Directed Planar Reachability reduces to Grid Graph Reachability in Logspace

By
Tanvi Soni

Directed Planar Graph Reachability logspace reducible to Grid Graph Reachability

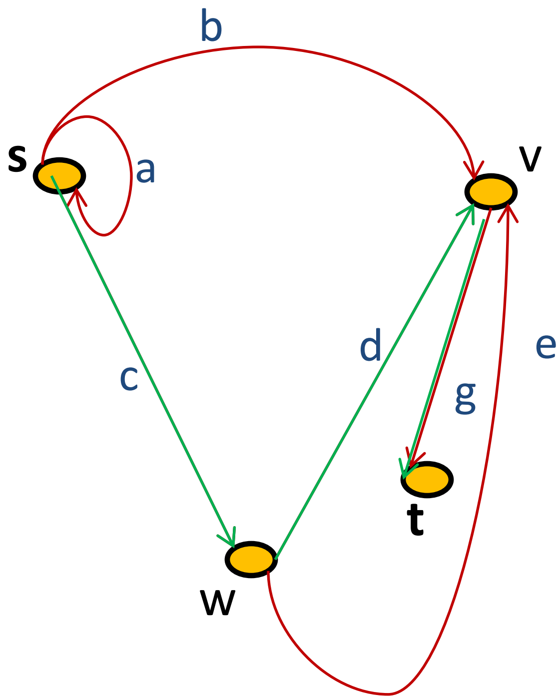
- Obtain undirected spanning tree T of directed planar graph G
- Impose T on grid graph
- Lay all non-tree edges on grid graph
- s lies on top left corner of grid, t lies on bottom right corner of grid
- Reduction is complete!!!

Grid graph – Vertices lie on a grid and edges are between two adjacent vertices on grid

We assume that vertices s and t lie on the same external face.

i. If s and t already lie on same face, then there is no problem. Any face can be assumed to be external face.

ii. If s and t lie on different face, then we may need to reorient the edges of the graph.

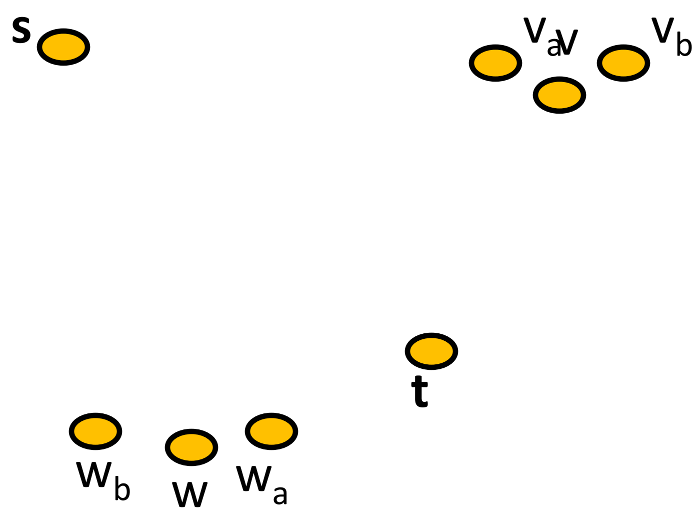


$P = s \rightarrow w \rightarrow v \rightarrow t$

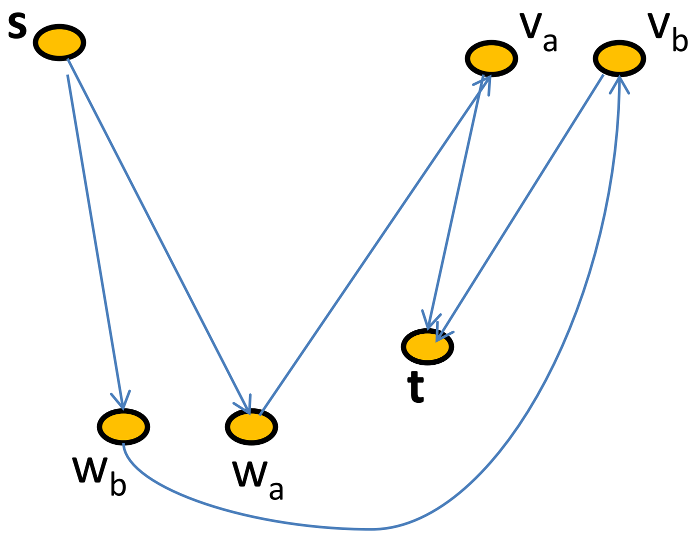
If s and t do not lie on the same face, then

i. Determine undirected path from s to t. - Can be done in logspace
ii. If there is no such path, then there cannot exist directed path from s to t.
iii. Else, let P be the required undirected path.

