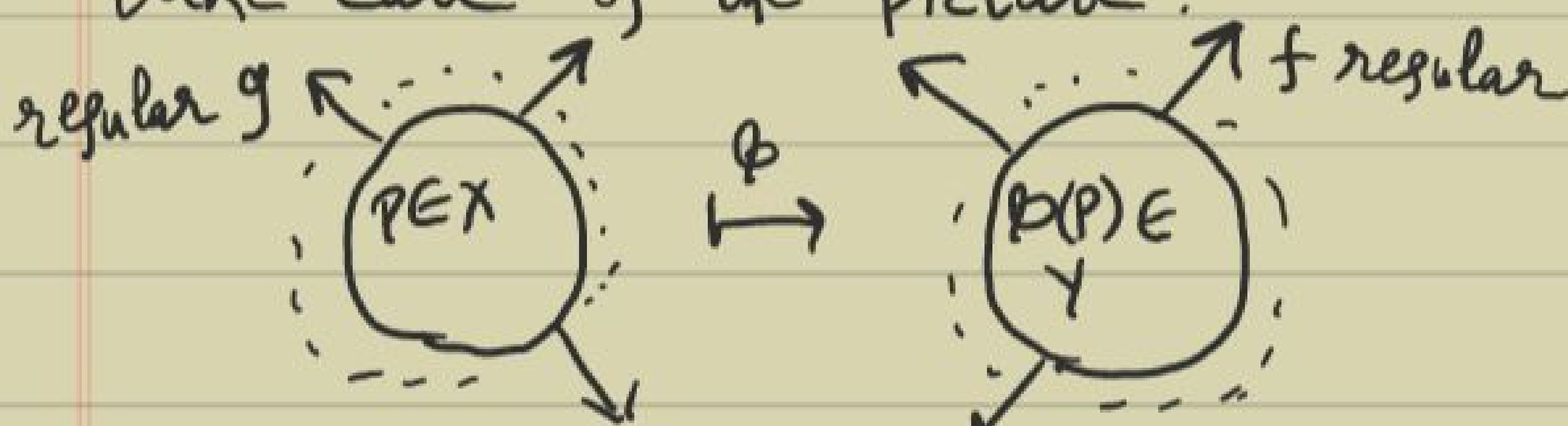
- From now on, we use <u>variety</u> to refer to AV, quasi-AV, PV or quasi-PV.

- A "good" map ϕ between varieties X, Y should not only be continuous but also "take care" of the picture:



- <u>Defn</u>: For varieties X, Y, a <u>morphism</u> $\phi : X \to Y$ is a continuous map satisfying: $\forall$ open $V \subseteq Y$, $\forall$ regular $f : Y \to k$,

$$\underbrace{f \circ \phi}_{\text{pullback of } f} : \underbrace{\phi^{-1}(V)}_{\text{open in } X} \to k \text{ is a regular fn.}$$

- For a variety Y, <u>$\mathcal{O}(Y)$</u> is the <u>ring of regular fns. on Y</u>.

- eg. $\mathcal{O}(\mathbb{A}^1_{\mathbb{C}}) = \mathbb{C}[x_1] = A$.
  $\mathcal{O}(\mathbb{P}^1_{\mathbb{C}}) = \mathbb{C}$.

- But, there are many other fns. outside $\mathcal{O}(Y)$ that are regular on some <u>open</u> patch $U \subsetneq Y$.
- So, now on, we will use (U,f) to denote a regular fn. f on an open $U \subseteq Y$.
- They afford a natural equivalence relation:

- <u>Defn</u>: • $(U,f) \sim (V,g)$ if $f = g$ on $U \cap V$.
  • The set of equivalence classes:
  $\{(U,f) \mid \text{open } U \subseteq Y, \text{ regular } f \text{ on } U\} / \sim$
  is a field, denoted by <u>$K(Y)$</u>, called the <u>fn. field of Y</u>.
  • The elements of $K(Y)$ are called <u>rational fns. on Y</u>.

- For a pt. $P \in Y$, let us look at fns. that are regular on "arbitrarily small" nbds. of P.

- Defn: • The <u>germs on Y near P</u> is the set $\mathcal{O}_P(Y) :=$ $\{(U,f) \mid \text{open } U \subseteq Y,\ P \in U,\ \text{regular } f \text{ on } U\}/\sim$.
  • The <u>germs vanishing at P</u> is the subset $\mathfrak{m}_P :=$ $\{(U,f) \in \mathcal{O}_P(Y) \mid f(P) = 0\}$.

- The following properties can be shown as an exercise:

- <u>Proposition</u>: (1) $\mathcal{O}_P(Y)$ is a <u>k-algebra</u> with $\mathfrak{m}_P$ as an <u>ideal</u>.
  (2) $\mathcal{O}_P(Y)/\mathfrak{m}_P$ is a <u>field</u> extension of k.
  (I.e. $\mathfrak{m}_P$ is a <u>maximal</u> ideal.)
  (3) $\mathfrak{m}_P$ is the <u>unique</u> maximal ideal.
  (I.e. $\mathcal{O}_P(Y)$ is a <u>local ring</u>.)