

A Short, Fast, Post-quantum Multivariate Digital Signature Scheme

Nitin Saxena

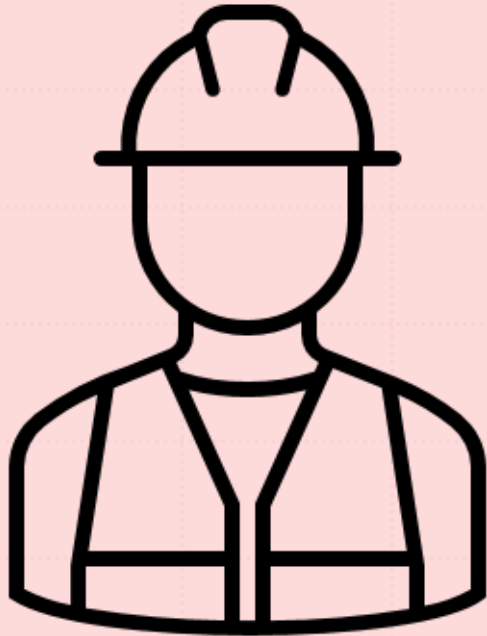
(joint with Anindya Ganguly, Angshuman Karmakar)

CSE & WSAIS, IIT Kanpur

IIT Kanpur

July-2025





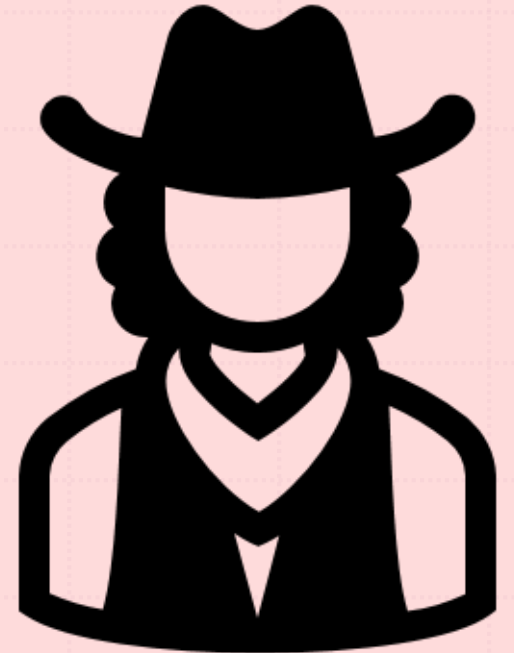
Party A



Malicious person



Insecure channel

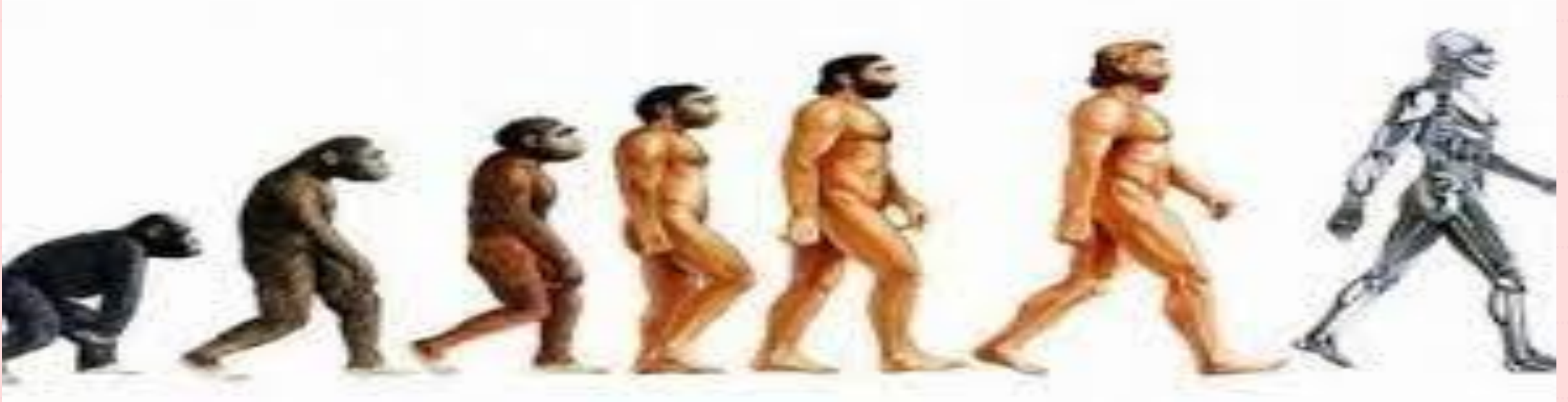


Party B

Enigma

Homomorphic encryption

Post-quantum cryptography



Playfair cipher

Public key cryptography

Computations on encrypted data

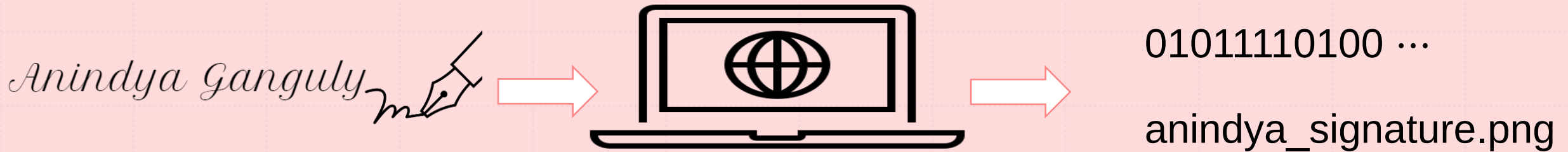
Quantum cryptography

Computational
power available



Attack: Mathematical
(hard?) problem

Digital Signature



Offline signatures are widely utilized for signing a variety of documents, such as contracts, checks, and legal forms

Adobe
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7 steps

1. In the email you received from the sender of the document to sign, click the link labeled "Cli...
