Idea: Given $v_1, v_2, ..., v_n$, compute $v'_1, v'_2, ..., v'_n$ and sort $v_1, v_2, ..., v_n$ in increasing order of $|v'_n|$. The reordered sequence $v'_1, v'_2, ..., v'_n$ is a reduced basis, but as we cannot claim $v'_1 = v'_1$, the proof of the earlier lemma about a reduced basis does not go through. Hence, we cannot get the shortest vector in this manner.

**First Algorithm proposed**

**Input:** $v_1, v_2, ..., v_n$

**Step 1:** Compute $u_1, u_2, ..., u_n$ from $v_1, v_2, ..., v_n$ using ’approximate orthogonalization’ process

**Step 2:** Check if $u_1, u_2, ..., u_n$ is a reduced basis
If not suppose the first violation occurs at index $i$.

**Step 3:** Swap $u_i$ and $u_{i+1}$, rename the sequence $v_1, v_2, ..., v_n$ and goto Step 1

This algorithm stops only if we have a reduced basis.

**Analysis of the above algorithm**

$$u'_i = u_i - \sum_{j<i} [\mu_{ij}] u_j$$

Denote the sequence as $\hat{u}_1, \hat{u}_2, ..., \hat{u}_n$ after the swap.
But we want $\hat{u}'_j = u'_j$ for all $j < i$ and $j > i$
Therefore we modify the above algorithm.

**Modified Algorithm**

**Input:** $v_1, v_2, ..., v_n$

**Step 0:** Let $u_i = v_i$

**Step 1:** for($i=1; i \leq n$; ) {

**Step 2:** Compute $u_i = v_i - \sum_{j<i} [\mu_{ij}] u_j$
& $u'_i = v'_i - \sum_{j<i} [\mu_{ij}] u'_j$

**Step 3:** Check if $|u'_{i+1}| \leq 2|u'_i|^2$
Step 4: If not, swap $u_{i+1}$ and $u_i$ and let $i = i - 1$
Step 5: else let $i = i + 1$

The analysis of the modified algorithm will follow in the next class.