CS738: Advanced Compiler Optimizations

Interprocedural Data Flow Analysis

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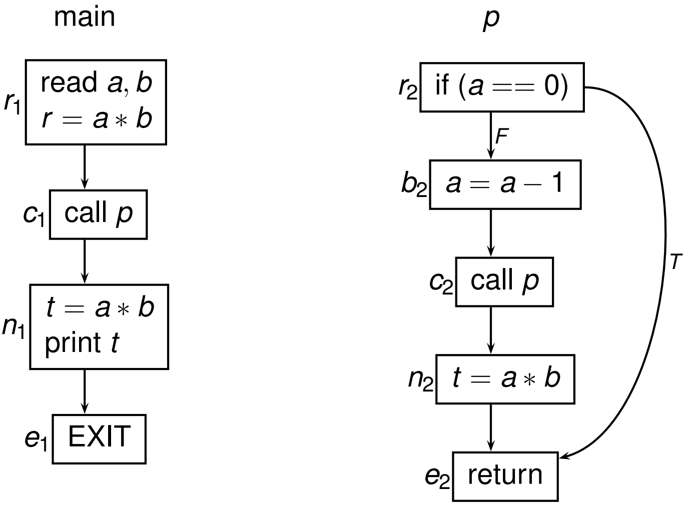
Interprocedural Analysis: WHY?

Is $a * b$ available at IN of n_1?



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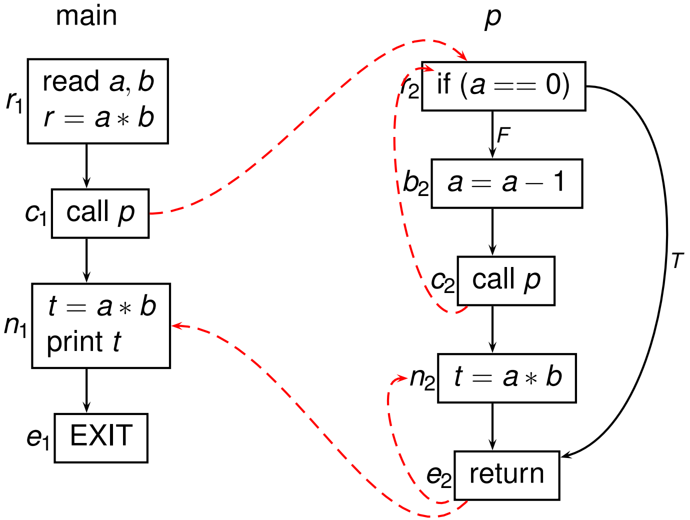
- Function pointers and virtual functions
- Dynamic functions (functional programs)

Infeasible Paths

How to avoid data flowing along invalid paths?

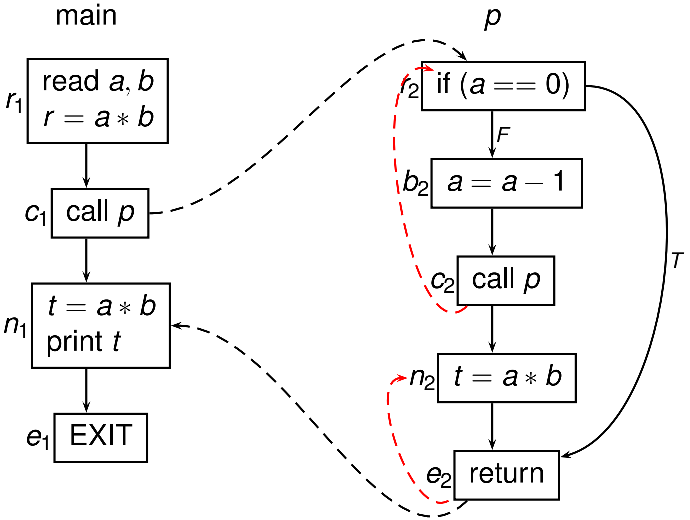
$r_1 \to c_1 \to r_2 \to b_2 \to c_2 \to r_2 \to e_2 \to n_1$



Recursion

How to handle Infinite paths?

$$\ldots \rightarrow r_2 \rightarrow c_2 \rightarrow r_2 \rightarrow c_2 \rightarrow r_2 \ldots$$



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- Dynamically created functions (in functional languages)
- No static control flow graph!