2. Click on the "Click here to sign" field in the document to sign.
3. A pop-up window will open to let you create your electronic signature in the signature field.

Signaturely
<https://signaturely.com/online-signature>

Free Online Signature Generator (Type or Draw)

A **signature** generator (or **signature** maker/**signature** creator) is an online tool you can use to **create** an online **signature** to sign documents. You can draw or type ...

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How to create digital signatures

Creating a **digital signature** is easy · Open the email with a request to digitally sign your document. · Click the link. · Agree to **electronic signing**. · Click each ...

Smallpdf.com
<https://smallpdf.com/eSign-PDF/How-To-eSign-PDF>

How to Create a Digital Signature Online

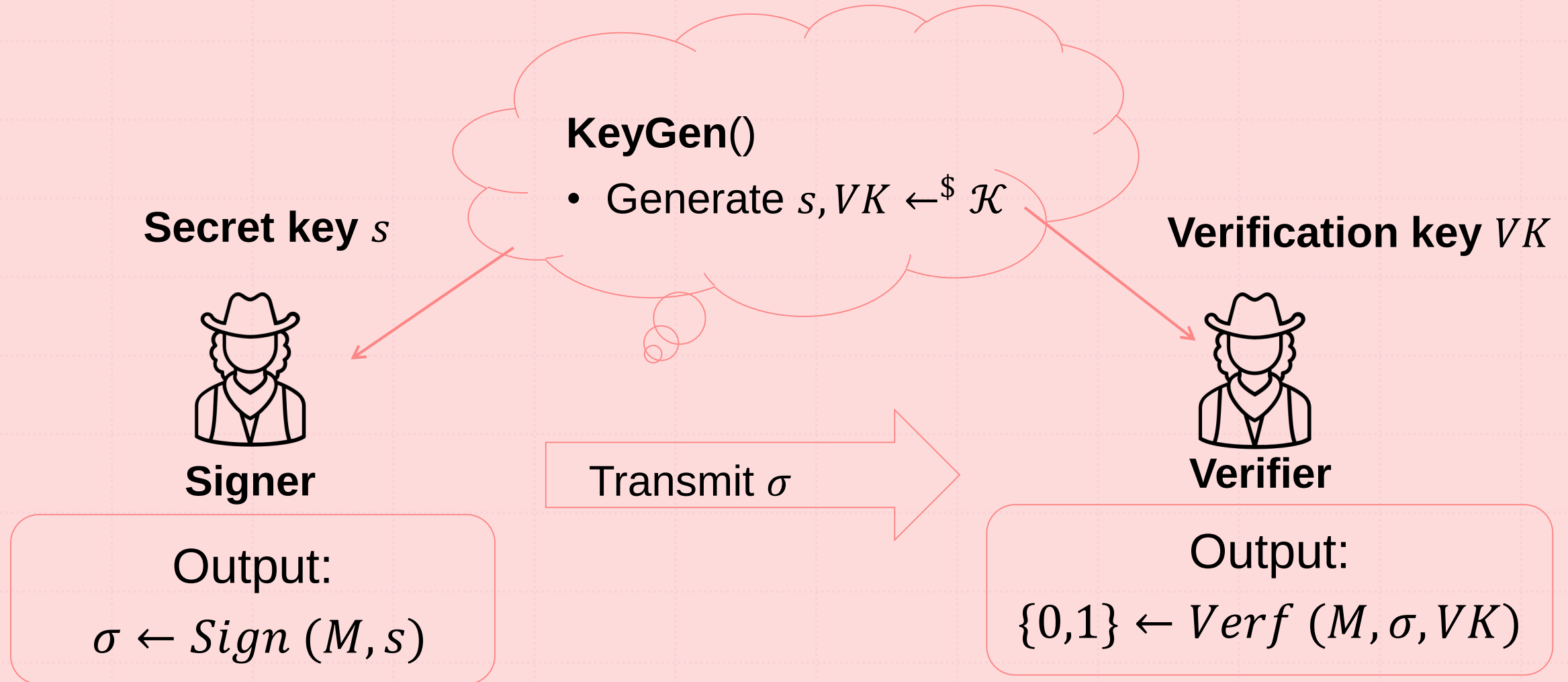
❑ The ease of copying a digitized handwritten signature makes it susceptible to forgery.

