CS345– ALGORITHMS-II

NITIN SAXENA

ASSIGNMENT 1

POINTS: 50

DATE GIVEN: 02-AUG-2019 DUE: 17-AUG-2019 (11PM)

<u>Rules</u>:

- You are strongly encouraged to work *independently*. That is the best way to understand & master the subject.
- Write the solutions on your own and honorably *acknowledge* the sources if any. `http://cse.iitk.ac.in/pages/AntiCheatingPolicy.html`
- Submit your solutions, before time, to your specific TA:
  13000 to 160620 – **Pranav Bisht** `pbisht@cse`
  160624 to 170195 – **Ashish Dwivedi** `ashish@cse`
  170201 to 170409 – **Abhibhav Garg** `abhibhav@cse`
  170412 to 170646 – **Nikhil Shagrithaya** `nshagri@cse`
  170648 to 170830 – **Abhimanyu** `abhimanu@iitk` .
  Preferably, give the TA a printed copy of your LaTeXed or Word processed solution sheet.
- There will be a penalty if you write unnecessary or unrelated details in your solution. (Remember that in the exams you'll get limited space.)
- Clearly express the fundamental idea of your algorithm; sketch the main steps of the pseudocode; give a proof of correctness and that of the claimed time complexity. This decides the distribution of partial marks.
- Problems marked '0 points' are for practice.

Question 1: *[3 points]* Consider a subset S of the planar square of *unit* sides. Suppose no two points in S are closer than distance 0.71. How large is S? Give a formal proof.

Question 2: *[5 points]* Show that if one can compute the convex hull of n points in the plane in $o(n \log n)$ time then one can also sort n numbers in $o(n \log n)$ time.

Question 3: *[10 points]* Show the following properties of the convex hull CH(S) of points $S \subset \mathbb{R}^2$:

- CH(S) is unique,
- the vertices of CH(S) are in S.

Question 4: *[15 points]* You saw a divide-conquer algorithm to compute the non-dominated points for a given set $S \subset \mathbb{R}^2$. It has complexity $O(n \log n)$, where $n := |S|$. Suppose the number of non-dominated points is h.

Modify the algorithm, or the analysis, so that the time complexity becomes $O(n \log h)$ time.

Question 5: *[17 points]* You are given two sets A and B of integers. Each has size n and the integers are in the range 0 to $10n$. Let $A + B$ denote the *cartesian-sum* multiset

$$\{a + b \mid a \in A, b \in B\}.$$

We want to compute this multiset $A + B$. Give an algorithm to do this in $O(n \log n)$ time.

Question 6: *[0 points]* Given an array $A[0 \ldots n-1]$ of integers, a pair (i, j) is an *inversion* if $i < j$ and $A[i] > A[j]$. Design an $O(n \log n)$ time algorithm to count the number of inversions in A.

Question 7: *[0 points]* In a set of points S, is it true that in the closest pair either the x-distance or the y-distance is minimized?

Question 8: *[0 points]* In closest pair algorithm done in the class– could we avoid median-finding by x-sorting S only once in the beginning?

Question 9: *[0 points]* Could 1-dim closest pair be solved faster than $O(n \log n)$ time?

Question 10: *[0 points]* For positive real numbers α, β, consider the recurrence

$$T(n) = T(\alpha \cdot n) + T(\beta \cdot n) + O(n).$$

Characterize the possible function T that you can get.

□□□