Similarity-based Learning via Data Driven Embeddings*

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Outline

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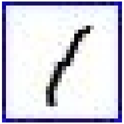

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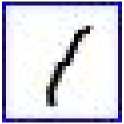

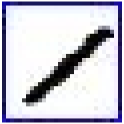

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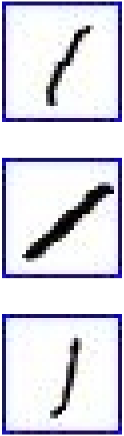

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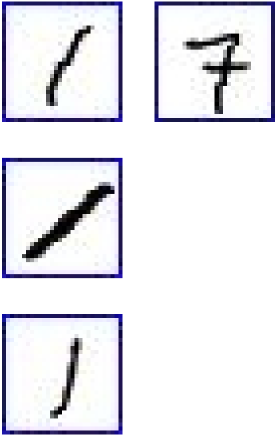

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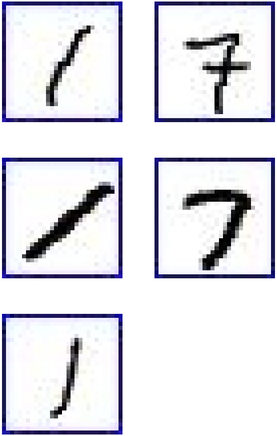

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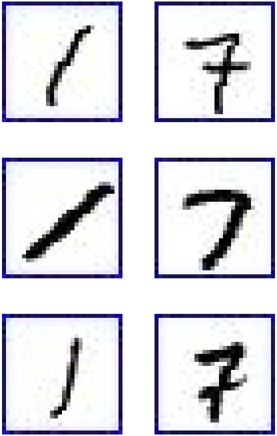

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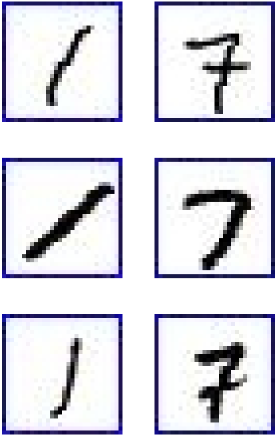

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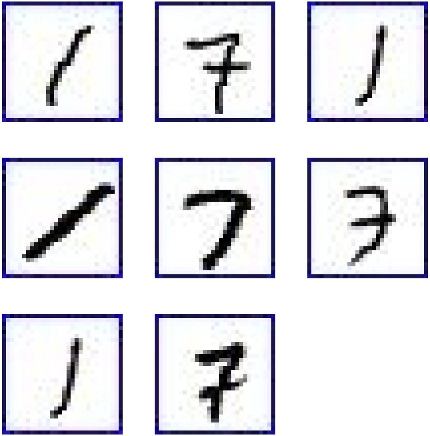

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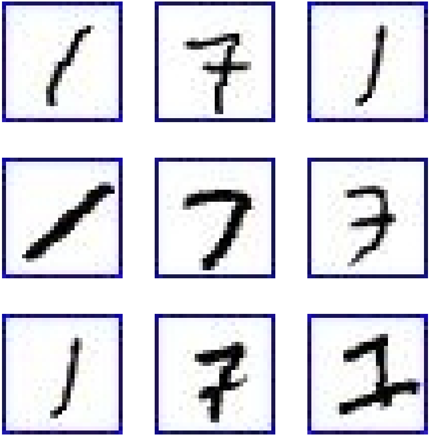

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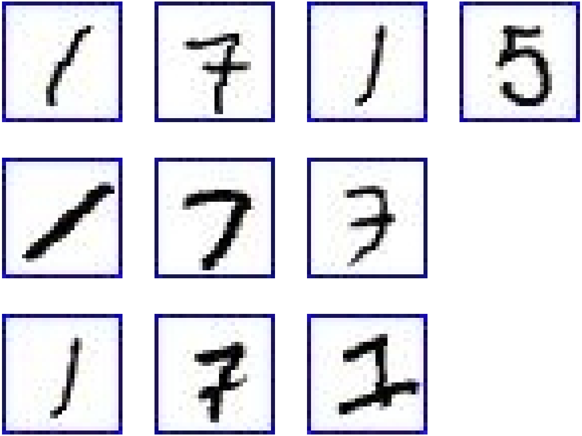

