + b) \geq 1 - \xi_i, \\
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- Again a *Linearly-constrained Quadratic program*
- More insight gained by looking at the *dual program*

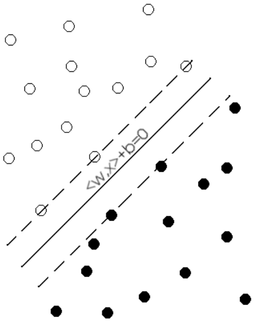
The Dual Program for the Hard Margin SVM

$$\text{maximize} \quad \sum_{i=1}^{l} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{l} y_i y_j \alpha_i \alpha_j \langle \vec{x_i} \cdot \vec{x_j} \rangle,$$

$$\text{subject to} \quad \sum_{i=1}^{l} y_i \alpha_i = 0,$$

$$\alpha_1 \geq 0, i = 1, \ldots, l.$$

Some properties of the optimum $(\vec{w}^*, b^*, \vec{\alpha}^*)$:



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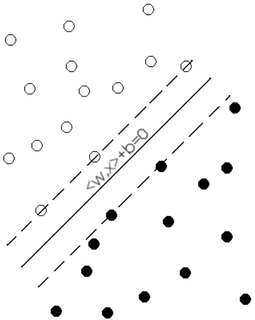

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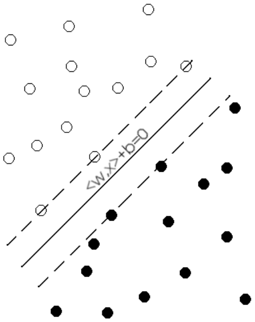

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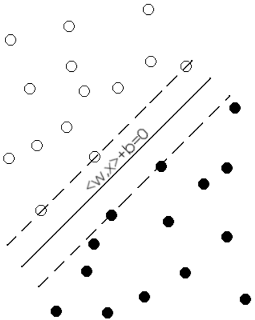

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- $\vec{w}^* = \sum_{i=1}^{l} y_i \alpha_i^* \vec{x_i}$
- $f(\vec{x}, \vec{\alpha}^*, b^*) = \sum_{i=1}^{l} y_i \alpha_i^* \langle \vec{x_i} \cdot \vec{x} \rangle + b^*$

The Support

- Consider each vector applying a force of α_i on the hyperplane in the direction $y_i \frac{\vec{w}}{\|\vec{w}\|_2}$.

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- Hence the vectors lying on the margin, for whom $\alpha_i \neq 0$, "support" the hyperplane.

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- To do this model selection has to be done carefully
- The classifier family should not be too powerful to prevent overfitting

Bounds for Linear Classifiers

Theorem (Vapnik and Chervonenkis)

For any probability distribution on the input domain $\mathbb{R}^d$*, and any target function* g*, with probability no less than* $1-\delta$*, any linear hyperplane* f *classifying a randomly chosen training set of size* l *perfectly cannot disagree with the target function on more than* ϵ *fraction of the input domain (with respect to the underlying distribution) where*

$$\epsilon = \frac{2}{l}\left((d+1)\log\frac{2el}{d+1} + \log\frac{2}{\delta}\right)$$

Bounds for Large Margin Classifiers

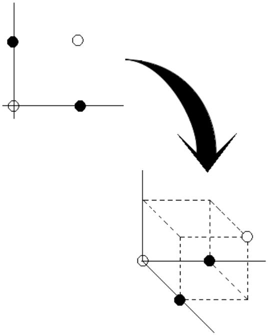
Theorem (Vapnik)

For any probability distribution on the input domain - a ball of radius R, and any target function g, with probability no less than $1-\delta$, any linear hyperplane f classifying a randomly chosen training set of size l perfectly with margin $\geq \gamma$ cannot disagree with the target function on more than ϵ fraction of the input domain (with respect to the underlying distribution) where

$$\epsilon = \tilde{O}\left(\frac{1}{l}\left(\frac{R^2}{\gamma^2} + \log\frac{1}{\delta}\right)\right)$$

NOTE : Error bound Independent of the dimension !

The XOR problem



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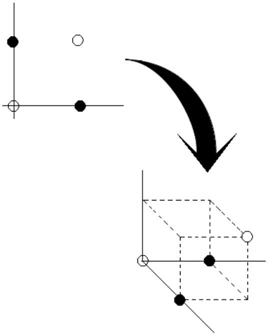

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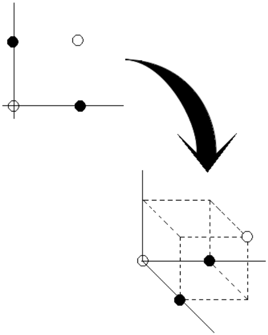

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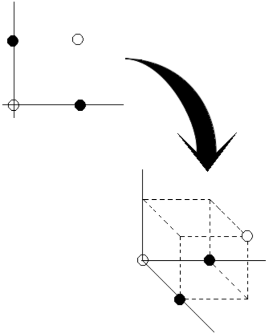

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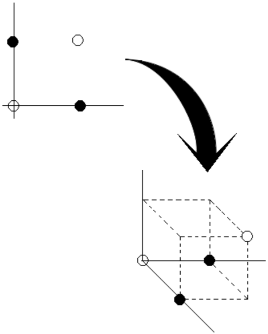

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- Note that not all kernels correspond to feature maps

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- General Convex Optimization Solvers - CVX, SeDuMi - compatible with Matlab©

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- Use of RBF kernels improved performance to $> 86\%$

Image based Gender Identification

- Varma-Babu - application *Kernel Learning*

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Topic Drift in Page-ranking Algorithms

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Topic Drift in Page-ranking Algorithms

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- Use SVDD to obtain representative pages for a topic and prune irrelevant pages
- Use a kernel based on both link and content information
- Performance gains in terms of precision and recall observed over other existing topic distillation techniques

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- `http://www.support-vector.net/`