CS738: Advanced Compiler Optimizations

Data Flow Analysis

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Department of CSE, IIT Kanpur

Agenda

- *Intraprocedural* Data Flow Analysis: Classical Examples

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  - Last lecture: Reaching Definitions

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  - Last lecture: Reaching Definitions
  - Today: Available Expressions

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- *Intraprocedural* Data Flow Analysis: Classical Examples
  - Last lecture: Reaching Definitions
  - Today: Available Expressions
  - Discussion about the similarities/differences

Available Expressions Analysis

- An expression e is available at a point p if

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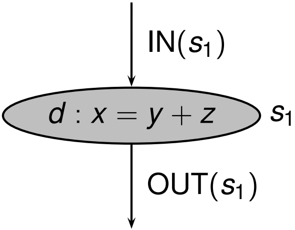
  - There is no assignment to any component variable of *e* **after the last evaluation** of *e* prior to *p*
- Expression e is *generated* by its evaluation
- Expression e is *killed* by assignment to its component variables

AvE Analysis of a Structured Program



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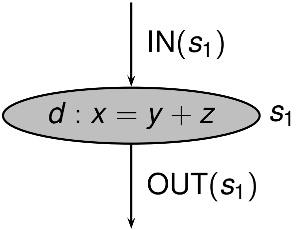

$$\text{OUT}(s_1) \quad = \quad \text{IN}(s_1) - \text{KILL}(s_1) \cup \text{GEN}(s_1)$$

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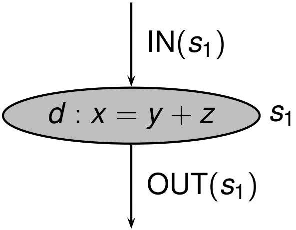

$$
\begin{aligned}
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\end{aligned}
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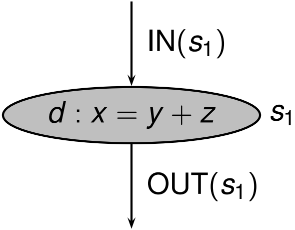
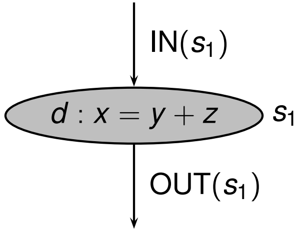
AvE Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{OUT}(s_1) &= \mathrm{IN}(s_1) - \mathrm{KILL}(s_1) \cup \mathrm{GEN}(s_1) \\
\mathrm{GEN}(s_1) &= \{y + z\}
\end{aligned}
$$

AvE Analysis of a Structured Program



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\end{aligned}$$

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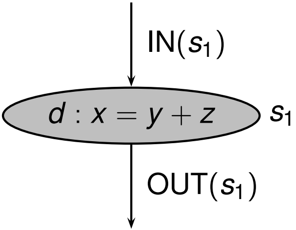

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where E_x: set of all expression having x as a component

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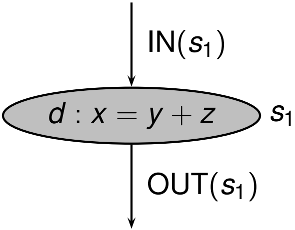

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This may not work in general – WHY?

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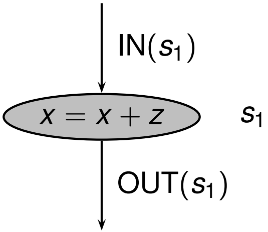

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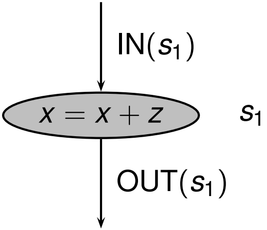
Incorrectly marks $x + z$ as available after s_1

AvE Analysis of a Structured Program



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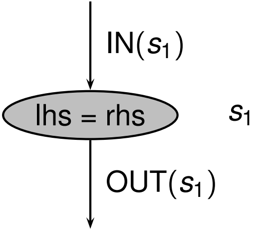
Incorrectly marks $x + z$ as available after s_1

