CS747 - RANDOMIZED METHODS IN COMPUTATIONAL COMPLEXITY NITIN SAXENA

MID-SEMESTER EXAMINATION (2018-19/I)

POINTS: 45

DATE GIVEN: 13-SEP

DUE: 18-SEP-2018

<u>Rules</u>:

- You are not allowed to discuss.
- Write the solutions on your own and honorably *acknowledge* the sources if any. http://cse.iitk.ac.in/pages/AntiCheatingPolicy. html
- Clearly express the fundamental *idea* of your proof/ algorithm before going into the other proof details. The distribution of partial marks is according to the proof steps.
- There will be a penalty if you write unnecessary or unrelated details in your solution. Also, do not repeat the proof details covered before.

Question 1: [10 points] Show that $P \neq BPP$ implies $P \neq NP$.

Question 2: [10 points] Recall Razborov-Smolensky's approximator polynomial technique. Can it be generalized to prove $\operatorname{mod}_p \notin \operatorname{ACC}^0[q^m]$, where p, q are distinct primes and $m \in \mathbb{N}$? Give a detailed response.

Question 3: [10 points] Let G be an undirected non-complete regular graph with self-loops. Let A be its normalized adjacency matrix. Show that the eigenvalue of A with the second-largest *value* (note- we are not considering the absolute value) is positive.

Question 4: [12+3 points] Let G be an (n, d, λ) -expander. Prove the following combinatorial expansion property: For all $S, T \subseteq V(G)$,

$$\left| |E(S,T)| - \frac{d}{n} |S| \cdot |T| \right| \leq \lambda d \cdot \sqrt{|S| \cdot |T|}.$$

E(S,T) denote the set of edges from S to T.

Deduce an upper bound on the size of independent sets in G.

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