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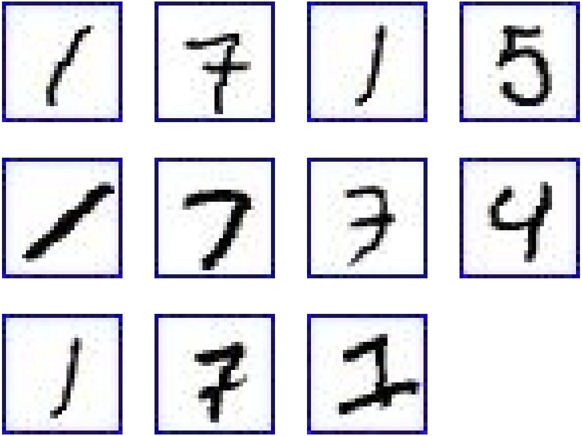

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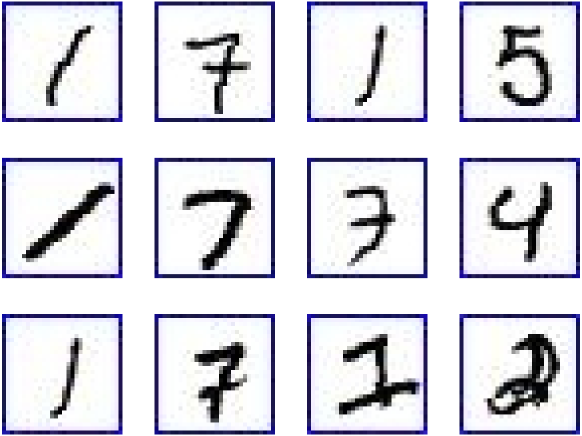

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Spam mail detection

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DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING SEMINAR SERIES
Departmental Colloquium