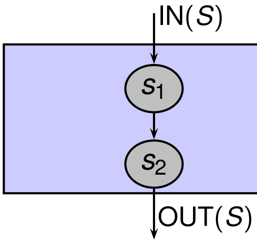
$$\text{GEN}(s_1) = \emptyset \text{ for this case}$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{OUT}(s_1) &= \mathrm{IN}(s_1) - \mathrm{KILL}(s_1) \cup \mathrm{GEN}(s_1) \\
\mathrm{GEN}(s_1) &= \{\text{rhs} \mid \text{lhs is not part of rhs}\} \\
\mathrm{KILL}(s_1) &= E_{\text{lhs}}
\end{aligned}
$$

AvE Analysis of a Structured Program



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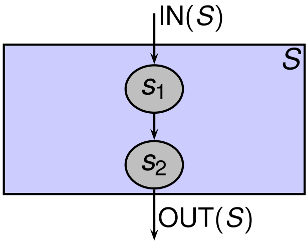
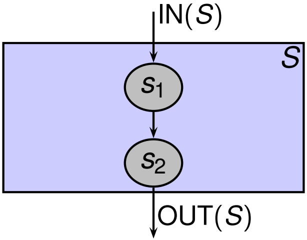


$$\text{GEN}(S) \quad =$$

AvE Analysis of a Structured Program



$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1) - \text{KILL}(s_2) \cup \text{GEN}(s_2)$$

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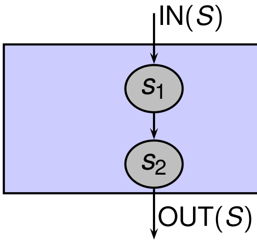

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\end{aligned}
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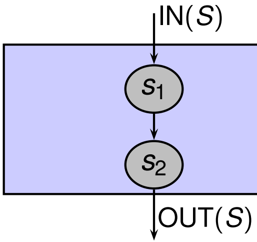

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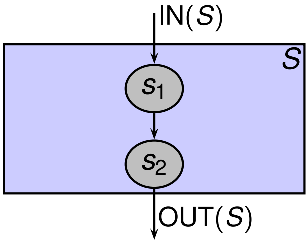

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\mathrm{IN}(s_1) &=
\end{aligned}
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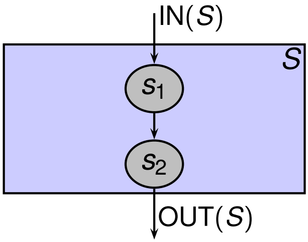

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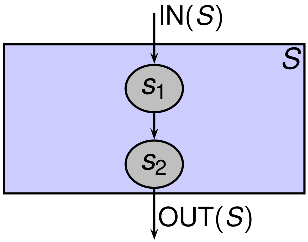

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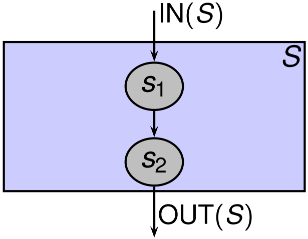

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\end{aligned}
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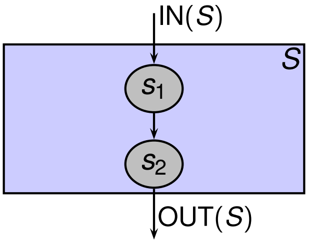

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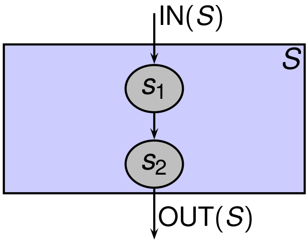

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\mathrm{IN}(s_2) &= \mathrm{OUT}(s_1) \\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_2)
\end{aligned}
$$

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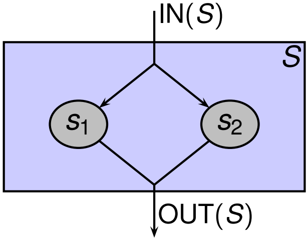

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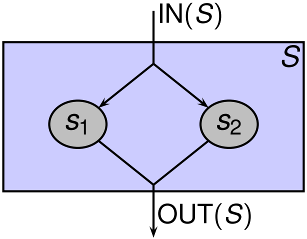
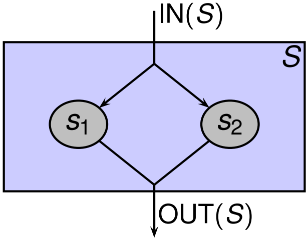


