CS618: Program Analysis
2016-17 Ist Semester

Liveness based Garbage Collection

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Ideal Garbage Collection

> *. . . garbage collection (GC) is a form of automatic memory management. The garbage collector, or just collector, attempts to reclaim garbage, or memory occupied by objects that are no longer in use by the program. . . .*

From Wikipedia

`https://en.wikipedia.org/wiki/Garbage_collection_(computer_science)`

Real Garbage Collection

> *. . . All garbage collectors use some efficient approximation to liveness. In tracing garbage collection, the approximation is that an object can't be live unless it is reachable. . . .*

From Memory Management Glossary

`www.memorymanagement.org/glossary/g.html#term-garbage-collection`

Liveness based GC

- During execution, there are significant amounts of heap allocated data that are *reachable but not live*.
  - Current GCs will retain such data.
- Our idea:
  - We do a liveness analysis of heap data and provide GC with its result.
  - Modify GC to mark data for retention only if it is live.
- Consequences:

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The language analyzed

- First order eager Scheme-like functional language.
- In Administrative Normal Form (ANF).

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p \in Prog & ::= & d_1 \ldots d_n \; e_{\text{main}} \\
d \in Fdef & ::= & (\textbf{define}\ (f\ x_1\ \ldots\ x_n)\ e) \\
e \in Expr & ::= & \left\{ \begin{array}{l} (\textbf{if}\ x\ e_1\ e_2) \\ (\textbf{let}\ x \leftarrow a\ \textbf{in}\ e) \\ (\textbf{return}\ x) \end{array} \right. \\
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An Example

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 (if (null? l1) l2
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(let z ←(cons (cons 4 (cons 5 nil))
         (cons 6 nil)) in
 (let y ← (cons 3 nil) in
   (let w ← (append y z) in
      π:(car (cdr w)))))
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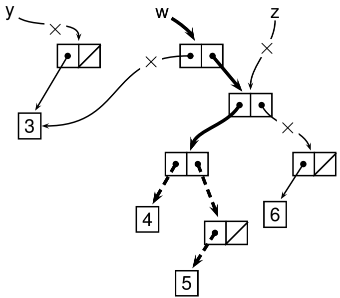

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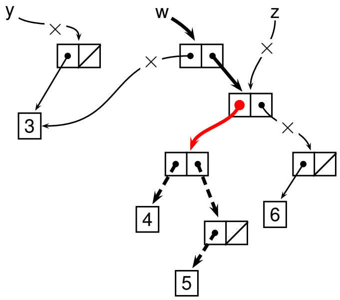

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Liveness – Basic Concepts and Notations

- *Access paths:* Strings over {**0**, **1**}.

  0 – access **car** field
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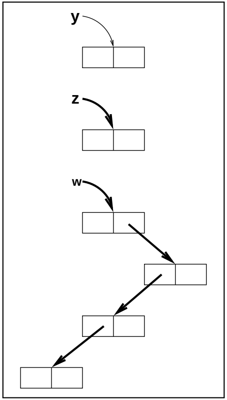
- Denote traversals over the heap graph
- *Liveness environment:*



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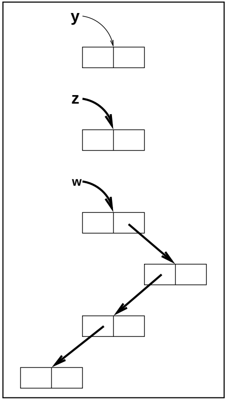

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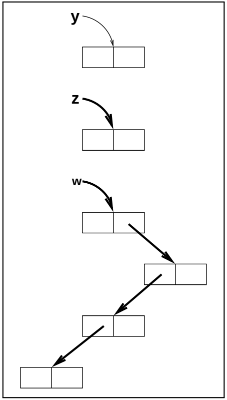
- *Liveness environment:* Maps root variables to set of access paths.

$$\mathsf{L}_i \quad : \quad \begin{cases} y \mapsto \emptyset \\ z \mapsto \{\epsilon\} \\ w \mapsto \{\epsilon, 1, 10, 100\} \end{cases}$$



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- *Liveness environment:* Alternate representation.