Title:  Similarity-based Learning via Data Driven Embeddings

Speaker:  Purushottam Kar

Affiliation:  Ph.D. Scholar, CSE Dept., IIT Kanpur
```

To each his own ...

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- Our goal is to output a classifier $\hat{\ell} : \mathcal{X} \rightarrow \{-1, +1\}$ such that it mostly gives out the true labels.

$$\Pr_{x \sim \mathcal{D}} \left[\hat{\ell}(x) \neq \ell(x) \right] < \epsilon$$

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- SVM allows use of arbitrary Positive semi-definite kernels

$$\hat{\ell}(x) = \mathrm{sgn}\left(\sum_{x' \in S} \alpha_{\mathrm{SVM}}(x') K(x, x') \ell(x')\right)$$

Learning with Similarities

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- Some very nice work involving isometric embeddings to (pseudo)Hilbert / Banach spaces [Gottlieb et al., 2010, von Luxburg and Bousquet, 2004, Haasdonk, 2005]
- However, none addressed the issue of suitability of the similarity/distance measure to the learning task

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- Can we do the same for non-PSD similarities ?

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Definition ([Balcan and Blum, 2006])

A similarity $K : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ is said to be (ϵ, γ)-good for a classification problem if for some weighing function $w : \mathcal{X} \rightarrow [-1, 1]$, at least a $(1 - \epsilon)$ probability mass of examples $x \sim \mathcal{D}$ satisfies

$$\mathop{\mathbb{E}}_{\substack{x' \sim \mathcal{D}, \ell(x') = \ell(x) \\ x'' \sim \mathcal{D}, \ell(x'') \neq \ell(x)}} \left[w\left(x'\right) K(x, x') - w\left(x''\right) K(x, x'') \right] \geq \gamma$$

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- In other words, according to the similarity function, most points, on an average, are more similar to points of the same label

Learning with a good similarity function

Theorem ([Balcan and Blum, 2006])

Given an (ϵ, γ)-good similarity function, for any $\delta > 0$, given $n = \frac{16}{\gamma^2} \lg \frac{2}{\delta}$ labeled points $(x_i)_{i=1}^n$, the classifier $\hat{\ell}$ defined below has error at margin $\frac{\gamma}{2}$ no more than $\epsilon + \delta$ with probability greater than $1 - \delta$,

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- Consequently one can use these algorithms to learn this classifier as well

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Definition ([Wang et al., 2007])

A distance function $d : \mathcal{X} \times \mathcal{X} \rightarrow \mathbb{R}$ is said to be (ϵ, γ, B)-good for a classification problem if there exist two class conditional probability distributions $\tilde{\mathcal{D}}_+$ and $\tilde{\mathcal{D}}_-$ such that for all $x \in \mathcal{X}$, $\frac{\tilde{\mathcal{D}}_+(x)}{\mathcal{D}(x)} < \sqrt{B}$ and $\frac{\tilde{\mathcal{D}}_-(x)}{\mathcal{D}(x)} < \sqrt{B}$, such that at least a $(1 - \epsilon)$ probability mass of examples $x \sim \mathcal{D}$ satisfies

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- Yet again this yields a classifier with guaranteed generalization properties

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Theorem ([Wang et al., 2007])

Given an (ϵ, γ, B)-good distance function, for any $\delta > 0$, given $n = \frac{4B^2}{\gamma^2} \lg \frac{1}{\delta}$ pairs of positive and negatively labeled points $\left(x_i^+, x_i^-\right)_{i=1}^n$, the classifier $\hat{\ell}$ defined below has error at margin $\frac{\gamma}{B}$ no more than $\epsilon + \delta$ with probability greater than $1 - \delta$,

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- This naturally lends itself to a boosting-like implementation
- Each of the pairs yields a stump $\mathrm{sgn}\left(d(x, x_i^+) - d(x, x_1^-)\right)$