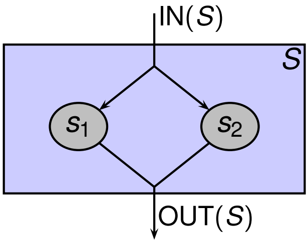
$$\mathrm{GEN}(S) \quad =$$

AvE Analysis of a Structured Program



$$\text{GEN}(S) \quad = \quad \text{GEN}(s_1) \cap \text{GEN}(s_2)$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \cap \mathrm{GEN}(s_2) \\
\mathrm{KILL}(S) &=
\end{aligned}
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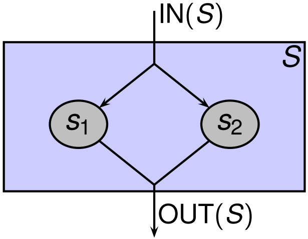

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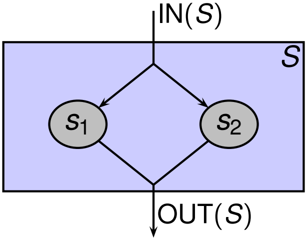

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\end{aligned}
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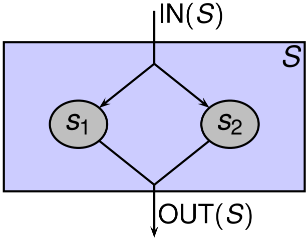

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\mathrm{IN}(s_1) &= \mathrm{IN}(s_2) = \mathrm{IN}(S)
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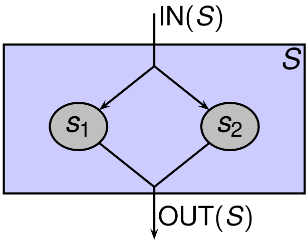

$$
\begin{array}{rcl}
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\mathrm{KILL}(S) & = & \mathrm{KILL}(s_1) \cup \mathrm{KILL}(s_2) \\
\mathrm{IN}(s_1) & = & \mathrm{IN}(s_2) \quad = \quad \mathrm{IN}(S) \\
\mathrm{OUT}(S) & = &
\end{array}
$$

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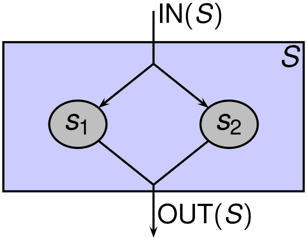

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\mathrm{OUT}(S) &= \mathrm{OUT}(s_1) \cap \mathrm{OUT}(s_2)
\end{aligned}
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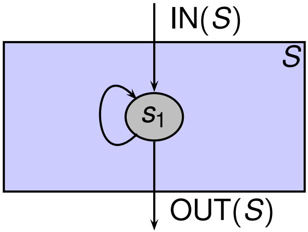

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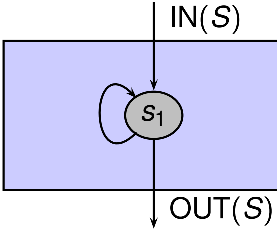

$$\mathrm{GEN}(S) \quad =$$

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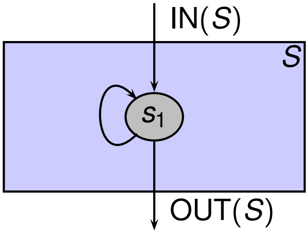
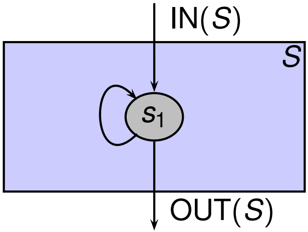


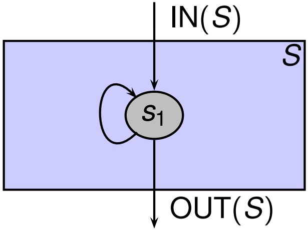
$$\mathrm{GEN}(S) \quad = \quad \mathrm{GEN}(s_1)$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) &=
\end{aligned}
$$

