CS 771A: Intro to Machine Learning, IIT Kanpur				**Quiz I** (24 Jan 2024)
Name	MELBO			**20 marks**
Roll No	240007	Dept.	AWSM	Page **1** of **2**

Instructions:

1. This question paper contains 1 page (2 sides of paper). Please verify.
2. Write your name, roll number, department above in **block letters neatly with ink**.
3. Write your final answers neatly **with a blue/black pen**. Pencil marks may get smudged.
4. Don't overwrite/scratch answers especially in MCQ – such cases may get straight 0 marks.
5. Do not rush to fill in answers. You have enough time to solve this quiz.

Q1. (**True-False**) Write **T** or **F** for True/False (write **only in the box on the right-hand side**). You must also give a brief justification for your reply in the space provided below.(**3 x (1+2) = 9 marks**)

1	The two hyperplane classifiers $\mathbf{a}^\top\mathbf{x}+b$ and $\mathbf{p}^\top\mathbf{x}+q$ with $\mathbf{a},\mathbf{p}\in\mathbb{R}^2, b,q\in\mathbb{R}$ have the same decision boundary if $\mathbf{a}+\mathbf{p}=\mathbf{0}$ and $b+q=0$. Give a brief proof if your answer is T else give a concrete counter example if your answer is F.	T

Say a point $\mathbf{x}_0$ is on the decision boundary for the first hyperplane. This implies $\mathbf{a}^\top\mathbf{x}_0+b=0$ which in turn implies that $-\mathbf{a}^\top\mathbf{x}_0-b=0=\mathbf{p}^\top\mathbf{x}_0+q$ which implies that the point $\mathbf{x}_0$ lies on the decision boundary for the second hyperplane as well. Similarly vice-versa.

2	Melbo has learnt a classifier $\mathbf{w}^\top\mathbf{x}+b$ with $\mathbf{w}\in\mathbb{R}^2, b\in\mathbb{R}$. If $\text{sign}(\mathbf{w}^\top\mathbf{x}_0+b)>0$ for some $\mathbf{x}_0\in\mathbb{R}^2$ then it must always be the case that $\text{sign}(\mathbf{w}^\top(-\mathbf{x}_0)+b)<0$. Give brief proof if answer is T else a clear counter example of $\mathbf{w},\mathbf{x}_0,b$ if answer is F.	F

Say $\mathbf{w}=(1,1)\in\mathbb{R}^2, \mathbf{x}_0=(1,1), b=10$. We have $\mathbf{w}^\top\mathbf{x}_0+b=22$ but $\mathbf{w}^\top(-\mathbf{x}_0)+b=8$.

3	Consider $f,g:\mathbb{R}\to\mathbb{R}$ of the form $f(x)=ax+b, g(x)=bx+a$ with $a,b>0$. If $a\neq b$, there must exist $x_0,x_1\in\mathbb{R}$ such that $f(x_0)<g(x_0)$ and $f(x_1)>g(x_1)$. If your answer is T, give example of x_0,x_1 in terms of a,b, else give a counter example.	T

An example in terms of a,b and universal constants is perfectly admissible. However, examples that use specific values of a,b will not get full credit.

For the first part we wish $(f-g)(x_0)=(a-b)(x_0-1)<0$. Letting $x_0=b-a+1$ gives us $(f-g)(x_0)=(a-b)(b-a)=-(a-b)^2<0$ since $a\neq b$.

For the next part we wish $(f-g)(x_0)=(a-b)(x_1-1)>0$. Letting $x_1=a-b+1$ gives us $(f-g)(x_0)=(a-b)(a-b)=(a-b)^2>0$ since $a\neq b$.

Q2. (Sliding parabolas) Consider $f(x) = (x-a)^2 + b, g(x) = -(x-p)^2 + q$ and $h(x) = x^3/2$.

Find values of $a, b \in \mathbb{R}$ such that f and h share a tangent at $x = 1$.	$a = 1/4$	$b = -1/16$
Find values of $p, q \in \mathbb{R}$ such that g and h share a tangent at $x = 1$.	$p = 7/4$	$q = 17/16$
Find the value of $f + g$ at $x = 1$ i.e., $(f+g)(1)$	$(f+g)(1) = 1$	
Find the first derivative of $f + g$ at $x = 1$ i.e., $(f+g)'(1)$	$(f+g)'(1) = 3$	
Find second derivative of $f + g$ at $x = 1$ i.e., $(f+g)''(1)$	$(f+g)''(1) = 0$	

Write your answers only in the space provided. **(2 + 2 + 1 + 1 + 1 = 7 marks)**

Q4. (Vector line-up) Give examples of 4D vectors (fill-in the 4 boxes) with the following properties. Any example will get full marks so long as it satisfies all the properties mentioned in the question. Your answers to the parts a, b, c, d, e may be same/different. (**4 x 1 = 4 marks**)

a. A vector $\mathbf{v} \in \mathbb{R}^4$ such that $\mathbf{v} \neq \mathbf{0}$ and $\mathbf{v}$ is perpendicular to both the vectors $(1,0,1,1)$ and $(0,1,0,0)$.
b. A vector $\mathbf{v} \in \mathbb{R}^4$ with only integer coordinates (at least 2 non-zero coordinates) whose L_2 norm is also an integer.
c. A vector $\mathbf{v} \in \mathbb{R}^4$ that is perpendicular to its own negative i.e., $\mathbf{v} \perp -\mathbf{v}$.
d. A point $\mathbf{v} \in \mathbb{R}^4$ with equal L_2 distance from the vectors $(1,2,3,4)$ and $(4,3,2,1)$.

0	0	-1	1
3	4	0	0
0	0	0	0
0	0	0	0

Note: for part b., something like a Pythagorean triplet is needed

Anything written here will not be graded

ROUGH WORK