Outline

A unified notion of what is a good similarity/distance

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- This notion allows us to tune the goodness notion itself, allowing for better classifiers
- The resulting model subsumes the previous two models
- Consequently, the model does not require separate treatment for similarity and distance functions either

What is a good similarity/distance function

Definition (K. and Jain, 2011)

A similarity function $K : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is said to be (ϵ, γ, B)-good for a classification problem if for some antisymmetric *transfer* function $f : \mathbb{R} \to [-C_f, C_f]$ and some weighing function $w : \mathcal{X} \times \mathcal{X} \to [-B, B]$, at least a $(1 - \epsilon)$ probability mass of examples $x \sim \mathcal{D}$ satisfies

$$\mathop{\mathbb{E}}_{\substack{x' \sim \mathcal{D}, \ell(x') = \ell(x) \\ x'' \sim \mathcal{D}, \ell(x'') \neq \ell(x)}} \left[w(x', x'') f(K(x, x') - K(x, x'')) \right] \geq 2C_f \gamma$$

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- With appropriate setting of the weighing function and the transfer function, the previous two models can be recovered.

Outline

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- Let us first see how, given a (good) transfer function, can we learn a (good) classifier.
- We will later on plug in the routines to learn the transfer function as well.

Learning with data-sensitive notions of suitability

Algorithm 1 LEARN-DISSIM

Require: A similarity function K, landmark pairs $\mathcal{L} = \left(x_i^+, x_i^-\right)_{i=1}^n$, a good transfer function f.

Ensure: A classifier $\hat{\ell} : \mathcal{X} \rightarrow \{-1, +1\}$

1: Define $\Phi_{\mathcal{L}} : \mathcal{X} \rightarrow \mathbb{R}^n$ as $\Phi_{\mathcal{L}} : x \mapsto \left(f(K(x, x_i^+) - K(x, x_i^-))\right)_{i=1}^n$

2: Get a labeled training set $T = \{t_j\}_{j=1}^{n'} \subset \mathcal{X}$ sampled from $\mathcal{D}$.

3: $T' \leftarrow \{\Phi_{\mathcal{L}}(t_j)\}_{j=1}^{n'} \subset \mathbb{R}^n$ be the data set embedded in $\mathbb{R}^n$

4: Learn a linear hyperplane over $\mathbb{R}^n$ using T', $\ell_{\text{lin}} \leftarrow$ LEARN-LINEAR(T')

5: Let $\hat{\ell} : \mathcal{X} \rightarrow \{-1, +1\}$ be defined as $\hat{\ell} : x \mapsto \ell_{\text{lin}}\left(\Phi_{\mathcal{L}}(x)\right)$

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- LEARN-LINEAR may be taken to be any linear hyperplane learning algorithm such as Perceptron, SVM.
- The above procedure essentially creates a *data-driven*, problem specific embedding of the domain $\mathcal{X}$ into a Euclidean space

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- Not amenable to efficient algorithms as hyperplane classification error is NP-hard to minimize [Garey and Johnson, 1979, Arora et al., 1997]
- We provide our guarantees in terms of smooth Lipschitz losses like hinge-loss, log-loss etc that can be efficiently minimized over large datasets.

Working with surrogate loss functions

Definition (K. and Jain, 2011)

A similarity function is said to be (ϵ, B)-good with respect to a loss function $L : \mathbb{R} \rightarrow \mathbb{R}^+$ if for some transfer function $f : \mathbb{R} \rightarrow \mathbb{R}$ and some weighing function $w : \mathcal{X} \times \mathcal{X} \rightarrow [-B, B]$, $\underset{x \sim \mathcal{D}}{\mathbb{E}} [L(G(x))] \leq \epsilon$ where

$$G(x) = \underset{\substack{x' \sim \mathcal{D}, \ell(x') = \ell(x) \\ x'' \sim \mathcal{D}, \ell(x'') \neq \ell(x)}}{\mathbb{E}} \left[w(x', x'') \, f(K(x, x') - K(x, x'')) \right]$$

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Theorem (K. and Jain, 2011)