AvE Analysis of a Structured Program



$$\begin{aligned} \mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\ \mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \end{aligned}$$

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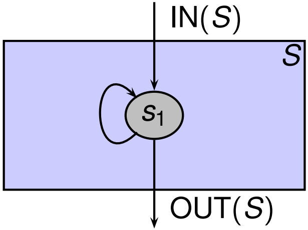

$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \\
\mathrm{OUT}(S) &=
\end{aligned}
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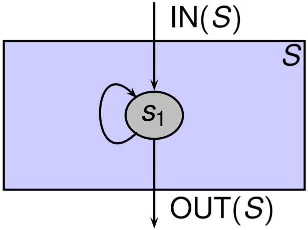

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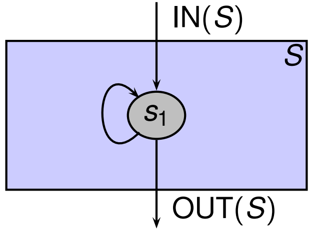

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\end{aligned}
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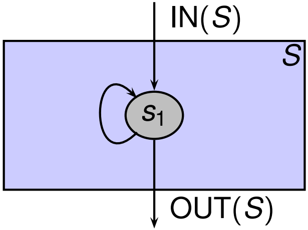
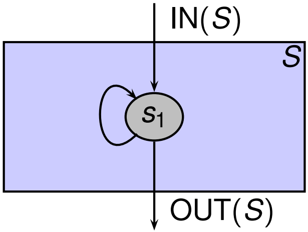


$$
\begin{aligned}
\text{GEN}(S) &= \text{GEN}(s_1) \\
\text{KILL}(S) &= \text{KILL}(s_1) \\
\text{OUT}(S) &= \text{OUT}(s_1) \\
\text{IN}(s_1) &= \text{IN}(S) \cap \text{GEN}(s_1)
\end{aligned}
$$

AvE Analysis of a Structured Program



$$
\begin{aligned}
\mathrm{GEN}(S) &= \mathrm{GEN}(s_1) \\
\mathrm{KILL}(S) &= \mathrm{KILL}(s_1) \\
\mathrm{OUT}(S) &= \mathrm{OUT}(s_1) \\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cap \mathrm{GEN}(s_1)\ \textcolor{red}{?} \\
\mathrm{IN}(s_1) &= \mathrm{IN}(S) \cap \mathrm{OUT}(s_1)\textcolor{red}{??}
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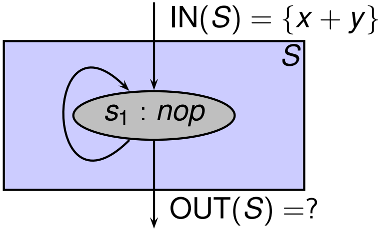

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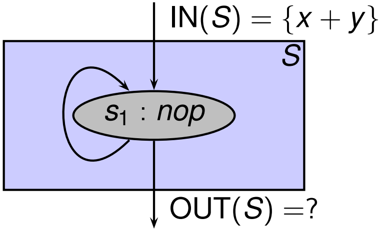
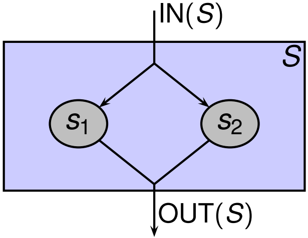


Is x + y available at OUT(S)?

AvE Analysis is Approximate



- Assumption: All paths are feasible.

