Variational Inference (Wrap-up), Inference via Sampling

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Topics in Probabilistic Modeling and Inference (CS698X)

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Recap: VI using Monte-Carlo based Gradients of ELBO

- VI = ELBO optimization. Requires ELBO gradients: $\nabla_\phi \mathcal{L}(\phi) = \nabla_\phi \mathbb{E}_q[\log p(\mathbf{X}, \mathbf{Z}) - \log q(\mathbf{Z}|\phi)]$

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- Note: We can use minibatches of data (instead of all $\mathbf{X}$) to compute the above gradients

Automatic Differentiation Variational Inference (ADVI)

- Auto. Diff. (AD): A way to automate differentiation of functions with unconstrained variables

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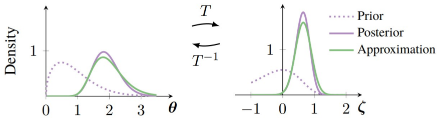

(a) Latent variable space **(b)** Real coordinate space

$$T : \text{supp}(p(\boldsymbol{\theta})) \to \mathbb{R}^K$$

$$\boldsymbol{\zeta} = T(\boldsymbol{\theta})$$

$$p(\mathbf{x}, \boldsymbol{\zeta}) = p\left(\mathbf{x}, T^{-1}(\boldsymbol{\zeta})\right) \left| \det J_{T^{-1}}(\boldsymbol{\zeta}) \right|$$

Transformed density — Original density — Jacobian of inverse of T

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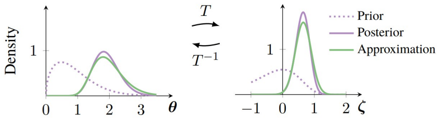

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- ADVI transforms the variables to real-valued and then does VI with Gaussian variational approx.

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- Popular in deep probabilistic models such as variational autoencoders, deep Gaussian Processes, etc

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  - Normalizing flows:' Turn a simple $q(\boldsymbol{z})$ into a complex one via series of invertible transformations

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- Many recent inference algorithms are based on minimizing such divergences

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- Still a very active area of research, especially for doing VI in complex models
  - Models with discrete latent variables
  - Reducing the variance in Monte-Carlo estimate of ELBO gradients

Inference via Sampling

(Note that we have already seen Gibbs sampling)

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Sampling for Approximate Inference

- Some typical inference tasks
  - Compute a (possibly intractable) posterior distribution: $p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})} = \frac{p(\mathcal{D}|\theta)p(\theta)}{\int p(\mathcal{D}|\theta)p(\theta)d\theta}$
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- Sampling methods provide a general way to (approximately) solve these problems

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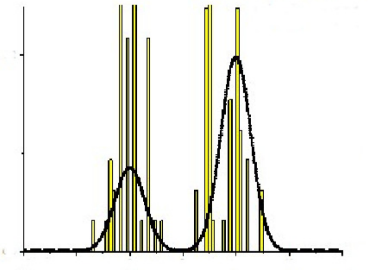
- Can approximate any distribution using a set of randomly drawn samples from it



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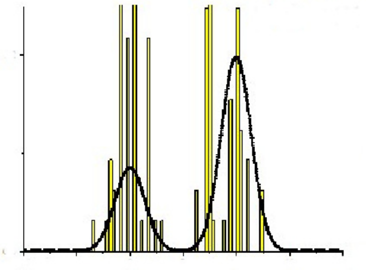

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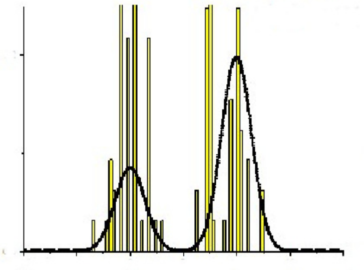

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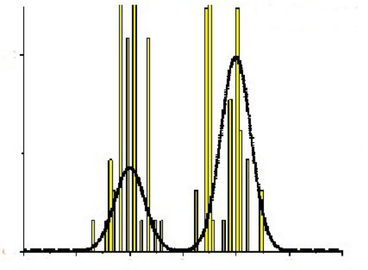

- The samples can also be used for computing expectations (Monte-Carlo averaging)
- Usually straightforward to generate samples if it is a simple/standard distribution
- **The interesting bit:** Even if the distribution is "difficult" (e.g., an intractable posterior), it is often possible to generate random samples from such a distribution, as we will see..

