CS738: Advanced Compiler Optimizations

Foundations of Data Flow Analysis

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Agenda

- Poset, Lattice, and Data Flow Frameworks: Review
- Connecting Tarski Lemma with Data Flow Analysis
- Solutions of Data Flow Analysis constraints

Knaster-Tarski Fixed Point Theorem

- Let f be a monotonic function on a complete lattice $(S, \bigwedge, \bigvee)$. Define
  - $\text{red}(f) = \{v \mid v \in S, f(v) \leq v\}$, pre fix-points
  - $\text{ext}(f) = \{v \mid v \in S, f(v) \geq v\}$, post fix-points
  - $\text{fix}(f) = \{v \mid v \in S, f(v) = v\}$, fix-points

  Then,
  - $\bigwedge \text{red}(f) \in \text{fix}(f)$. Further, $\bigwedge \text{red}(f) = \bigwedge \text{fix}(f)$
  - $\bigvee \text{ext}(f) \in \text{fix}(f)$. Further, $\bigvee \text{ext}(f) = \bigwedge \text{fix}(f)$
  - $\text{fix}(f)$ is a complete lattice

Application of Fixed Point Theorem

- $f : S \to S$ is a **monotonic** function
- $(S, \bigwedge)$ is a **finite height** semilattice
- $\top$ is the top element of $(S, \bigwedge)$
- Notation: $f^0(x) = x, f^{i+1}(x) = f(f^i(x)), \forall i \geq 0$
- The greatest fixed point of f is

$$f^k(\top), \text{ where } f^{k+1}(\top) = f^k(\top)$$

Fixed Point Algorithm

```
// monotonic function f on a meet semilattice
x := ⊤;
while (x ≠ f(x)) x := f(x);
return x;
```

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    `for (each block` B`) {`
        $\mathrm{IN}(B) = \bigwedge_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$`;`
        $\mathrm{OUT}(B) = f_B(\mathrm{IN}(B))$`;`
    `}`
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- In other words: just “invert” the flow graph!!

Acknowledgement

Rest of the slides based on the material at
`http://infolab.stanford.edu/~ullman/dragon/w06/w06.html`

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- **MFP** *(maximal fixedpoint)* = result of iterative algorithm.

Maximum Fixedpoint

- **Fixedpoint** = solution to the equations used in the iteration:

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- $\leq$: carries less information.

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- But try not to include too many more.
- Less "ignorance," but we "know too much."

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  - Finite height
    no infinite chains $\ldots < x_2 < x_1 < x < \ldots$

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- But the MFP (iterative algorithm) alternates compositions and meets arbitrarily.

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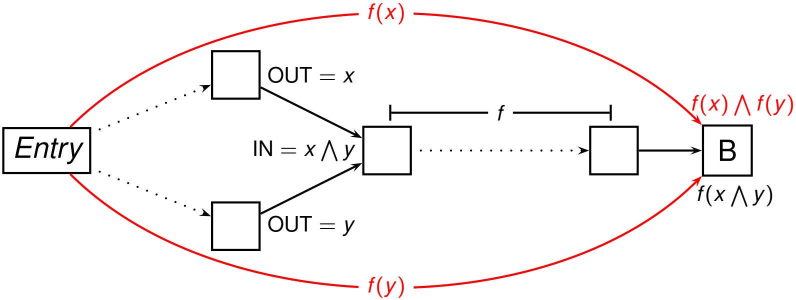
  - Easy proof for functions in Gen-Kill form.
- And they have finite height.
  - Only a finite number of defs, variables, etc. in any program.

Two Paths to *B* that Meet Early



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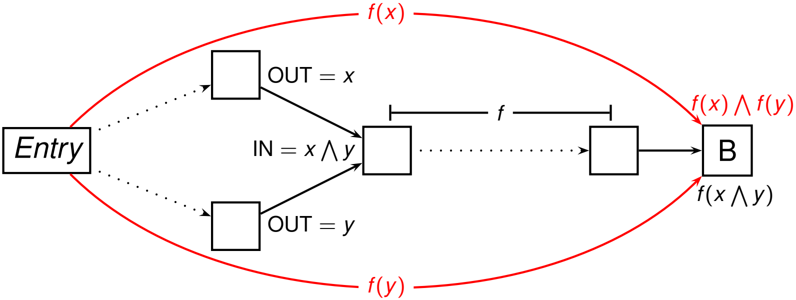

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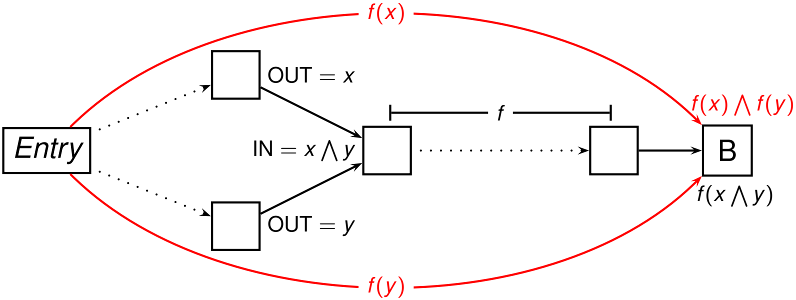

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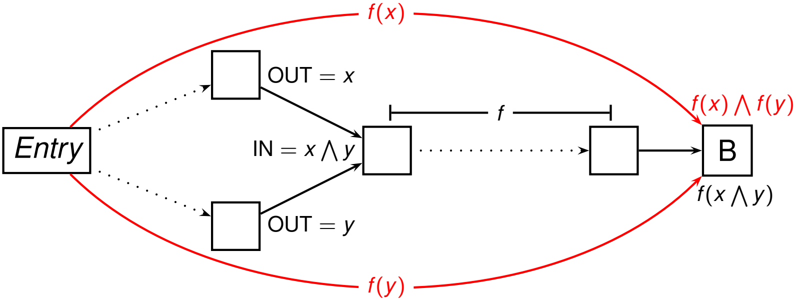

- MOP considers paths independently and combines at the last possible moment.
- In MFP, Values x and y get combined too soon.
- Since $f(x \bigwedge y) \leq f(x) \bigwedge f(y)$, it is as we added non-existent paths.

Distributive Frameworks

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  - But the reverse is not true

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- If a framework is distributive, then combining paths early doesn't hurt.
  - MOP = MFP.
  - That is, the iterative algorithm computes a solution that takes into account all and only the physical paths.