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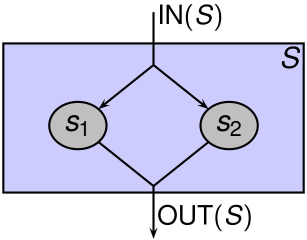

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- Example:

```
if (true) s1;
else      s2;
```

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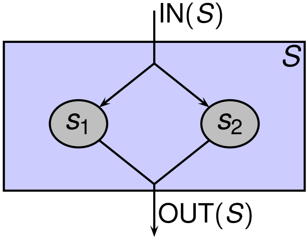

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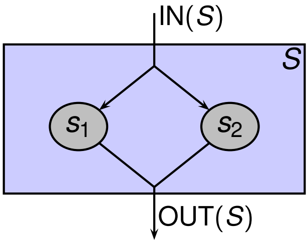

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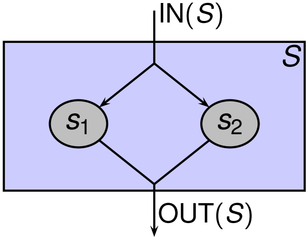
Fact		**Computed**		**Actual**
GEN(S)	$=$	GEN(s_1) $\cap$ GEN(s_2)	$\subseteq$	GEN(s_1)

AvE Analysis is Approximate



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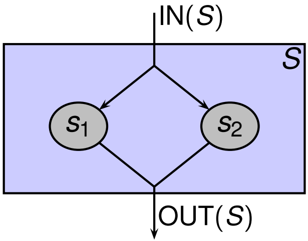
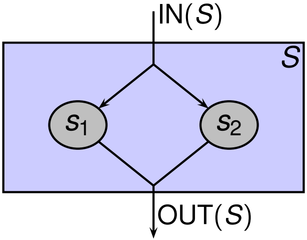
Fact		**Computed**		**Actual**
GEN(S)	$=$	GEN(s_1) $\cap$ GEN(s_2)	$\subseteq$	GEN(s_1)
KILL(S)	$=$	KILL(s_1) $\cup$ KILL(s_2)	$\supseteq$	KILL(s_1)

AvE Analysis is Approximate



- Thus,

AvE Analysis is Approximate



- Thus,

$$\text{true GEN}(S) \supseteq \text{analysis GEN}(S)$$

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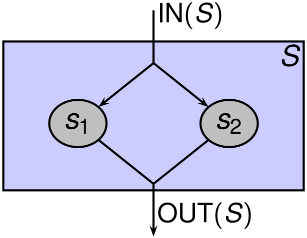

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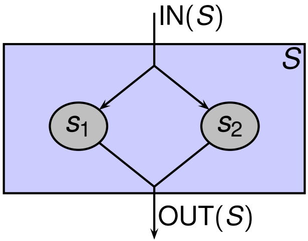

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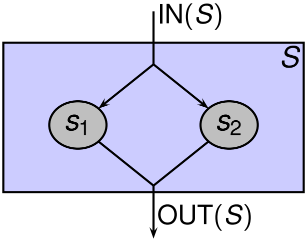

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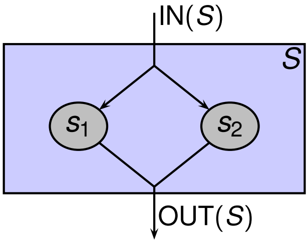

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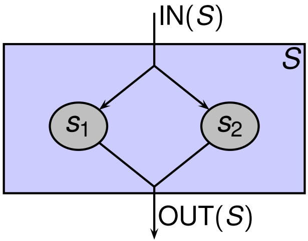

- Thus,

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- Fewer expressions marked available than actually do!
- Later we shall see that this is SAFE approximation
  - prevents optimizations
  - but NO wrong optimization

AvE for Basic Blocks

- Expr e is available at the start of a block if

$$\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

AvE for Basic Blocks

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  - It is available at the end of all predecessors

$$\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$$

- Expr e is available at the end of a block if
  - Either it is generated by the block
  - Or it is available at the start of the block and not killed by the block

$$\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$$

Solving AvE Constraints

- KILL & GEN known for each BB.