$$P = (s, v_1, v_2, \ldots, v_m, t)$$

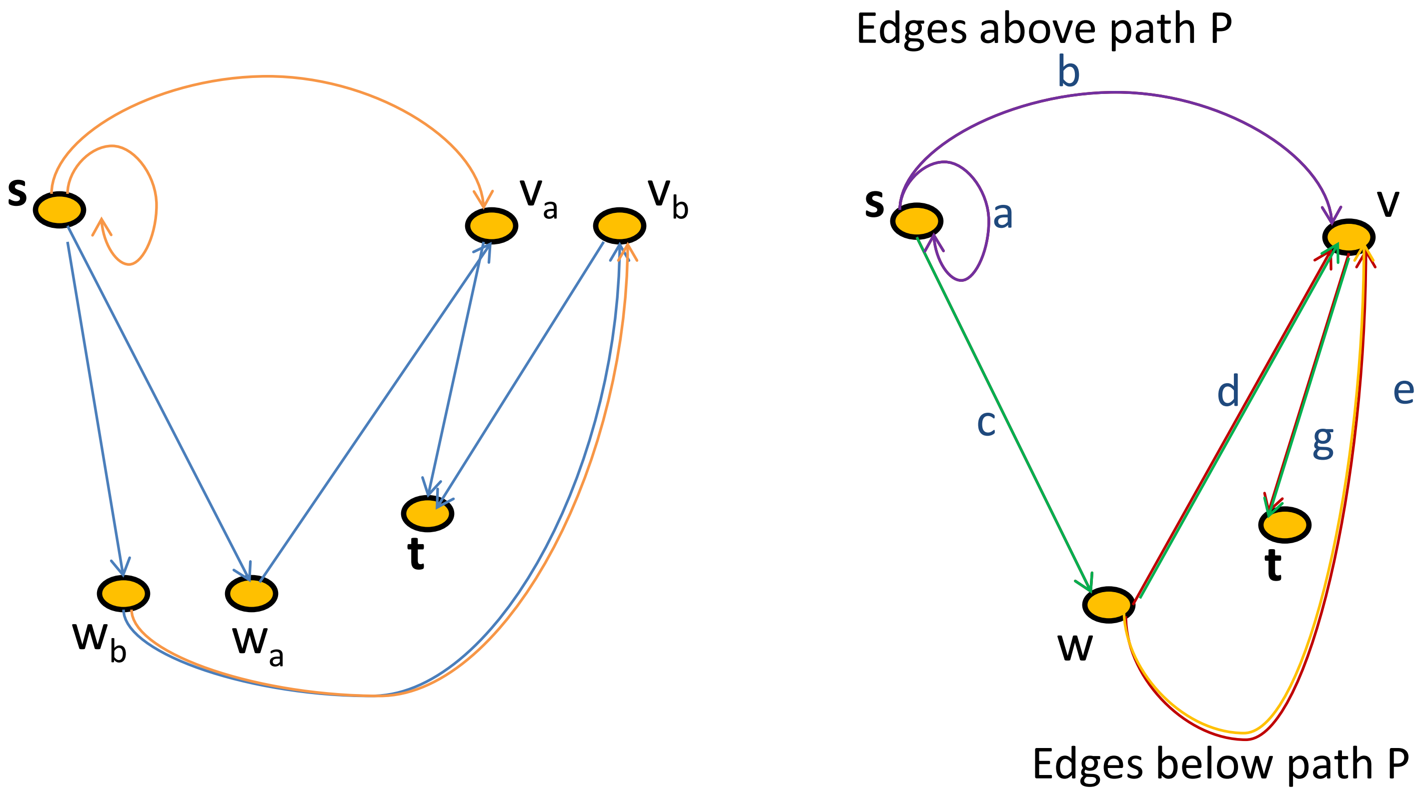


Duplicate all the v_i's on path P = (s, v_1, v_2, … , v_m, t)
Therefore, for each v_i, we have $v_{i,a}$ and $v_{i,b}$