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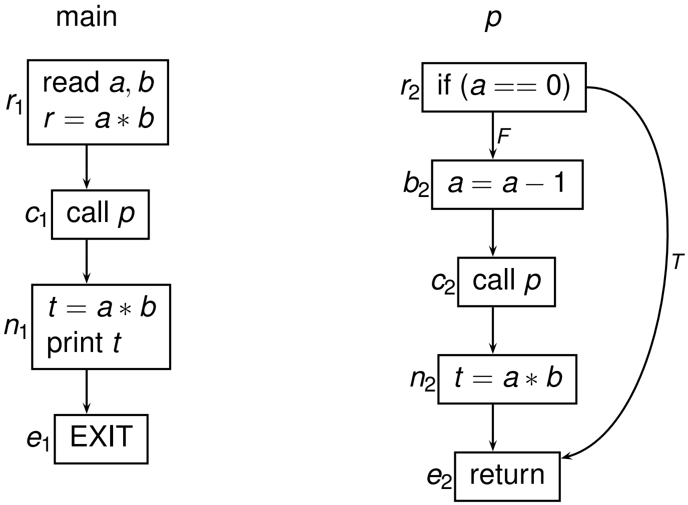
M. Sharir, and A. Pnueli. **Two Approaches to Inter-Procedural Data-Flow Analysis**. In Jones and Muchnik, editors, Program Flow Analysis: Theory and Applications. Prentice-Hall, 1981.

Notations and Terminology

Control Flow Graph

One per procedure



Control Flow Graph for Procedure *p*

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  - $\text{path}_G(m, n)$: Set of all path in graph $G = (N, E)$ leading from m to n

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  - recursion stack for formal parameters
- No procedure variables (pointers, virtual functions etc.)

Data Flow Framework

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- $f_{(m,n)} \in F$ represents propagation function for edge (m, n) of control flow graph $G = (N, E)$
  - Change of DF values from the *start* of m, through m, to the *start* of n

Data Flow Equations

$$
\begin{aligned}
x_r &= \mathit{BoundaryInfo} \\
x_n &= \bigwedge_{(m,n)\in E} f_{(m,n)}(x_m) \qquad n \in N - r
\end{aligned}
$$

- MFP solution, approximation of MOP

$$
y_n = \bigwedge \{ f_p(\mathit{BoundaryInfo}) : p \in \mathrm{path}_G(r, n) \} \quad n \in N
$$

Functional Approach
to
Interprocedural Analysis

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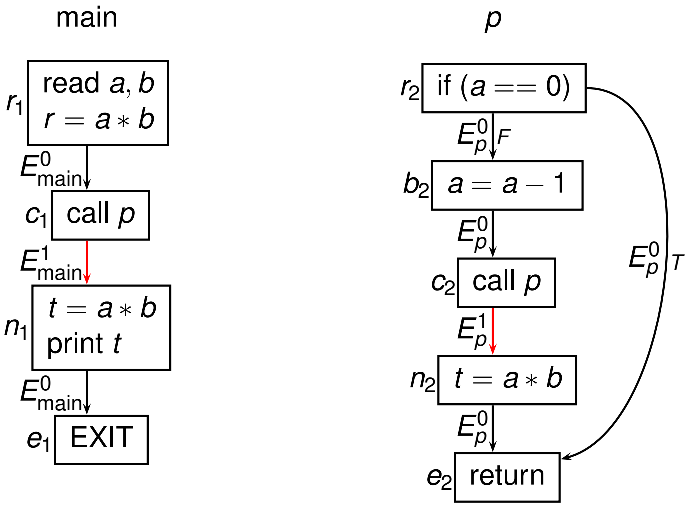
- Computes relationship between DF value at entry node and related data at *any* internal node of procedure
- At call site, DF value propagated directly using the computed relation

