- To do this, we start with a trdeg one field $k \subset K$. Consider the set $\underline{\underline{C_K}}$ of all dvr's of K. We make C_K into a topological space.

- <u>Defn</u>: • Let $C_K := \{ v \mid \text{valuation } v \text{ on } K \text{ wrt dvr } R_v \text{ \& max. ideal } \mathfrak{m}_v \}$.
  • Let <u>closed sets</u> of C_K be defined as those subsets that are finite or C_K itself.
  • The <u>open sets</u> of C_K are complement of closed.
  • For open $U \subseteq C_K$ define the <u>ring of regular fns.</u> $\underline{\mathcal{O}(U)} := \bigcap_{v \in U} R_v$.

▷ Each $f \in \mathcal{O}(U)$ defines a distinct fn. $U \to k$.

<u>Pf</u>: • Consider the fn. $\begin{array}{l} U \to k \\ v \mapsto f \bmod \mathfrak{m}_v \end{array}$.

• Fns. corresponding to f, g are the same if $f \equiv g \bmod \mathfrak{m}_v$, for all $v \in U$.
• This means (since $\mathfrak{m}_v$ is principal & U is infinite) that $f = g$. $\square$

▷ For every $f \in K$, $\exists$ open $U \subseteq C_K$ s.t. $f \in \mathcal{O}(U)$.

Pf: • Let $f = g/h$ for polynomials g, h (over k).

• f is not defined over a pt. $v \in C_K$ iff $h \equiv 0 \pmod{m_v}$.

• Again, since h is a polynomial & m_v principal, the above can happen only for finitely many v.

$\Rightarrow f$ is defined on some open $U \subseteq C_K$. □

- Defn: • We call C_K (together with the regular fns. & its fn. field K) an abstract curve.

• A morphism $\phi : X \to Y$ between abstract curves or varieties is a continuous map s.t. $\forall$ open $V \subseteq Y$, $\forall f \in \mathcal{O}_Y(V)$, $f \circ \phi \in \mathcal{O}_X(\phi^{-1}(V))$.

- They arise naturally as:

- Exercise: every non-singular curve is isomorphic to an abstract curve.

- Next, we show a converse!