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  - Solution is possible.
  - Iterative approach (on the next slide).

```
for each block B {
```

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    OUT(B) = U; U = "universal" set of all exprs
```

```
for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
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OUT(Entry) = ∅; // remember reaching defs?
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```
for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
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change = true;
while (change) {
    change = false;
```

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for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
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while (change) {
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    for each block B other than Entry {
```

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for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
}
OUT(Entry) = ∅; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋂_{P∈PRED(B)} OUT(P);
```

```
for each block B {
    OUT(B) = 𝒰; 𝒰 = "universal" set of all exprs
}
OUT(Entry) = ∅; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        IN(B) = ⋂_{P∈PRED(B)} OUT(P);
        oldOut = OUT(B);
        OUT(B) = IN(B) − KILL(B) ∪ GEN(B);
```

for each block B {
    $\mathrm{OUT}(B) = \mathcal{U}$; $\mathcal{U}$ = "universal" set of all exprs
}
$\mathrm{OUT}(\mathit{Entry}) = \emptyset$; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        $\mathrm{IN}(B) = \bigcap_{P \in \mathrm{PRED}(B)} \mathrm{OUT}(P)$;
        oldOut = $\mathrm{OUT}(B)$;
        $\mathrm{OUT}(B) = \mathrm{IN}(B) - \mathrm{KILL}(B) \cup \mathrm{GEN}(B)$;
        if ($\mathrm{OUT}(B) \neq$ oldOut) then {
            change = true;
        }
    }
}

Some Issues

- What is $\mathcal{U}$ – the set of *all* expressions?

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- How to compute it efficiently?

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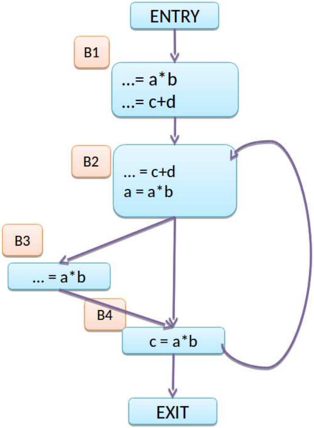
- What is $\mathcal{U}$ – the set of *all* expressions?
- How to compute it efficiently?
- Why *Entry* block is initialized differently?

Available Expressions: Example



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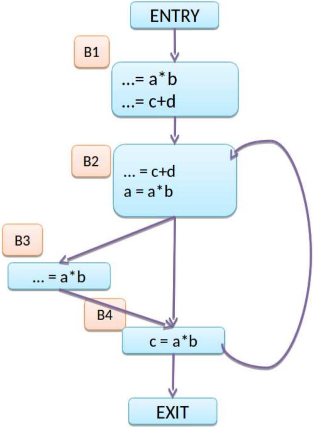
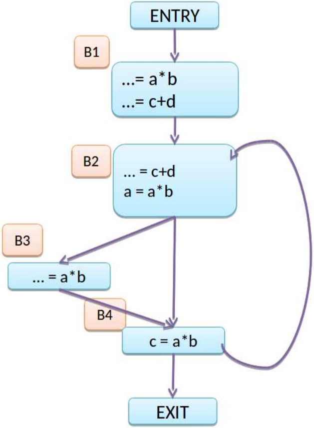


BB	GEN	KILL
B1		
B2		
B3		
B4		

