– Exercise: $A(X)_{(0)} = K(X)$.

▷ Thus, $\dim X = \operatorname{trdeg} A(X) = \operatorname{trdeg} K(X)$.

– Eg. In $\mathbb{A}^2_k$, $X = Z(x_2^2 - x_1^3)$ has
$\dim = \operatorname{trdeg}_k \text{fr-field}(k[x_1, x_2]/(x_2^2 - x_1^3))$
$= 1$

$\parallel$ $k(x_1)[x_2]/\langle x_2^2 - x_1^3\rangle$

– Let us now look at the projective case:

Exercise: The result $\mathcal{O}_X(X_f) = S(X)_f$ holds for projective X as well with the definitions: $S(X) := S/I(X)$, for a homogeneous $f \in S$,
$$S(X)_f := \{ g/f^r \mid r \in \mathbb{N}_{\geq 0},\ \deg g = \deg f^r, \ \& \ g \text{ is homogeneous} \}.$$

– Proposition 1: $\mathcal{O}_X(X) = k$ for projective X.

Pf: $\mathcal{O}_X(X) = \mathcal{O}_X(X_1) = S(X)_1 = k$. $\square$

Similarly,

- **Proposition 2**: For a pt. P in a projective variety X, $\mathcal{O}_{X,P} = S(X)_{m_P}$.
- Pf: <the affine pf, modified> $\square$

- Thus, although $\mathcal{O}_X(X)$ trivializes for a PV X, the local information at a pt. is more or less preserved.

A more "local" kind of Morphism

- Definitions: • For varieties X, Y, a rational map $\varphi: X \to Y$ is given as:

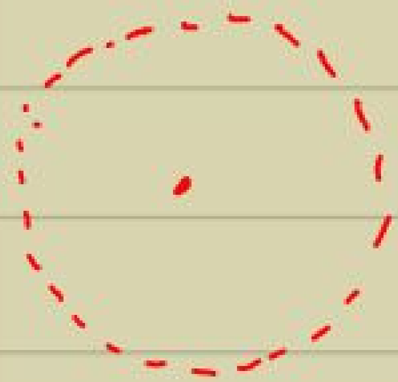
$$\{(U, \varphi_U) \mid \text{open } U \subseteq X, \text{morphism } \varphi_U : U \to Y\}/\sim$$

where, $(U, \varphi_U) \sim (V, \varphi_V)$ if $\varphi_U = \varphi_V$ on $U \cap V$.

• A rational map φ is dominant if $\exists$ open $U \subseteq X$ s.t. $\varphi_U(U)$ is dense in Y.

W is dense if the smallest closed set containing it is Y.

- In the real case $\mathbb{R}^2$ the open balls do not have their closure as the whole space.



- On the other hand, in Zariski topology every open set is <u>dense</u>, i.e. the closure is everything!

- <u>Claim</u>: Say, $U \subseteq \mathbb{A}^n_k$ is open and V is a closed set containing U. Then, $V = \mathbb{A}^n_k$.

<u>Pf</u>:
- Suppose $V \neq \mathbb{A}^n$.
- Since $\mathbb{A}^n \setminus V$ is open, we can show $U \cap (\mathbb{A}^n \setminus V) \neq \phi$. (Otherwise, $\mathbb{A}^n_k$ can be reduced!)

- Thus, $V = \mathbb{A}^n$ and U is <u>dense</u>. $\square$