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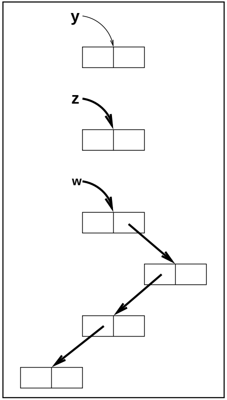

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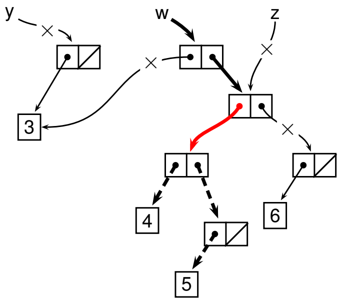
Demand

(car (cdr `w`**))**



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Liveness analysis – The big picture

π_{main}: (**let** `z` ← ... **in**
(**let** `y` ← ... **in**
π_9: (**let** `w` ← (**append** `y z`) **in**
π_{10}: (**let** `a` ← (**cdr** `w`) **in**
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Liveness environments:

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Function summaries:

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- **GOAL:** Compute Liveness Environment at various program points, statically.

$\mathcal{L}app(a, \sigma)$ – Liveness environment generated by an *application* a, given a demand σ.

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Liveness analysis of Expressions

$$\mathcal{L}exp((\textit{return } x), \sigma) = \{x.\sigma\}$$

$$\mathcal{L}exp((\textbf{if } x \;\; e_1 \;\; e_2), \sigma) = \{x.\epsilon\} \cup \mathcal{L}exp(e_1, \sigma) \cup \mathcal{L}exp(e_2, \sigma)$$

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Notice the similarity with:

$$live_{in}(B) = live_{out}(B) \setminus kill(B) \cup gen(B)$$

in classical dataflow analysis for imperative languages.

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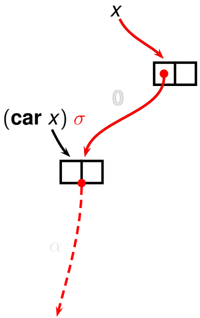

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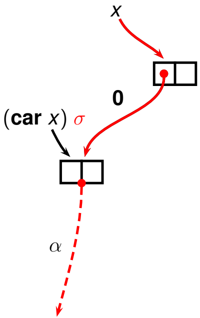

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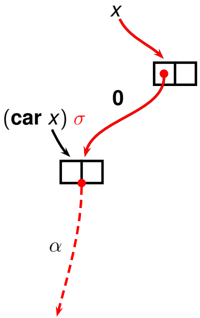

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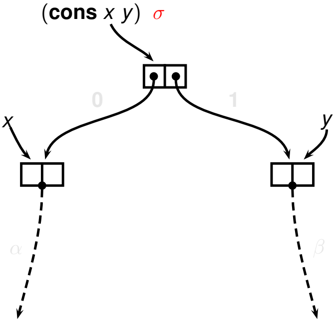

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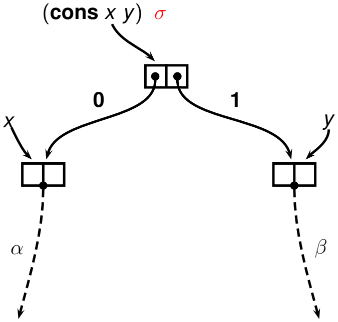

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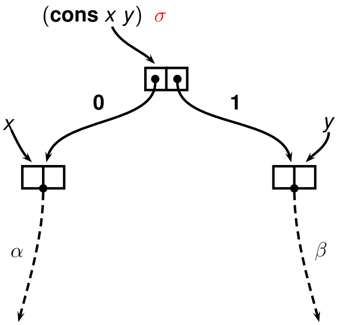