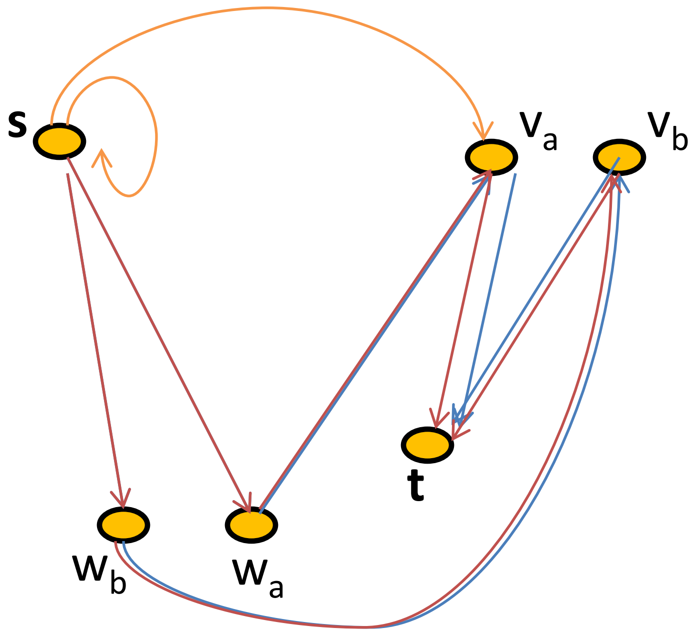
Connect vertices marked **a** with each other.

Similarly, for vertices marked with **b**.



Edges above (or left) of the path are connected to vertices marked with **a**

Edges below (or right) of the path are connected to vertices marked with **b**



Essentially, we cut the graph along path P and reoriented the edges around that path so that we could obtain s and t on the same face.

Now we reduce directed Planar s-t reachability to grid graph reachability

We assume the following:

- All edges are unidirectional. (Merge vertices joined by bidirectional edges).

- Assume no vertex in G has degree > 3.

- Vertex s has degree 2

1. Compute undirected spanning tree.   - Possible in logspace

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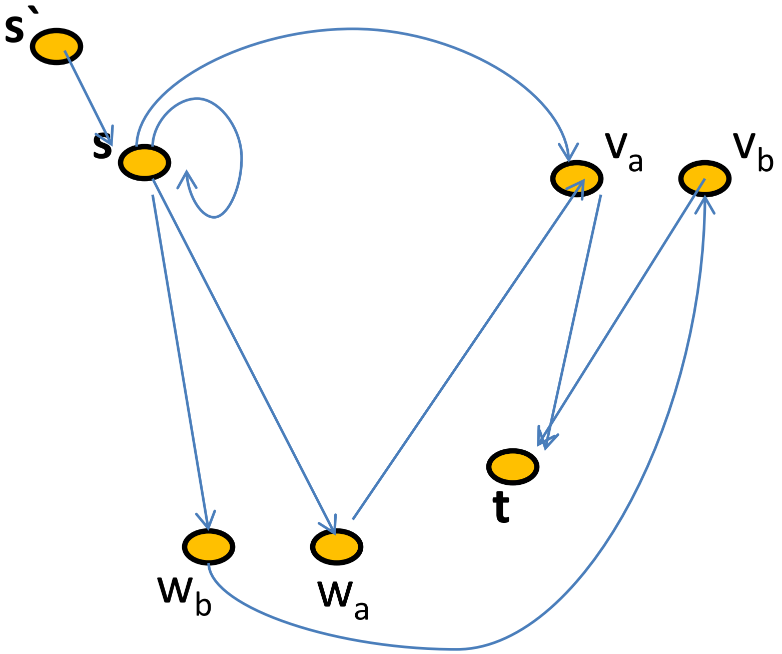
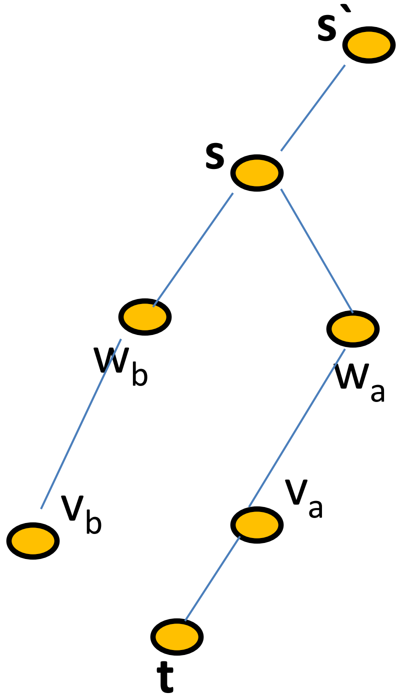
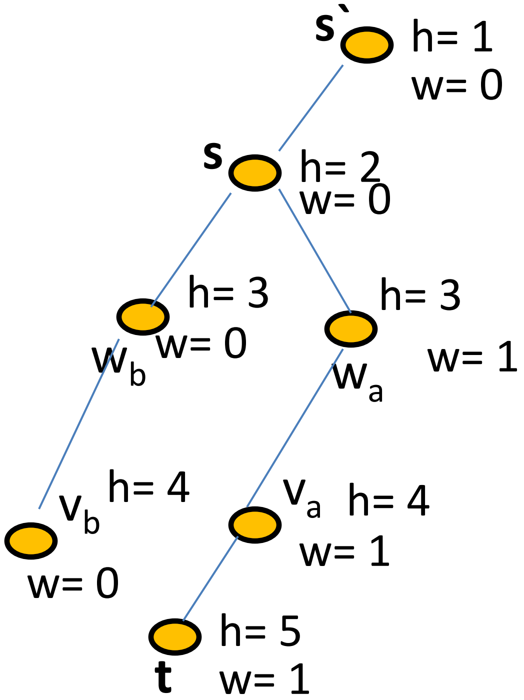