❑ Digital signature provides *integrity* : message authentication, non-repudiation

Signature schemes: Wide applications

- Social Media/ UPI
- Legal docs/ degree certificates
- Electronic voting m/c
- NFT/ Blockchain
- Authentication/ Data privacy
- Protection against alteration
- Non-repudiated transfer of information
- Unobstructed channel of communication

Digital Signature: Math modelling





Motivation for multivariate



❑ Design a secure signature scheme



- ❑ Lattices are crypto-friendly quantum-safe constructions
- ❑ Multivariate construction offers short signature size

- ❑ Quantum algorithms can efficiently solve problems, e.g. like IFP, DL
- ❑ Research community needs diversity in hardness assumptions
- ❑ Recent NIST submission has eleven multivariate candidates



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- ❑ **Quantum algorithms can
efficiently solve problems, e.g. like IF,
DL**

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diversity in hardness assumptions

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multivariate candidates



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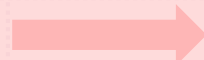
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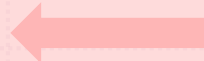
Motivation for multivariate



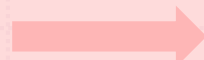
❑ Design a secure signature scheme



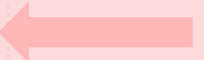
❑ Quantum algorithms can efficiently solve problems, e.g. like IFP, DL



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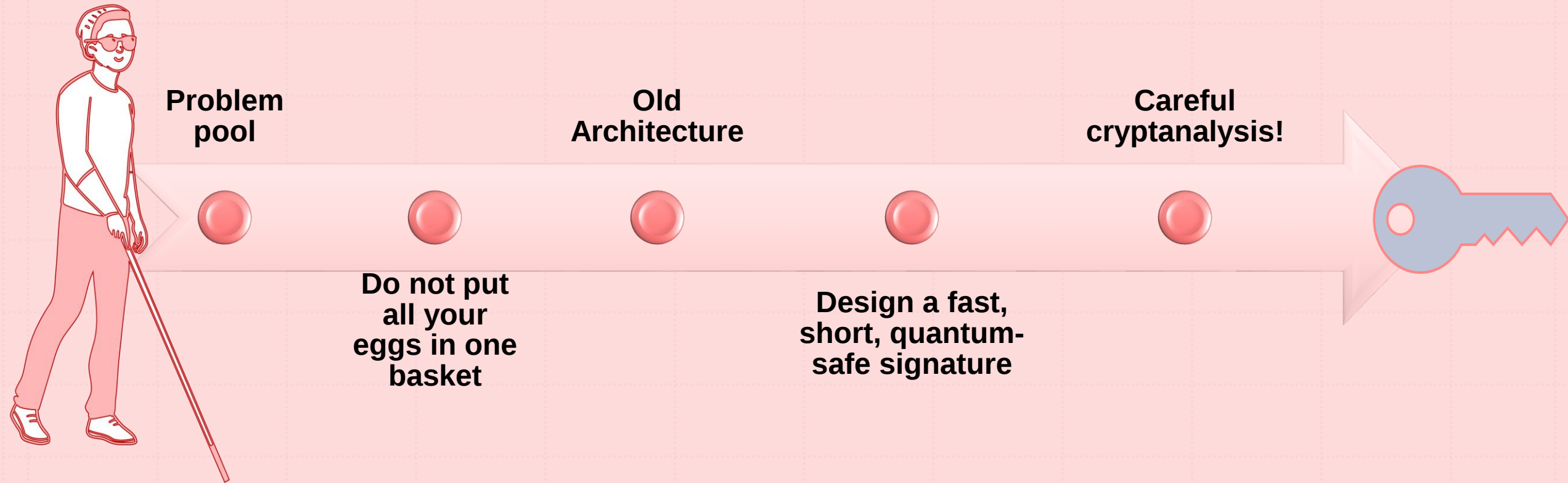