$\mathcal{U}$= {a*b, c+d}

Available Expressions: Example



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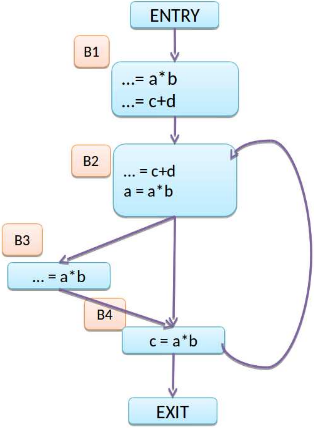

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B1	{a*b, c+d}	{ }
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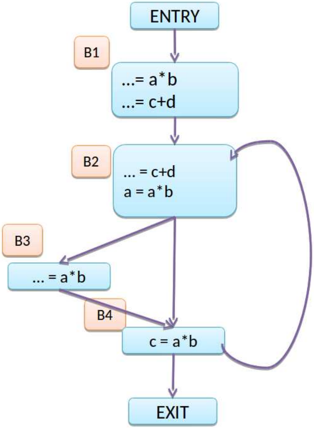

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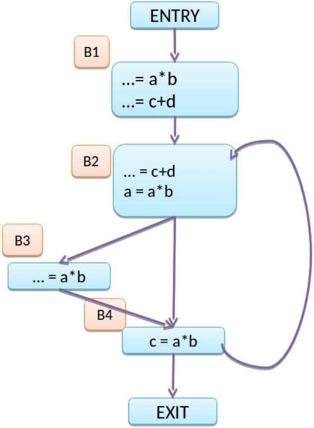

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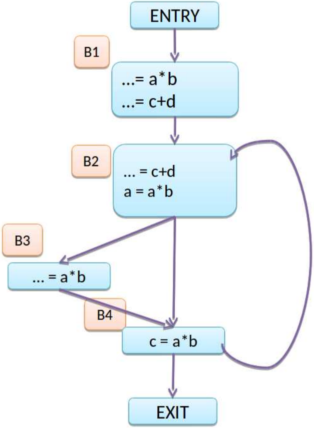

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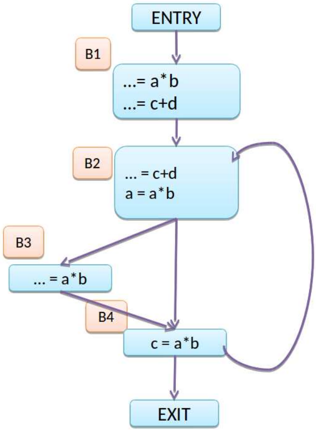

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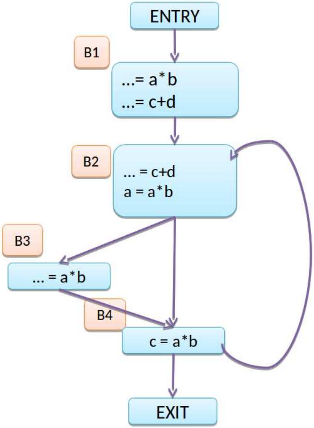

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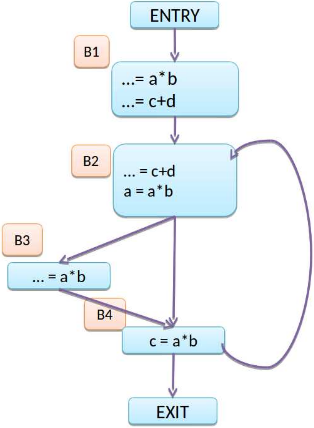

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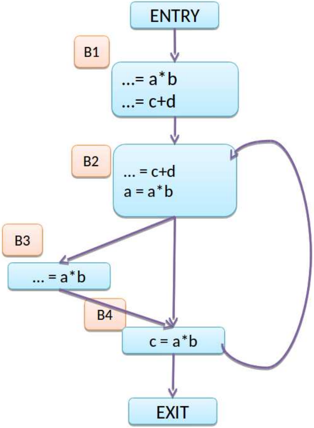
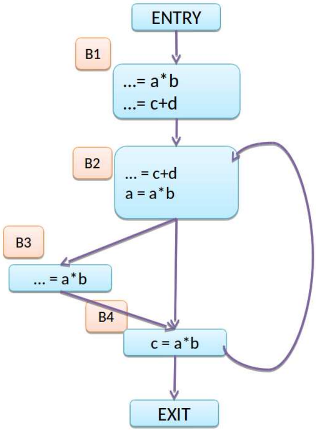


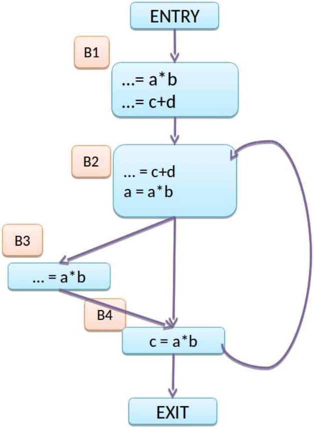
Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$

Available Expressions: Example



Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
1	IN	$\emptyset$	a*b, c+d	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

Available Expressions: Example



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Init	IN	-	-	-	-
	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
1	IN	$\emptyset$	a*b, c+d	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
2	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

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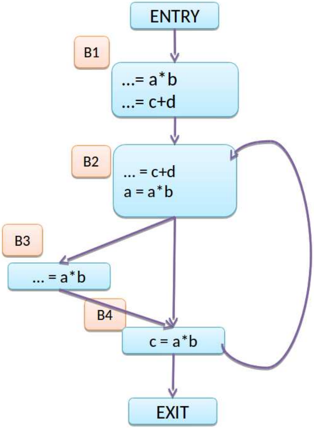

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	OUT	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$	$\mathcal{U}$
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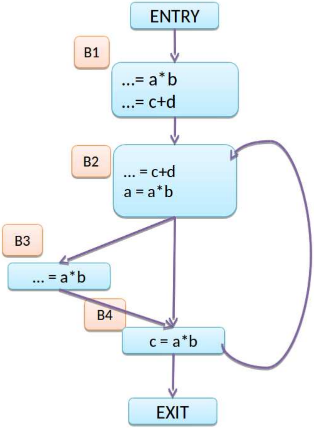
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
2	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b
3	IN	$\emptyset$	a*b	c+d	c+d
	OUT	a*b, c+d	c+d	a*b, c+d	a*b

Available Expressions: Bitvectors



a bit for each expression:

a*b	c+d

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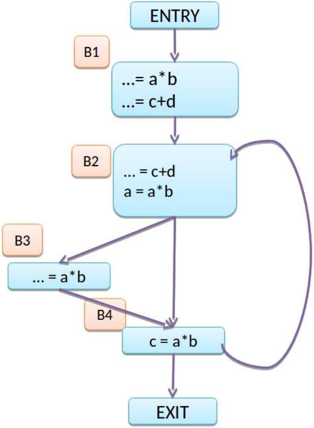

a bit for each expression:

a*b	c+d

Pass#	Pt	B1	B2	B3	B4
Init	IN	-	-	-	-
	OUT	11	11	11	11
1	IN	00	11	01	01
	OUT	11	01	11	10
2	IN	00	10	01	01
	OUT	11	01	11	10
3	IN	00	10	01	01
	OUT	11	01	11	10

Available Expressions: Bitvectors

- Set-theoretic definitions:

$$\text{IN}(B) = \bigcap_{P \in \text{PRED}(B)} \text{OUT}(P)$$