Interprocedural Flow Graph

First Representation:

$$
\begin{aligned}
G &= \bigcup\{G_p : p \text{ is a procedure in program}\} \\
G_p &= (N_p, E_p, r_p) \\
N_p &= \text{set of all basic block of } p \\
r_p &= \text{root block of } p \\
E_p &= \text{set of edges of } p \\
&= E_p^0 \cup E_p^1 \\
(m, n) \in E_p^0 &\Leftrightarrow \text{direct control transfer from } m \text{ to } n \\
(m, n) \in E_p^1 &\Leftrightarrow m \text{ is a call block, and } n \text{ immediately follows } m
\end{aligned}
$$

Interprocedural Flow Graph: 1^{st} Representation



Interprocedural Flow Graph

Second representation

$$
\begin{aligned}
G^* &= (N^*, E^*, r_1) \\
r_1 &= \text{root block of main} \\
N^* &= \bigcup_p N_p \\
E^* &= E^0 \cup E^1 \\
E^0 &= \bigcup_p E_p^0 \\
(m, n) \in E^1 &\Leftrightarrow (m, n) \text{ is either a } \textit{call} \text{ edge} \\
&\quad\; \text{or a } \textit{return} \text{ edge}
\end{aligned}
$$

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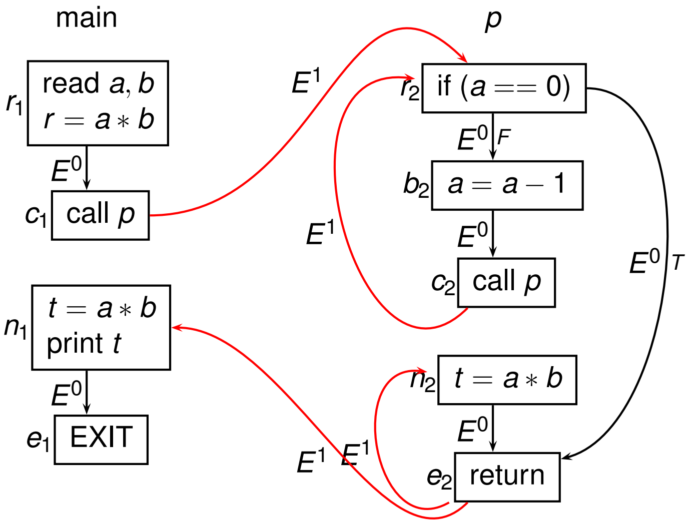
  - $(m, n) \in E_s^1$ for some procedure s

Interprocedural Flow Graph: 2^{nd} Representation



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- Path $q \in \text{path}_{G^*}(r_1, n)$ is in IVP(r_1, n)
  - iff sequence of all E^1 edges in q (denoted q_1)is *proper*

Proper sequence

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  - q_1' obtained from deleting $q_1[i-1]$ and $q_1[i]$ from q_1 is proper

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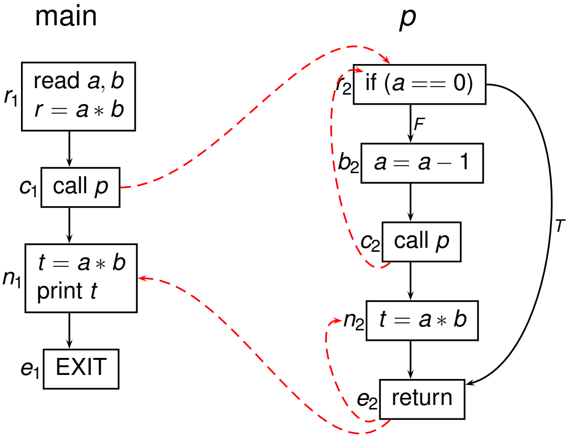
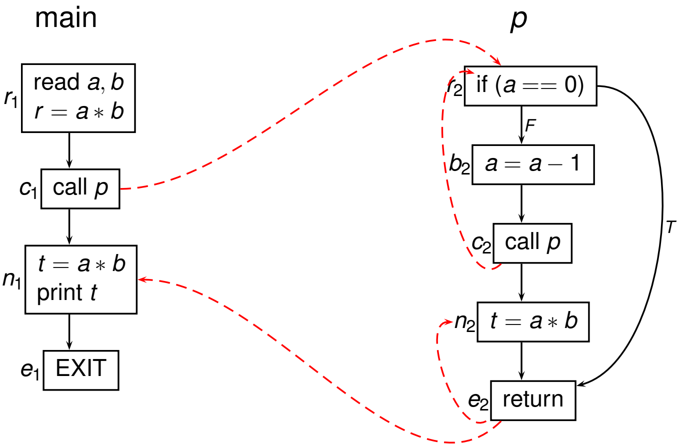
  - Each call edge has corresponding return edge in q restricted to E^1