❑ **Recent NIST submission has eleven multivariate candidates**

VDOO: Cause of Happiness

- ❖ **New design element:** introduced diagonal layers
- ❖ **Fastest:** size of linear system is **small**, so Gaussian Elimination is efficient
- ❖ **Secure:** against all existing classical and quantum attacks
- ❖ **Shortest:** **96 bytes**, which is one of the **smallest** signature size (including SPHINCS+, Dilithium, and Falcon)

Roadmap for Signature Design



Cryptography from Hard Problems

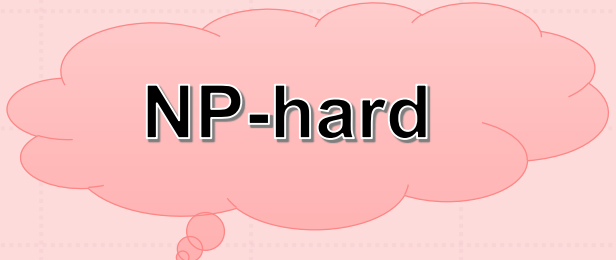
Hard problems	Example	Importance and drawbacks
Classical cryptography	RSA, ECDH, ECDSA, EdDSA	Small key and signature size. But quantum-insecure
Lattice-based cryptography	Crystals-Dilithium , Falcon, NTRU	Large key size and signature size. Fast. Most crypto friendly
Multivariate cryptography	Rainbow , UOV, Mayo	Small signature, large key size , simple construction
Hash-based cryptography	SPHINCS+, XMSS	Small public key size, large signature size and slow
Code-based cryptography	BIKE, Classical McEliece	Complex structure. Syndrome decoding; slow
Isogeny-based cryptography	SIKE , SQISign	Small signature and public key size but significantly slow



**Don't Put All
Your Eggs In
One Basket**

Multivariate Cryptography

Multivariate Quadratic (MQ) Problem



NP-hard

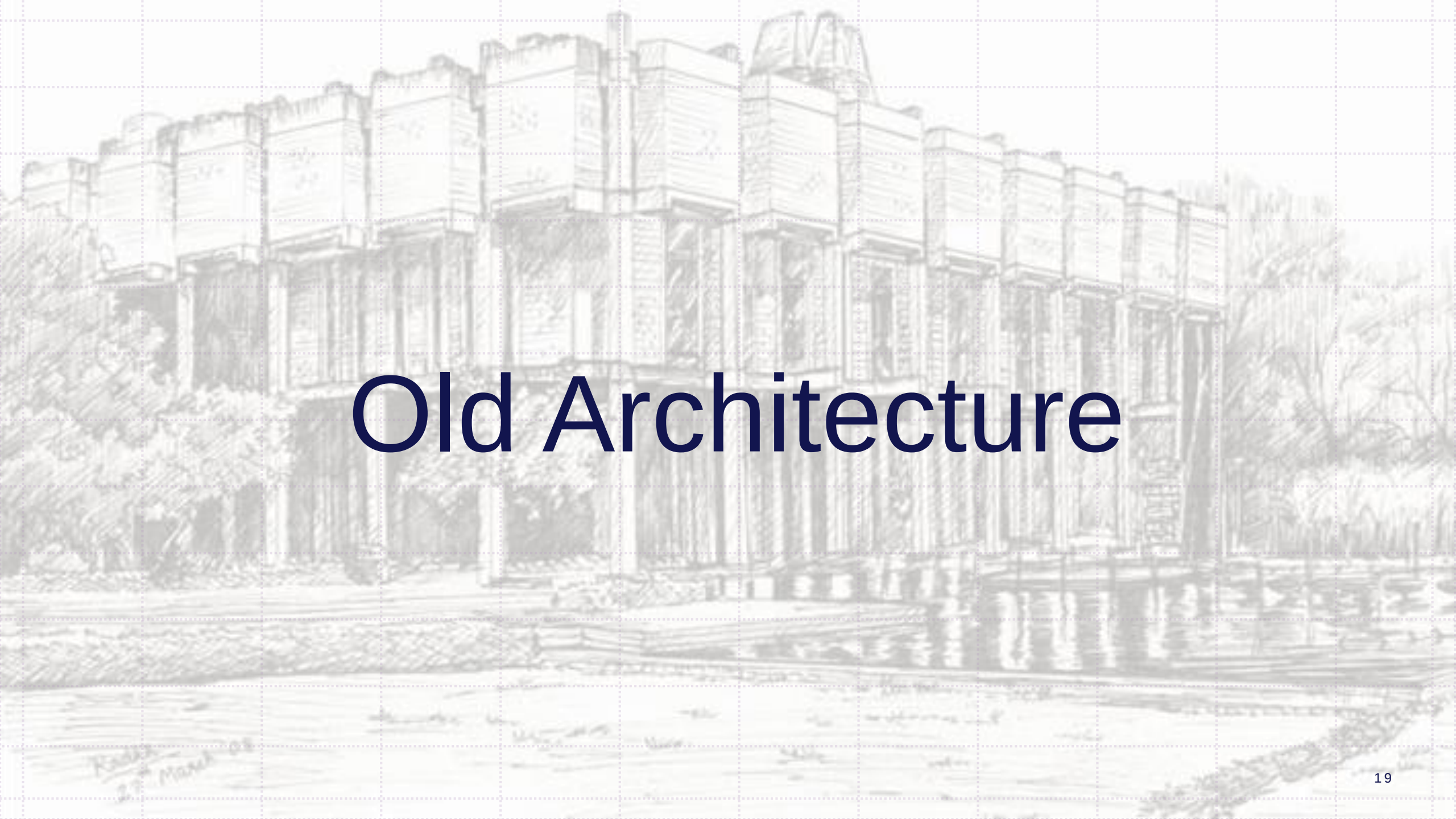
- Given a quadratic system of m homogeneous equations and n variables, find a solution in polynomial time.

