## CS747 - RANDOMIZED METHODS IN COMPUTATIONAL COMPLEXITY NITIN SAXENA

# MID-SEMESTER EXAMINATION (2018-19/I) <br> POINTS: 45 

DATE GIVEN: 13-SEP
DUE: 18-SEP-2018

## Rules:

- You are not allowed to discuss.
- Write the solutions on your own and honorably acknowledge the sources if any. http://cse.iitk.ac.in/pages/AntiCheatingPolicy. html
- Clearly express the fundamental idea of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Question 1: [10 points] Show that $\mathrm{P} \neq \mathrm{BPP}$ implies $\mathrm{P} \neq \mathrm{NP}$.
Question 2: [10 points] Recall Razborov-Smolensky's approximator polynomial technique. Can it be generalized to prove $\bmod _{p} \notin \mathrm{ACC}^{0}\left[q^{m}\right]$, where $p, q$ are distinct primes and $m \in \mathbb{N}$ ? Give a detailed response.

Question 3: [10 points] Let $G$ be an undirected non-complete regular graph with self-loops. Let $A$ be its normalized adjacency matrix. Show that the eigenvalue of $A$ with the second-largest value (note- we are not considering the absolute value) is positive.

Question 4: $[12+3$ points] Let $G$ be an $(n, d, \lambda)$-expander. Prove the following combinatorial expansion property: For all $S, T \subseteq V(G)$,

$$
\left||E(S, T)|-\frac{d}{n}\right| S|\cdot| T|\mid \leq \lambda d \cdot \sqrt{|S| \cdot|T|}
$$

$E(S, T)$ denote the set of edges from $S$ to $T$.
Deduce an upper bound on the size of independent sets in $G$.