IVPs


main
p
r_1
read a, b
$r = a * b$
c_1
call p
n_1
$t = a * b$
print t
e_1
EXIT
r_2
if ($a == 0$)
F
b_2
$a = a - 1$
c_2
call p
T
n_2
$t = a * b$
e_2
return

IVPs



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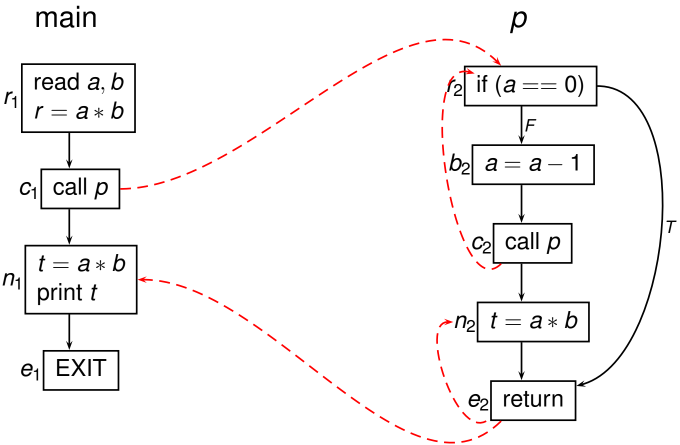

$$r_1 \rightarrow c_1 \rightarrow r_2 \rightarrow c_2 \rightarrow r_2 \rightarrow e_2 \rightarrow n_2 \rightarrow e_2 \rightarrow n_1 \rightarrow e_1 \in \text{IVP}(r_1, e_1)$$

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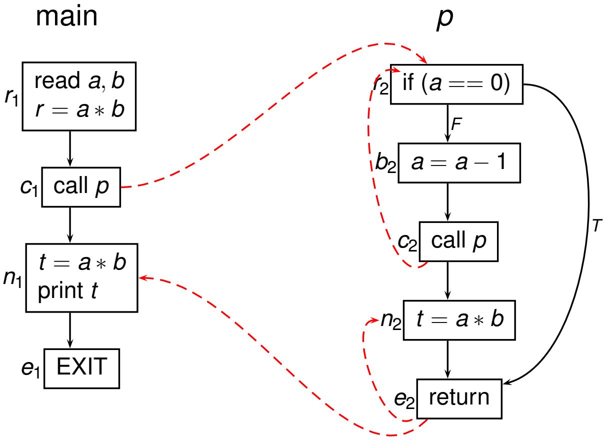

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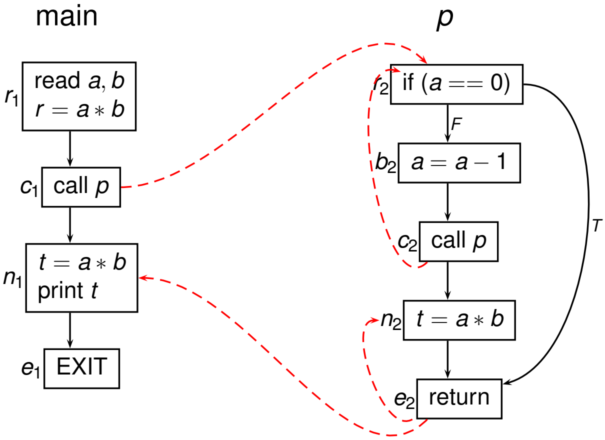

