CS748 - ARITHMETIC CIRCUIT COMPLEXITY

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END-SEMESTER EXAMINATION (2018-19/II)

POINTS: 80

DATE GIVEN: 19-APR DUE: 23-APR-2019 (5PM)

Rules:

- You are not allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Question 1: *[16 points]* Suppose a degree d polynomial f has a VP circuit of size s. For any $\Delta > 0$, show that f has a product-depth-Δ circuit of size $s^{O(d^{1/\Delta})}$.

Question 2: *[14 points]* Show that if there is a poly(s)-time blackbox PIT algorithm for size s, degree s circuits, then either E$\not\subseteq$ #P/poly OR there is a polynomial family $\{q_m\}_m$ in VNP that requires size $2^{\Omega(m)}$ VP circuits.

Question 3: *[20+30 points]* Recall *diagonal depth*-4 circuits, which is an important model to study in VP. Consider

$$C = \sum_{i\in[k]} c_i Q_i^{d_i} \in \mathbb{Q}[x_1, \ldots, x_n],$$

where $c_i \in \mathbb{Q}, d_i \in \mathbb{N}$, and Q_i's are polynomials of degree at most δ. In this problem we are interested in the case when δ is quite small (even the constant case is interesting!).

i) Prove that any such C computing the monomial $x_1 \cdots x_n$ satisfies $k = 2^{\Omega(n/\delta)}$.

[Hint: Use shifted partials.]

ii) Let C be given as a blackbox (with the promised parameters and size s). Construct a hitting-set in time poly$(s^{\delta \cdot \log s})$.

[Hint: Consider the trailing monomial in C.]

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