CS738: Advanced Compiler Optimizations

Flow Graph Theory

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Agenda

- Speeding up DFA
- Depth of a flow graph
- Natural Loops

Acknowledgement

Rest of the slides based on the material at http://infolab.stanford.edu/~ullman/dragon/w06/w06.html

Speeding up DFA

- Proper ordering of nodes of a flow graph speeds up the iterative algorithms: **depth-first ordering**.
- "Normal" flow graphs have a surprising property — **reducibility** — that simplifies several matters.
- Outcome: few iterations "normally" needed.

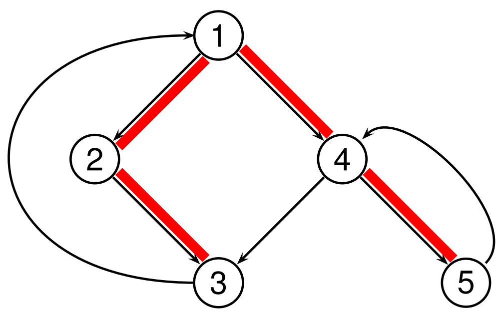
Depth-First Search

- Start at entry.
- If you can follow an edge to an unvisited node, do so.
- If not, backtrack to your *parent* (node from which you were visited).

Depth-First Spanning Tree (DFST)

- Root = *Entry*.
- Tree edges are the edges along which we first visit the node at the head.

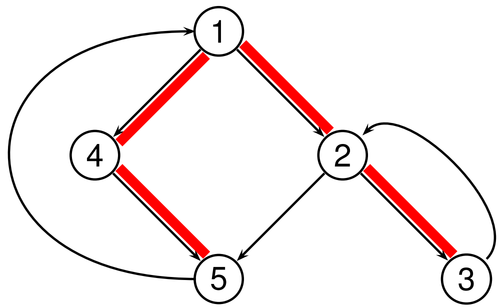
DFST Example



Depth-First Node Order

- The reverse of the order in which a DFS **retreats** from the nodes.
- Alternatively, reverse of postorder traversal of the tree.

DF Order Example



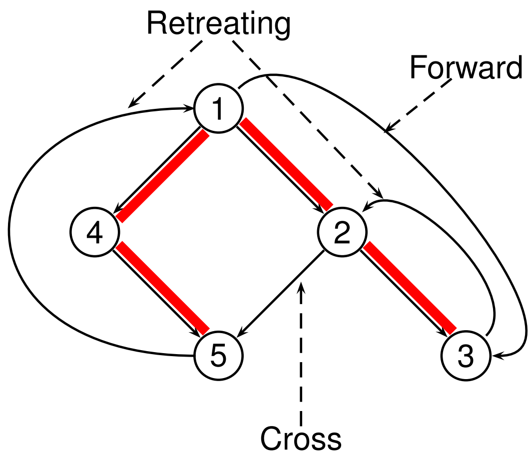
Four Kind of Edges

1. Tree edges.
2. **Forward edges**: node to proper descendant.
3. **Retreating edges**: node to ancestor.
4. **Cross edges**: between two node, neither of which is an ancestor of the other.

A Little Magic

- Of these edges, only retreating edges go from high to low in DF order.
- Most surprising: all cross edges go right to left in the DFST.
  - Assuming we add children of any node from the left.

Example: Non-Tree Edges



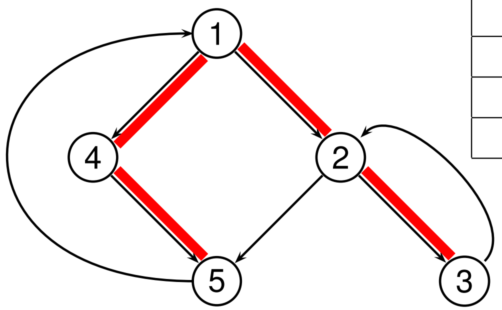
Roadmap

- "Normal" flow graphs are "**reducible**."
- "**Dominators**" needed to explain reducibility.
- In reducible flow graphs, loops are well defined, retreating edges are unique (and called "**back**" edges).
- Leads to relationship between DF order and efficient iterative algorithm.

