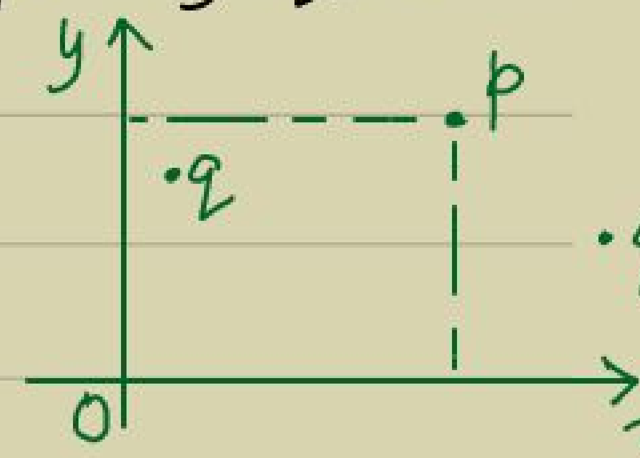
Non-dominated points problem

- Defn: A point p dominates q if $x(p) > x(q)$ & $y(p) > y(q)$.



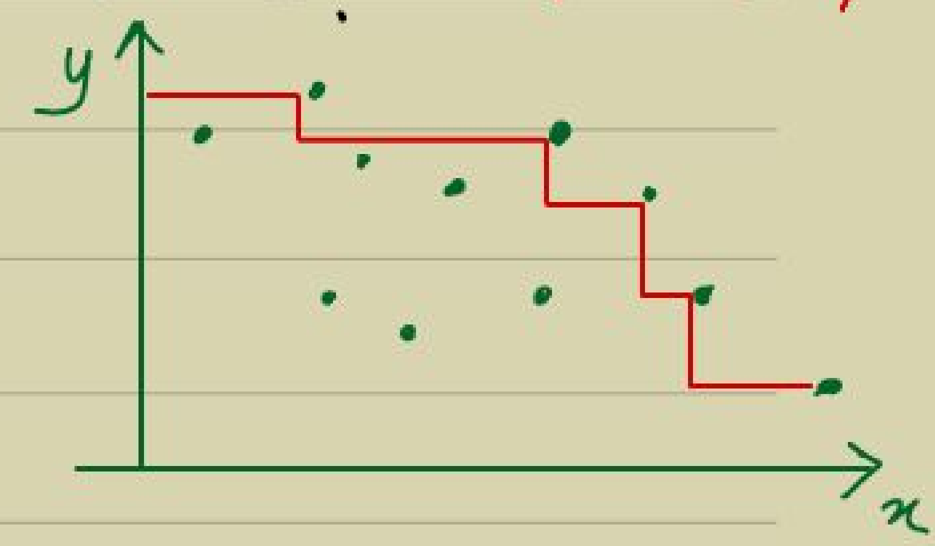
Input: Given n points $S \subset \mathbb{R}^2$.
Output: Points p in S that are not dominated by any point in S.

- Brute-force algorithm:
Go over every $p \in S$ & compare with each $q \in S$.
Takes $O(|S|^2)$ time.

- Can geometry help?

- What is the structure of non-dominated points?

- They form a staircase! (Exercise)



- Thus, these are extremal points in a sense.

▷ A point with max. x-coordinate is a non-dominated point.

- <u>Idea 1</u>: Among the points in S with the max. x-coordinate, pick the point p with max. y-coordinate.
    - Declare p non-dominated.
    - Delete all the points $q \in S$ with $y(q) < y(p)$.
    - Repeat till $S \neq \phi$.

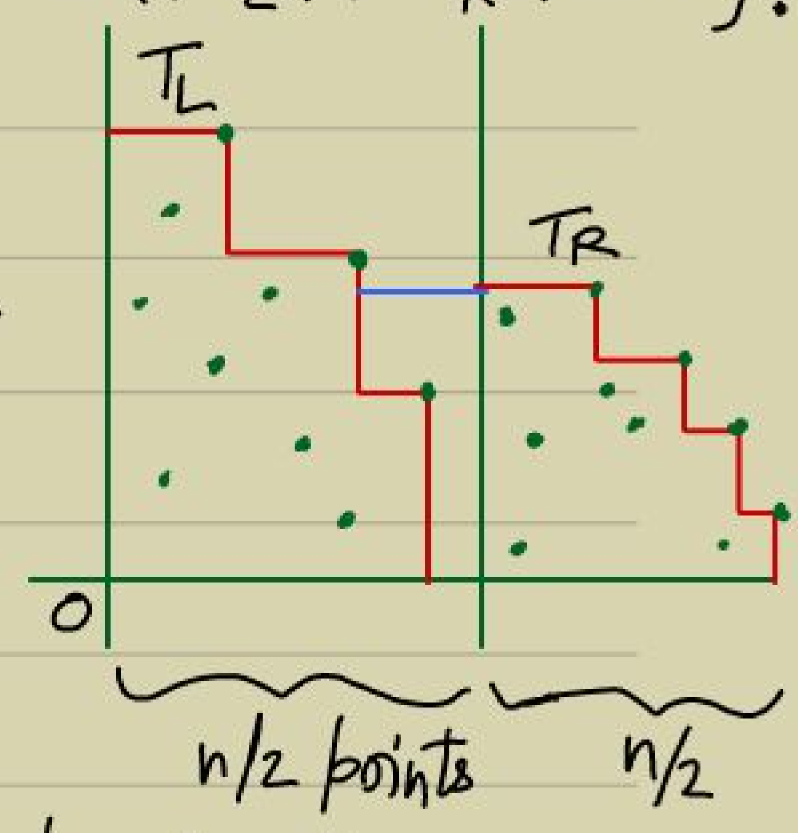
▷ If h = # non-dominated points in S, then the time complexity is $O(nh)$.

output-sensitive algorithm

- *Idea 2*: Divide S into two halves. Solve each & Merge the two staircases.
    - Let the two staircases be T_L & T_R.
    - During the merge step scan T_R to find the max-y-coordinate point $p \in T_R$.

      Scan T_L & delete those points q s.t. $y(q) < y(p)$.

      Take the union of T_L & T_R that remain.
    - Merge steps are $O(|T_L|+|T_R|)$ many.



Theorem: Non-dominated points in S can be computed in $O(|S| \lg |S|)$ time.

- by Kung, Luccio, Preparata (1975).