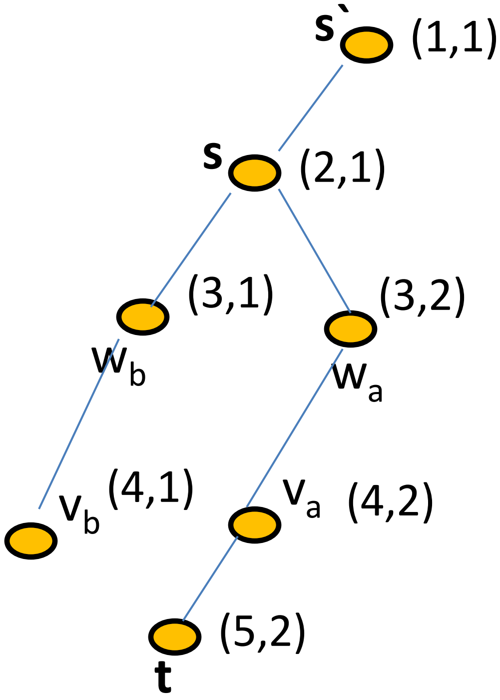


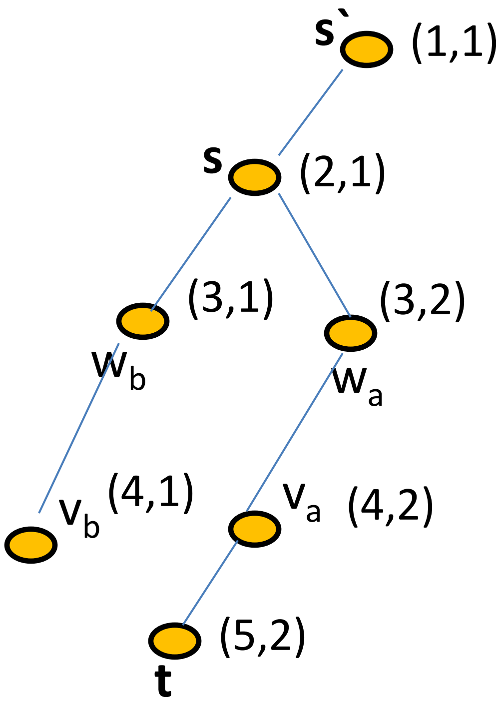
h(v) = height of vertex v

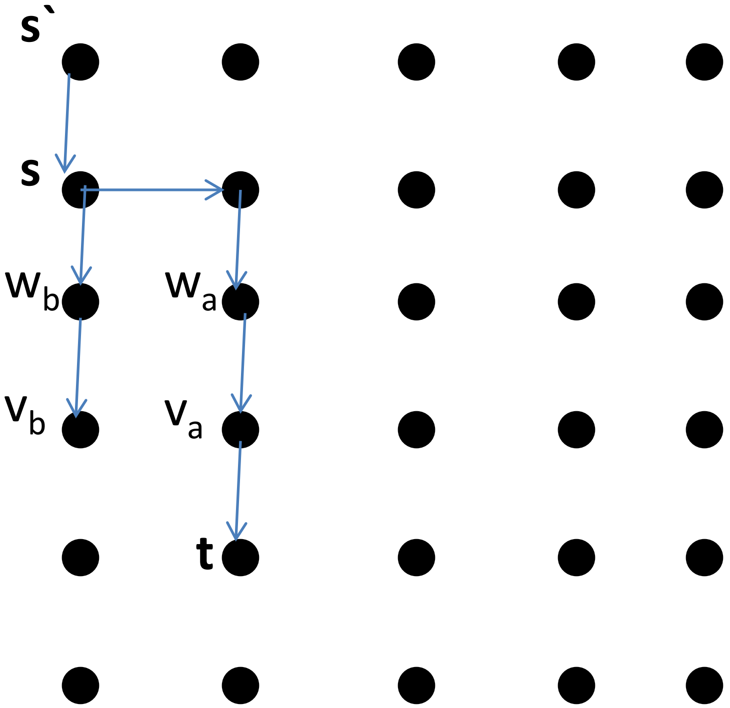
w(v) = number of leaf nodes strictly to the left of v

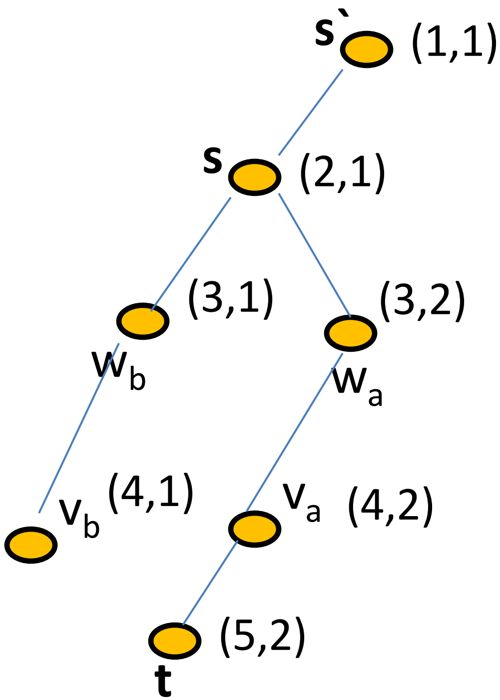


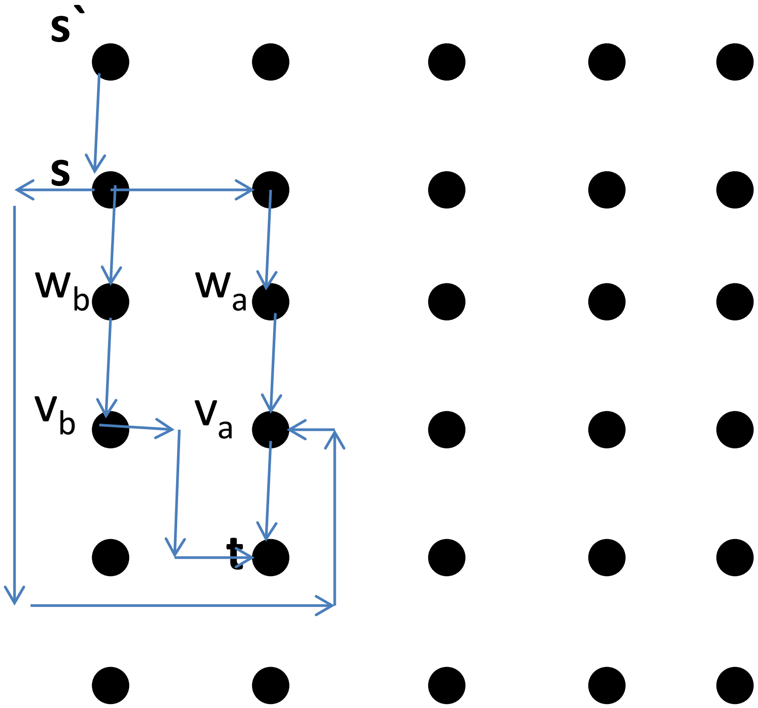
Coordinate of a vertex is (h(v), w(v)+1).




s`
(1,1)
s
(2,1)
(3,1)
(3,2)
w_b
w_a
v_b
(4,1)
v_a
(4,2)
(5,2)
t


s`
s
w_b
w_a
v_b
v_a
t





Hence, we obtain required grid graph that preserves s-t connectivity.

Reference:
The Directed Planar Reachability
By Eric Allender, Samir Datta and Sambuddha Roy