Dominators

- Node *d* **dominates** node *n* if every path from the *Entry* to *n* goes through *d*.
- [Exercise] A forward-intersection iterative algorithm for finding dominators.
- Quick observations:
  - Every node dominates itself.
  - The entry dominates every node.

Example: Dominators



Node	Dominators
1	1
2	1, 2
3	1, 2, 3
4	1, 4
5	1, 5

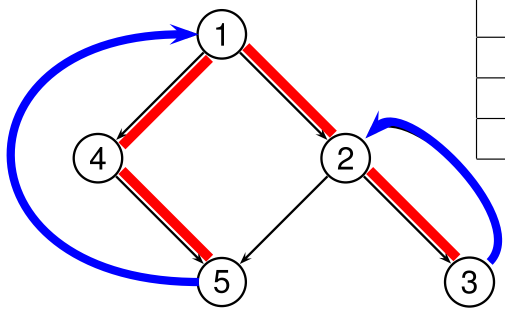
Common Dominator Cases

- The test of a while loop dominates all blocks in the loop body.
- The test of an if-then-else dominates all blocks in either branch.

Back Edges

- An edge is a **back edge** if its head dominates its tail.
- **Theorem:** Every back edge is a retreating edge in every DFST of every flow graph.
  - Proof? Discuss/Exercise
  - Converse almost always true, but not always.

Example: Back Edges

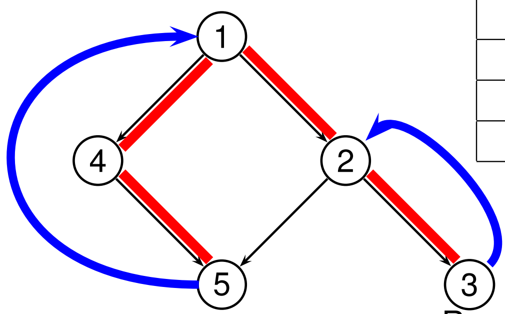


Node	Dominators
1	1
2	1, 2
3	1, 2, 3
4	1, 4
5	1, 5

Reducible Flow Graphs

- A flow graph is **reducible** if every retreating edge in any DFST for that flow graph is a back edge.
- **Testing reducibility:** Take any DFST for the flow graph, remove the back edges, and check that the result is acyclic.

Example: Remove Back Edges



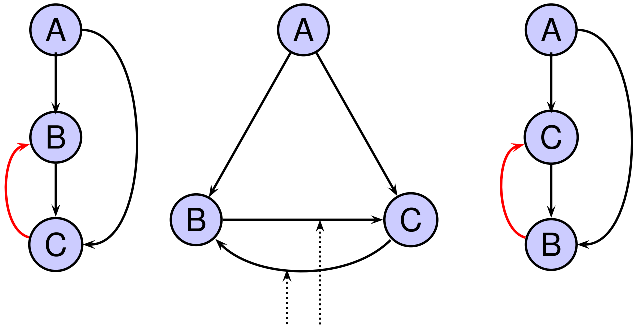
Node	Dominators
1	1
2	1, 2
3	1, 2, 3
4	1, 4
5	1, 5

Remaining graph is acyclic.

Why Reducibility?

- **Folk theorem:** All flow graphs in practice are reducible.
- **Fact:** If you use only while-loops, for-loops, repeat-loops, if-then(-else), break, and continue, then your flow graph **is** reducible.

Example: Nonreducible Graph



In any DFST, one of these edges will be a retreating edge.

Why Care About Back/Retreating Edges?

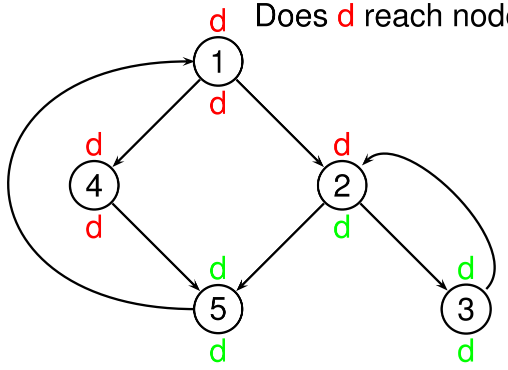
- Proper ordering of nodes during iterative algorithm assures number of passes limited by the number of “nested” back edges.
- Depth of nested loops upper-bounds the number of nested back edges.