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$$\text{IN}(B) = \bigwedge_{P \in \text{PRED}(B)} \text{OUT}(P)$$

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- Bitwise $\vee, \wedge, \neg$ operators

Available Expressions: Application

- Common subexpression elimination in a block *B*

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Available Expressions: Application

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  - Expression e available at the entry of B
  - e is also computed at a point p in B
  - Components of e are not modified from entry of B to p
- e is "upward exposed" in B
- Expressions generated in B are "downward exposed"

Comparison of RD and AvE

- *Some* vs. *All* path property

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- Meet operator: $\cup$ vs. $\cap$

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Comparison of RD and AvE

- *Some* vs. *All* path property
- Meet operator: $\cup$ vs. $\cap$
- Initialization of *Entry*: $\emptyset$
- Initialization of other BBs: $\emptyset$ vs. $\mathcal{U}$
- Safety: “More” RD vs. “Fewer” AvE

AvE: alternate Initialization

- What if we Initialize:

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- Otherwise x is dead at p

Live Variables: GEN

- GEN(B): Set of variables whose values may be used in block B prior to any definition
  - Also called "use(B)"
- "upward exposed use" of a variable in B

Live Variables: KILL

- KILL(B): Set of variables defined in block B prior to any use
  - Also called "def(B)"
- "upward exposed definition" of a variable in B

Live Variables: Equations

- Set-theoretic definitions:

$$\mathrm{OUT}(B) = \bigcup_{S \in \mathrm{SUCC}(B)} \mathrm{IN}(S)$$

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- Bitwise $\vee, \wedge, \neg$ operators

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- Also called *Anticipable expression*

QQ

- Expression e is very busy at a point p if
  - **Every** path from p to *Exit* has at least one evaluation of e and there is no assignment to any component variable of e before the first evaluation of e following p on these paths.
- Set up the data flow equations for Very Busy Expressions (VBE). You have to give equations for GEN, KILL, IN, and OUT.
- Think of an optimization/transformation that uses VBE analysis. Briefly describe it (2-3 lines only)
- Will your optimization be *safe* if we replace "*Every*" by "*Some*" in the definition of VBE?