Constructions based on MQ

~~Hidden Field Equation~~ [Patarin-96; Tao, Petzoldt, Ding-21]

Oil-Vinegar-based construction [Kipnis, Patarin, Goubin-99]

ZKP-based construction (5-round identification, MPCitH) [CHR+, Fen-22]



Old Architecture

Oil-Vinegar map

Quadratic map $\mathcal{F} :: (f^{(1)}, \dots, f^{(m)}): \mathbb{F}_q^n \rightarrow \mathbb{F}_q^m$

$$f^{(1)}(x_1, \dots, x_v, \dots, x_n) :: \sum_{i=1}^v \sum_{j=1}^v \alpha_{i,j}^{(1)} x_i x_j + \sum_{i=1}^v \sum_{j=v+1}^n \beta_{i,j}^{(1)} x_i x_j = t_1$$

$$f^{(2)}(x_1, \dots, x_v, \dots, x_n) :: \sum_{i=1}^v \sum_{j=1}^v \alpha_{i,j}^{(2)} x_i x_j + \sum_{i=1}^v \sum_{j=v+1}^n \beta_{i,j}^{(2)} x_i x_j = t_2$$

$$\begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{array}$$

$$f^{(m)}(x_1, \dots, x_v, \dots, x_n) :: \sum_{i=1}^v \sum_{j=1}^v \alpha_{i,j}^{(m)} x_i x_j + \sum_{i=1}^v \sum_{j=v+1}^n \beta_{i,j}^{(m)} x_i x_j = t_m$$

Oil-Vinegar map

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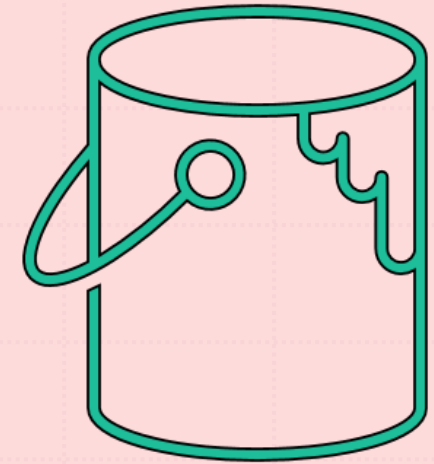
Construct an Oil-Vinegar Polynomial