DF Order and Retreating Edges

- Suppose that for a RD analysis, we visit nodes during each iteration in DF order.
- The fact that a definition *d* reaches a block will propagate in one pass along any increasing sequence of blocks.
- When *d* arrives along a retreating edge, it is too late to propagate *d* from OUT to IN.

Example: DF Order

Node 2 generates definition d.
Other nodes "empty" w.r.t. d.
Does d reach node **4**?



Depth of a Flow Graph

- The **depth** of a flow graph is the greatest number of retreating edges along any acyclic path.
- For RD, if we use DF order to visit nodes, we converge in depth+2 passes.
  - Depth+1 passes to follow that number of increasing segments.
  - 1 more pass to realize we converged.

Example: Depth = 2

Similarly . . .

- AE also works in depth+2 passes.
  - Unavailability propagates along retreat-free node sequences in one pass.
- So does LV if we use reverse of DF order.
  - A use propagates backward along paths that do not use a retreating edge in one pass.

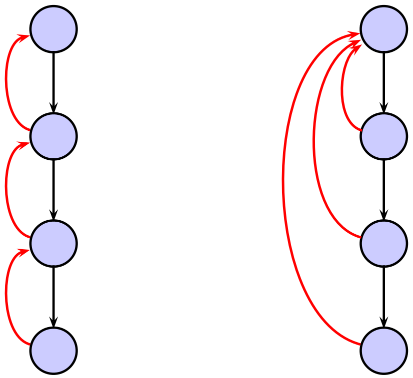
In General . . .

- The depth+2 bound works for any monotone bit-vector framework, as long as information only needs to propagate along acyclic paths.
  - Example: if a definition reaches a point, it does so along an acyclic path.

Why Depth+2 is Good?

- Normal control-flow constructs produce reducible flow graphs with the number of back edges at most the nesting depth of loops.
  - Nesting depth tends to be small.

Example: Nested Loops



3 nested while loops.

depth = 3.

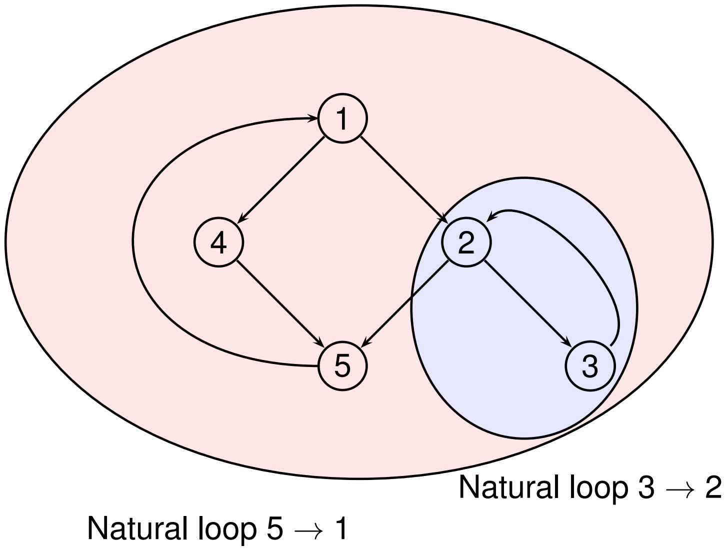
3 nested do-while loops.

depth = 1.

Natural Loops

- The **natural loop** of a back edge $a \rightarrow b$ is $\{b\}$ plus the set of nodes that can reach a without going through b.
- **Theorem:** two natural loops are either disjoint, identical, or nested.
- Proof: Discuss/Exercise

Example: Natural Loops



Reading Assignment

- New Dragon Book (Aho, Lam, Sethi, Ullman)
  - Chapter 9