If K is an (ϵ, B)-good similarity function with respect to a C_L-Lipschitz loss function L then for any $\epsilon_1 > 0$, with probability at least $1 - \delta$ over the choice of $d = (16B^2 C_L^2 / \epsilon_1^2) \ln(4B/\delta\epsilon_1)$ landmark pairs, the expected loss of the classifier $\hat{\ell}(x)$ returned by LEARN-DISSIM with respect to L satisfies $\underset{x}{\mathbb{E}} \left[L(\hat{\ell}(x)) \right] \leq \epsilon + \epsilon_1$.

Outline

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- For any transfer function f and arbitrary set of landmarks $\mathcal{L}$, let $L(f) = \underset{x\sim\mathcal{D}}{\mathbb{E}}\left[L(G(x))\right]$ and let $L(f, \mathcal{L})$ denote the generalization loss of the best classifier that uses the embedding $\Phi_{\mathcal{L}}$ defined by the landmarks $\mathcal{L}$.

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- The earlier result shows that for a *fixed* f, for a large enough random $\mathcal{L}$, $L(f, \mathcal{L}) \leq L(f) + \epsilon_1$.

Learning the transfer function

Theorem (K. and Jain, 2011)

Let $\mathcal{F}$ be a compact class of transfer functions with respect to the infinity norm and $\epsilon_1, \delta > 0$. *Let* $\mathcal{N}(\mathcal{F}, r)$ *be the size of the smallest* ϵ*-net over* $\mathcal{F}$ *with respect to the infinity norm at scale* $r = \frac{\epsilon_1}{4C_L B}$. *Taking* $n = \frac{64B^2C_L^2}{\epsilon_1^2}\ln\left(\frac{16B\cdot\mathcal{N}(\mathcal{F},r)}{\delta\epsilon_1}\right)$ *random landmark pairs, we have with probability greater than* $(1-\delta)$

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Algorithm 2 FTUNE

Require: A family of transfer functions $\mathcal{F}$, a similarity function K, a loss function L.
Ensure: An optimal transfer function $f^* \in \mathcal{F}$.

1: Select d landmark pairs $\mathcal{L}$.
2: **for all** $f \in \mathcal{F}$ **do**
3: $w_f \leftarrow$ LEARN-DISSIM$(K, \mathcal{L}, f)$, $L_f \leftarrow L(f, \mathcal{L})$
4: **end for**
5: $f^* \leftarrow \arg\min_{f\in\mathcal{F}} L_f$
6: **return** f^*.

Intelligent choice of landmark points

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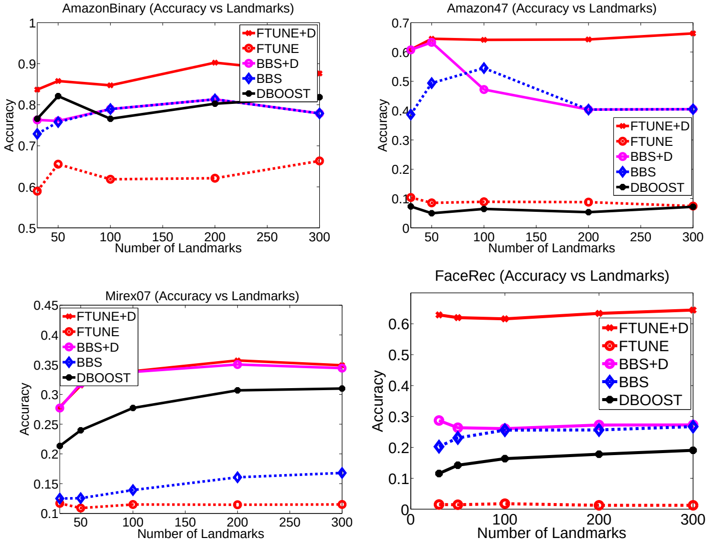
Algorithm 3 DSELECT

Require: A training set T.
Ensure: A set of n landmark pairs.

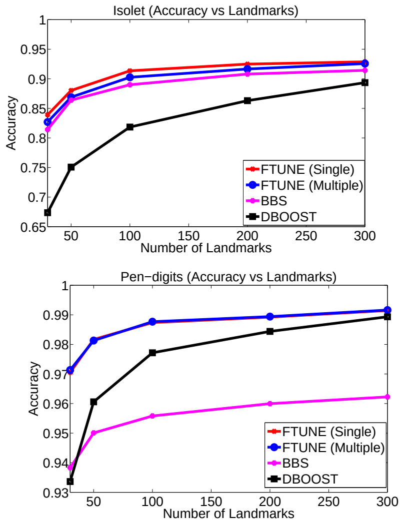
$S \leftarrow \textsc{random-element}(T), \mathcal{L} \leftarrow \emptyset$
for $j = 2$ **to** n **do**
  $z \leftarrow \arg\min_{x \in T} \sum_{x' \in S} K(x, x')$.
  $S \leftarrow S \cup \{z\}, \quad T \leftarrow T \backslash \{z\}$
end for
for $j = 1$ **to** n **do**
  Sample z_1, z_2 from S with replacement s.t. $\ell(z_1) = 1,\ \ell(z_2) = -1$
  $\mathcal{L} \leftarrow \mathcal{L} \cup \{(z_1, z_2)\}$
end for
return $\mathcal{L}$

Outline

Results


AmazonBinary (Accuracy vs Landmarks)
Accuracy
Number of Landmarks
FTUNE+D
FTUNE
BBS+D
BBS
DBOOST
Amazon47 (Accuracy vs Landmarks)
Mirex07 (Accuracy vs Landmarks)
FaceRec (Accuracy vs Landmarks)

Results


Isolet (Accuracy vs Landmarks)
Accuracy
Number of Landmarks
Letters (Accuracy vs Landmarks)
Pen−digits (Accuracy vs Landmarks)
Opt−digits (Accuracy vs Landmarks)
FTUNE (Single)
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Discussion

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- In contrast, our method consistently outperforms the existing methods in both the scenarios.
- Since FTUNE selects its output by way of validation, it is susceptible to over-fitting on small datasets.
- In these cases, DSELECT (intuitively) removes redundancies in the landmark points thus allowing FTUNE to recover the best transfer function.

Thanks

Preprint available at
`http://www.cse.iitk.ac.in/users/purushot/`

Outline

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