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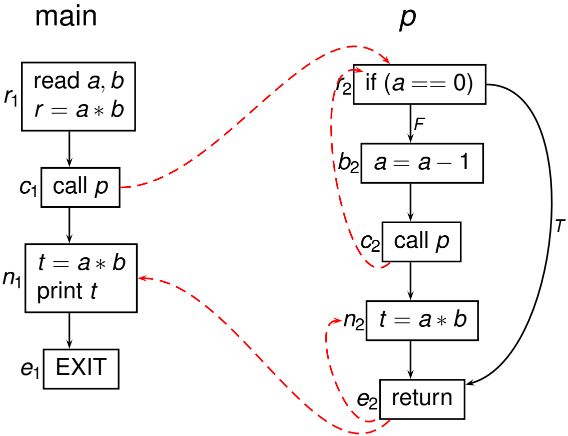
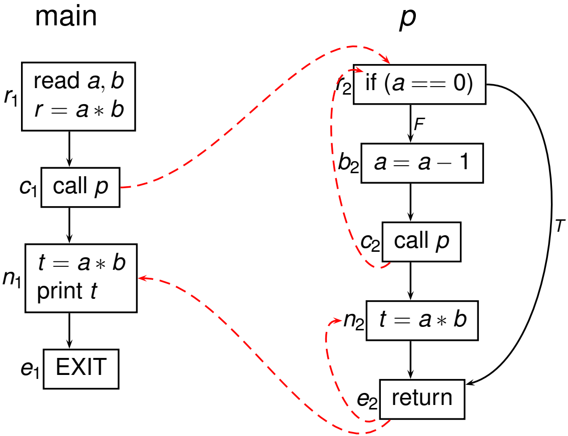


$$r_2 \rightarrow c_2 \rightarrow r_2 \rightarrow e_2 \rightarrow n_2$$

IVPs



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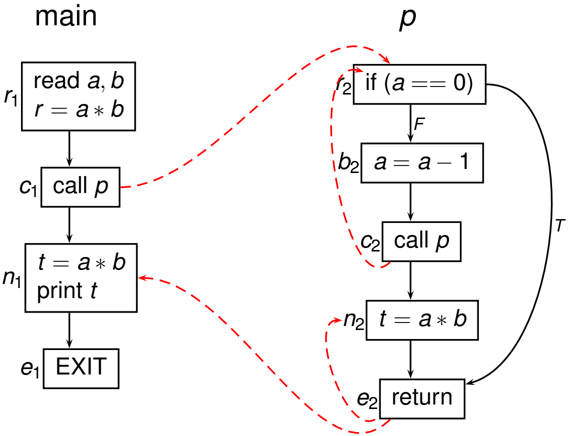

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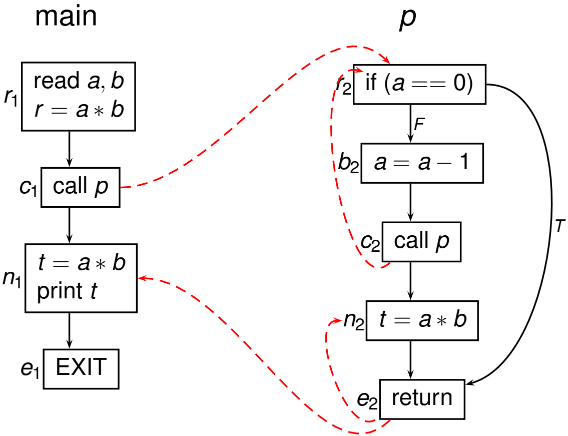

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Path Decomposition

$$
\begin{aligned}
q &\in \mathrm{IVP}(r_{\mathrm{main}}, n) \\
&\Leftrightarrow \\
q &= q_1 \parallel (c_1, r_{p_2}) \parallel q_2 \parallel \cdots \parallel (c_{j-1}, r_{p_j}) \parallel q_j \\
&\quad \text{where for each } i < j, q_i \in \mathrm{IVP}_0(r_{p_i}, c_i) \text{ and } q_j \in \mathrm{IVP}_0(r_{p_j}, n)
\end{aligned}
$$