Vinegar $\times$ Vinegar



Vinegar

Vinegar $\times$ Oil



Oil

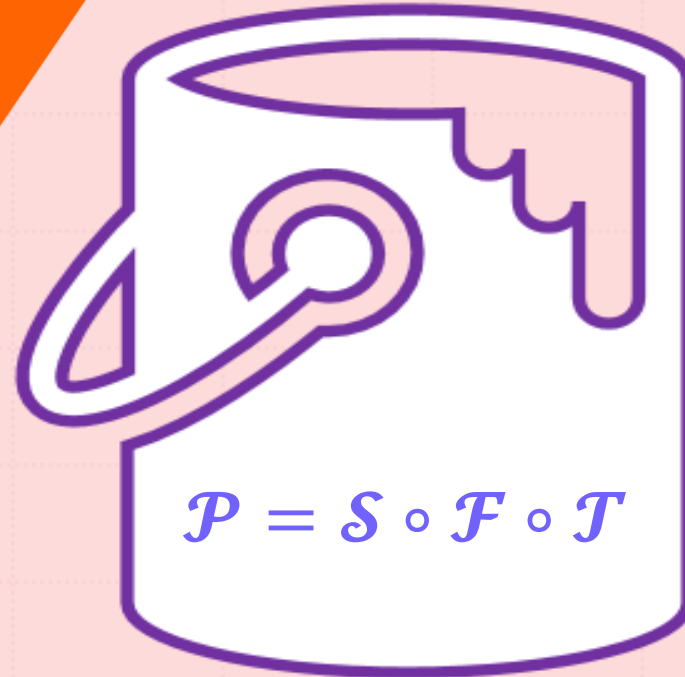
No Oil $\times$ Oil terms

Variables Bucket

Construct a (random) Multivariate Polynomial



Vinegar



$$\mathcal{P} = \mathcal{S} \circ \mathcal{F} \circ \mathcal{T}$$

Oil

Variables mixed randomly

Multivariate Signature Scheme

$$d = \mathcal{H}(msg)$$

→ Signature Generation →

$$d \in \mathbb{F}_q^m \implies_{\mathcal{S}^{-1}} w \in \mathbb{F}_q^m \implies_{\mathcal{F}^{-1}} y \in \mathbb{F}_q^n \implies_{\mathcal{T}^{-1}} x \in \mathbb{F}_q^n$$

Private Key:

□ invertible linear map

$$\mathcal{S} : \mathbb{F}_q^m \rightarrow \mathbb{F}_q^m, \quad \mathcal{T} : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^n$$

□ quadratic map $\mathcal{F} : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^m$

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Signature = x

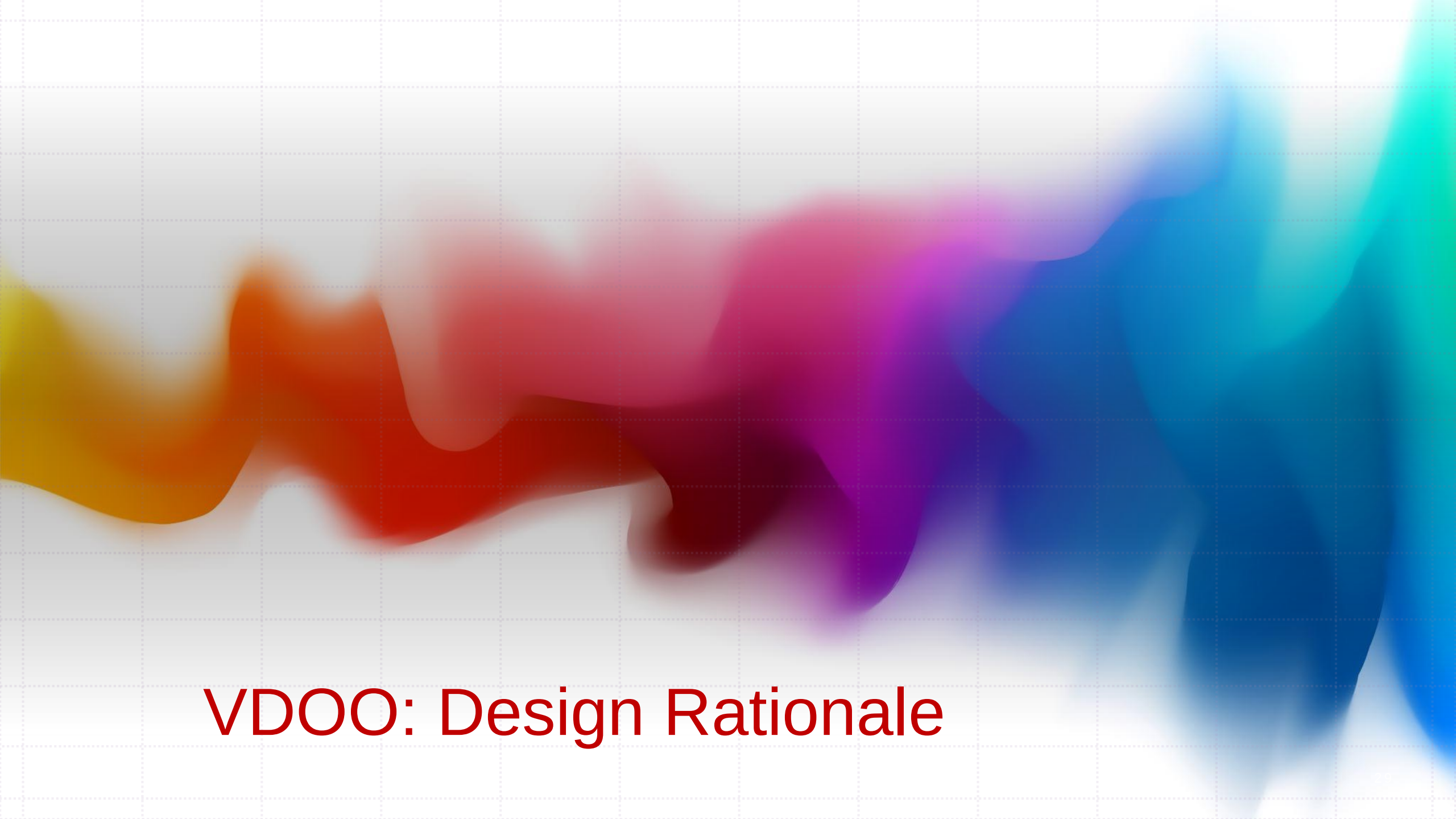
$$\begin{aligned} d &= \mathcal{H}(msg) \\ d' &= \mathcal{P}(x) \end{aligned}$$