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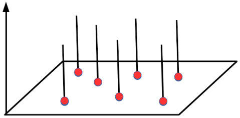

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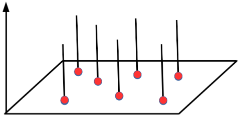

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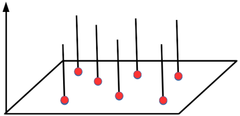

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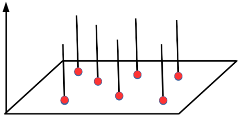

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- $p_L(A)$ is a discrete distribution with finite support $\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(L)}$ (can think of it as a histogram)

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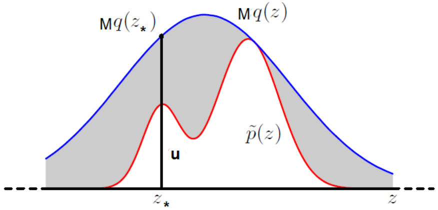

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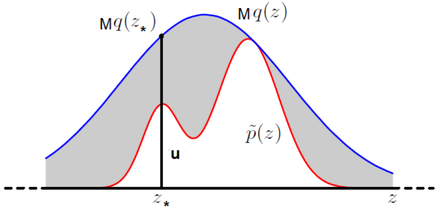

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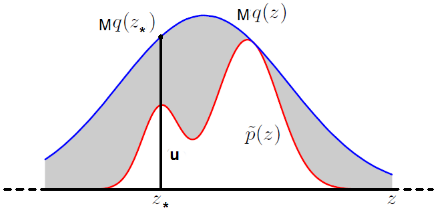

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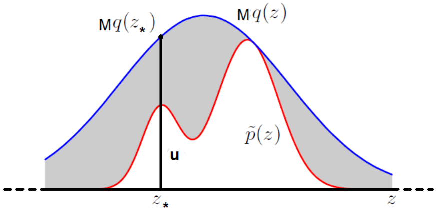

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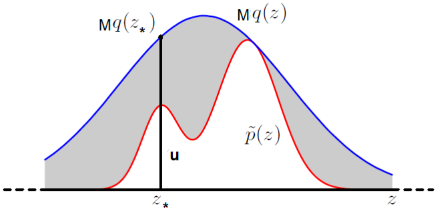

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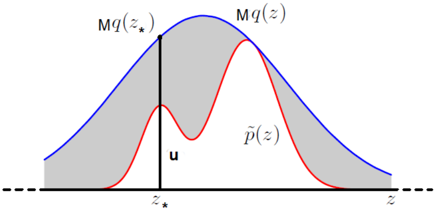

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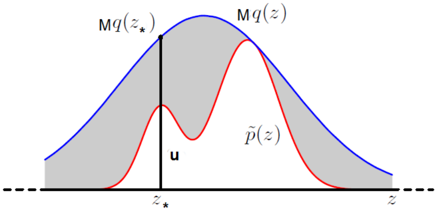

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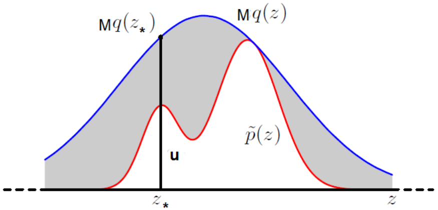

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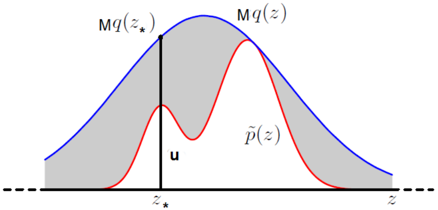

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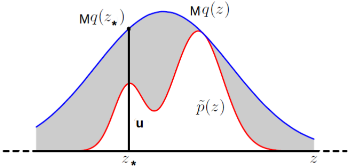

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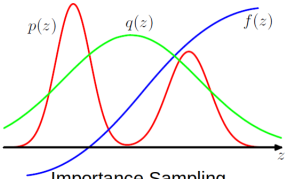

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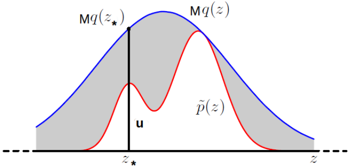

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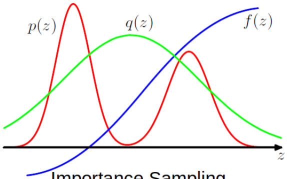

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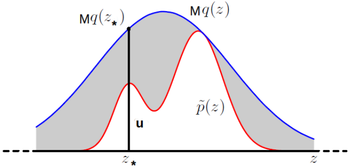

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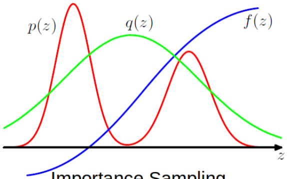

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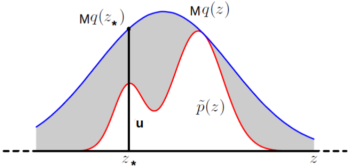

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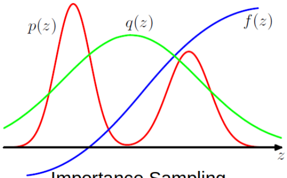

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- A solution to these: MCMC methods