CS345– ALGORITHMS-II

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ASSIGNMENT 3

POINTS: 50

DATE GIVEN: 26-SEP-2019 DUE: 19-OCT-2019

Rules:

- You are strongly encouraged to work *independently*. That is the best way to understand & master the subject.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Submit your solutions, before time, to your specific TA. The dynamic student-to-TA mapping would be announced by Pranav.
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  Preferably, give the TA a printed copy of your LaTeXed or Word processed solution sheet.
- There will be a penalty if you write unnecessary or unrelated details in your solution. (Remember that in the exams you'll get limited space.)
- Clearly express the fundamental idea of your algorithm; sketch the main steps of the pseudocode; give a proof of correctness and that of the claimed time complexity. This decides the distribution of partial marks.
- Problems marked '0 points' are for practice.

Question 1: *[7 points]* You are given k sorted lists containing overall n numbers. Design an $O(n \log k)$ time algorithm to merge them into a single sorted list.

[Hint: Use a tree.]

Question 2: *[6 points]* Suppose you are given a set of n processes $P_1, \dots, P_n$, requiring processing times $p_1, \dots, p_n$ respectively. You want to schedule them so that the *average completion time* gets minimized. Eg. the schedule (P_2, P_1) has the average completion time $(p_2 + (p_2 + p_1))/2$.

Solve this problem in $O(n \log n)$ time.

Question 3: *[5+5+5+5 points]* Let $G = (V, E)$ be an undirected connected graph with weight function $w : E \to \mathbb{R}$, and suppose that the edge weights are *distinct*.

- Show that, in G, the minimum spanning tree (mst) is unique, but that the second-best mst need not be unique.
- Let T be the mst of G. Prove that there exist edges $(u, v), (x, y) \in T$ such that $T - \{(u, v)\} \cup \{(x, y)\}$ is a second-best mst of G.
- Let T' be a spanning tree of G and, for any two vertices $u, v \in V$, let $\max[u, v]$ be an edge of maximum weight on the unique path between u and v in T'. Describe an $O(|V|^2)$-time algorithm that, given T', computes $\max[u, v]$ for all $u, v \in V$.
- Give an efficient algorithm to compute the second-best mst of G.

Question 4: *[3+3 points]* For a directed graph $G = (V, E)$ define the *reverse* graph as $G^{\mathrm{T}} := (V, E^{\mathrm{T}})$, where $E^{\mathrm{T}} := \{(u, v) \mid (v, u) \in E\}$. Define the *component* graph $G^{\mathrm{SCC}} = (V', E')$ as the graph where each vertex in V' corresponds to a strongly connected component of G; there is an edge $(u, v) \in E'$ iff in G there is an edge from the strongly connected component u to the strongly connected component v. Show that G^{SCC} is a dag.

Further, show $((G^{\mathrm{T}})^{\mathrm{SCC}})^{\mathrm{T}} = G^{\mathrm{SCC}}$.

Question 5: *[6+5 points]* An *Euler tour* of a directed graph $G = (V, E)$ is a cycle that traverses each *edge* of G exactly once (it may revisit a vertex). Show that G has an Euler tour iff indeg(v)=outdeg(v) for each $v \in V$.

Give a linear-time algorithm to find an Euler tour (or say that it does not exist).

Question 6: *[0 points]* Give a simple example of a directed graph with negative-weight edges for which Dijkstra's algorithm produces incorrect answers. Where does the proof fail?

Question 7: *[0 points]* Suppose that we are given a weighted directed graph $G = (V, E)$ in which edges that leave the source vertex s may have negative weights, all other edge weights are nonnegative, and there are no negative-weight cycles. Argue that Dijkstra's algorithm correctly finds shortest paths from s in this graph.

Question 8: *[0 points]* Let $G = (V, E)$ be a weighted directed graph with weight function $w : E \to \mathbb{R}$ and source vertex s. Then, for all edges $(u, v) \in E$, prove the *triangle inequality*: $\delta(s, v) \leq \delta(s, u) + w(u, v)$.

For a given v, does the equality hold?

Question 9: *[0 points]* In a dag we can count the number of paths in linear-time. How fast could you count simple-paths in a general graph?

Question 10: *[0 points]* Given a directed graph, are strongly connected components *unique*?

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