← Verification ←

$$d \neq d'$$

Verification/Public Key:

$$\mathcal{P} = \mathcal{S} \circ \mathcal{F} \circ \mathcal{T} : \mathbb{F}_q^n \rightarrow \mathbb{F}_q^m$$

The background of the slide features a light gray grid pattern. Overlaid on this grid are several large, overlapping, semi-transparent shapes in a variety of colors including yellow, orange, red, pink, purple, blue, and teal. These shapes are fluid and organic in form, creating a vibrant, abstract composition.

VDOO: Design Rationale

Diagonal Layer

Vinegar Variables: First randomly fix $x_1, x_2, \dots, x_v \in_U \mathbb{F}_q$

$$f_1(x_1, x_2, \dots, x_{v+1}) = x_{v+1} \cdot l_1(x_1, x_2, \dots, x_v) + g_1(x_1, x_2, \dots, x_v)$$

l_i is linear and g_i is quadratic

$$f_2(x_1, x_2, \dots, x_{v+2}) = x_{v+2} \cdot l_2(x_1, x_2, \dots, x_{v+1}) + g_2(x_1, x_2, \dots, x_{v+1})$$

$$\begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{array}$$

$$f_d(x_1, x_2, \dots, x_{v+d}) = x_{v+d} \cdot l_d(x_1, x_2, \dots, x_{v+d-1}) + g_d(x_1, x_2, \dots, x_{v+d-1})$$

Why Diagonal Layer?

Diagonal Layer

$$\gamma_1^{(1)} x_1 + c_1 = t_1$$

$$\gamma_2^{(2)} x_2 + c_2 = t_2$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\gamma_N^{(N)} x_N + c_N = t_N$$

Time Complexity: $\mathcal{O}(N)$

Oil Layer

$$\gamma_1^{(1)} x_1 + \gamma_2^{(1)} x_2 + \cdots + \gamma_N^{(1)} x_N = t_1$$

$$\gamma_1^{(2)} x_1 + \gamma_2^{(2)} x_2 + \cdots + \gamma_n^{(2)} x_N = t_2$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$$

$$\gamma_1^{(N)} x_1 + \gamma_2^{(N)} x_2 + \cdots + \gamma_N^{(N)} x_N = t_N$$

Time Complexity: $\mathcal{O}(N^3)$

Design Rationale (V-D-O-O)

Layer: I

Vinegar

Diagonal

Layer: II

Vinegar

Oil

Layer: III

Vinegar

Oil

Design Rationale (V-D)

Goal: Find $x \in \mathbb{F}_q^n$, from $t = \mathcal{F}(x)$; $t \in \mathbb{F}_q^m$

Layer: I

$x_1, x_2, \dots, x_v$

$x_{v+1}, \dots, x_{v+d}$

$$\gamma_{v+1}^{(1)} x_{v+1} + c_1 = t_1$$

$$\gamma_{v+2}^{(2)} x_{v+2} + c_2 = t_2$$

$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$

$\vdots \quad \quad \quad \vdots \quad \quad \quad \vdots$

$$\gamma_n^{(d)} x_{v+d} + c_d = t_d$$

Design Rationale (V-D-O)

Layer: I

Vinegar

Diagonal

Layer: II

Vinegar

Oil

Design Rationale (V-D-O)

Layer: II

$x_1, x_2, \dots, x_v, \dots, x_{v+d}$

$x_{v+d+1}, \dots, x_{v+d+o_1}$

$$\gamma_{v+d+1}^{(d+1)} x_{v+d+1} + \gamma_{v+d+2}^{(d+1)} x_{v+d+2} + \dots + \gamma_{v+d+o_1}^{(d+1)} x_{v+d+o_1} = t_{d+1}$$

$$\gamma_{v+d+1}^{(d+2)} x_{v+d+1} + \gamma_{v+d+2}^{(d+2)} x_{v+d+2} + \dots + \gamma_{v+d+o_1}^{(d+2)} x_{v+d+o_1} = t_{d+2}$$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$$\gamma_{v+d+1}^{(d+o_1)} x_{v+d+1} + \gamma_{v+d+2}^{(d+o_1)} x_{v+d+2} + \dots + \gamma_{v+d+o_1}^{(d+o_1)} x_{v+d+o_1} = t_{d+o_1}$$

Design Rationale (V-D-O-O)

Layer: I

Vinegar

Diagonal

Layer: II

Vinegar

Oil

Layer: III

Vinegar

Oil

Design Rationale (V-D-O-O)

Layer: III

$x_1, x_2, \dots, x_v, \dots, x_{v+d}, \dots, x_{v+d+o_1}$

$x_{v+d+o_1+1}, \dots, x_n$

$$\gamma_{v+d+o_1+1}^{(d+o_1+1)} x_{v+d+o_1+1} + \gamma_{v+d+o_1+2}^{(d+o_1+1)} x_{v+d+o_1+2} + \dots + \gamma_n^{(d+o_1+1)} x_n = t_{d+o_1+1}$$

$$\gamma_{v+d+o_1+1}^{(d+o_1+2)} x_{v+d+o_1+1} + \gamma_{v+d+o_1+2}^{(d+o_1+2)} x_{v+d+o_1+2} + \dots + \gamma_n^{(d+o_1+2)} x_n = t_{d+o_1+2}$$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$\vdots$

$$\gamma_{v+d+o_1+1}^{(m)} x_{v+d+o_1+1} + \gamma_{v+d+o_1+2}^{(m)} x_{v+d+o_1+2} + \dots + \gamma_n^{(m)} x_n = t_m$$

Parameters

Security Level	Parameters (q, v, d, o_1, o_2) + salt	Signature Size (B)	Public Key (KB)
SL-1 (128-bit)	(16,60,30,34,36)	96	236
SL-3 (192-bit)	(256,100,30,40,40)	226	2437
SL-5 (256-bit)	(256,120,50,60,70)	316	8127



Careful Cryptanalysis

Chabhi Kaha Hai.

Structural attacks -- Forgery

1. Kipnis-Shamir attack [KS98]
2. Intersection attack [Beullens-21]
 - Simple attack [Beu22]
3. Rectangular min-rank attack [Beu21]
 - Combine (simple + rectangular min-rank) attack [Beu22]

Find an equivalent composition

$$\mathcal{P} = \mathcal{S}' \circ \mathcal{F}' \circ \mathcal{T}'$$

Structural attacks -- Forgery

1. Kipnis-Shamir attack [KS98]
2. Intersection attack [Beullens-21]
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Find an oil vector

VDOO is Secure

Parameter set	Simple attack	Combine attack	Intersection attack
Security level-I (128-bit)	134	136	141
Security level-III (192-bit)	207	194	229
Security level-V (256-bit)	270	264	293

Provable Security?

- Traditional MQ signature algorithms often depend on *ad-hoc* assumptions.
- While UOV Problem is well understood.
- The *EUF-CMA security of VDOO* signature scheme reduces to its EUF-KOA security.
- EUF-KOA security of VDOO scheme reduces to the *hardness of UOV problem (+ VDOO problem)*.
- Implying: **VDOO is EUF-CMA secure.**

EUFCMA:: Existential Unforgeability under Chosen Message Attack

EUFKOA:: Existential Unforgeability under Key Only Attack

Comparison



VDOO is Short and Fast

Algorithm	Sign size (B)	Public key size (KB)	Computational bottleneck in signing
VDOO	96	238	$GE_{(16,34)} + GE_{(16,36)}$
Mayo	387	1	$GE_{(16,65)}$
Rainbow	128	861	$GE_{(256,32)} + GE_{(256,48)}$
Unbalanced Oil-Vinegar	134	335	$GE_{(256,64)}$
QR-UOV	331	21	$GE_{(7,100)}$
TUOV	80	65	$GE_{(16,64)} + GE_{(16,32)}$

$GE_{(q,m)}$: Gaussian elimination of a system of m equations over $\mathbb{F}_q$

w.r.t. SL-1 parameters

Shortest among Standardized Signatures

Algorithms	Signature size (B)	Public Key size (B)
VDOO	96	23813
Crystals Dilithium	2420	1312
Falcon	666	897
SPHINCS+	7856	32

w.r.t. SL-1 parameters

At the End...

Conclusion

1. VDOO offers 96 Bytes for 128-bit security level
2. Gaussian elimination is faster for VDOO central polynomial
3. No classical and quantum attacks are known
4. Thus, useful for practical purpose.

Future Scope

1. Can we further reduce public key size?
2. Can we prove the security in Quantum Random Oracle?
3. Implementation package?
4. Physical/ side-channel attacks?



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Any Questions?





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Thank You!