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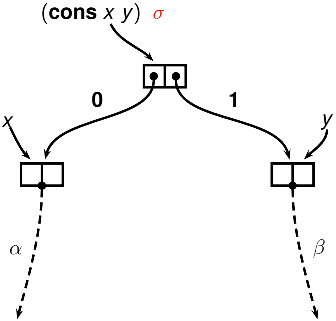
  $\bar{\mathbf{1}}$ – Removal of a leading **1**

$$\mathcal{L}app((\textbf{cons}\ x\ y), \sigma) = x.\bar{\mathbf{0}}\sigma \cup y.\bar{\mathbf{1}}\sigma$$

Liveness analysis of Primitive Applications

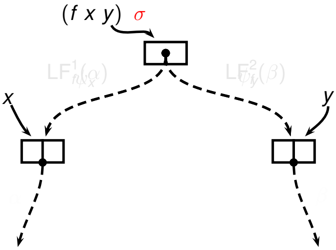


$$\mathcal{L}app((\textbf{cons}\ x\ y), \sigma) = \{x.\alpha \mid \mathbf{0}\alpha \in \sigma\} \cup \{y.\beta \mid \mathbf{1}\beta \in \sigma\}$$

- $\bar{\mathbf{0}}$ – Removal of a leading **0**
  $\bar{\mathbf{1}}$ – Removal of a leading **1**

$$\mathcal{L}app((\textbf{cons}\ x\ y), \sigma) = x.\bar{\mathbf{0}}\sigma \cup y.\bar{\mathbf{1}}\sigma$$

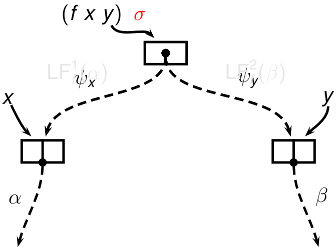
Liveness Analysis of Function Applications



$$\mathcal{L}app((f\ x\ y), \sigma) = x.\mathrm{LF}^1_f(\sigma) \cup y.\mathrm{LF}^2_f(\sigma)$$

- We use LF_f: context independent summary of f.
- To find $\mathrm{LF}^i_f(\ldots)$:
  - Assume a symbolic demand σ_{sym}.
  - Let e_f be the body of f.
  - Set $\mathrm{LF}^i_f(\sigma_{sym})$ to $\mathcal{L}exp(e_f, \sigma_{sym})(x_i)$.

Liveness Analysis of Function Applications



$$\mathcal{L}app((f\ x\ y), \sigma) = x.\overline{\psi_x}\sigma \cup y.\overline{\psi_y}\sigma$$

- We use LF_f: context independent summary of f.
- To find $\mathsf{LF}^i_f(\ldots)$:
  - Assume a symbolic demand σ_{sym}.
  - Let e_f be the body of f.
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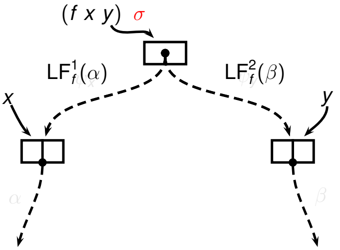
Liveness Analysis of Function Applications



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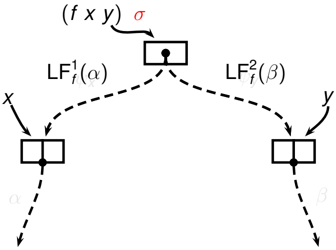
  - How to handle recursive calls?

Liveness Analysis of Function Applications



$$\mathcal{L}app((f\ x\ y), \sigma) = \ x.\mathrm{LF}_f^1(\sigma) \cup \ y.\mathrm{LF}_f^2(\sigma)$$

- We use LF_f: context independent summary of f.
- To find $\mathrm{LF}_f^i(\ldots)$:
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  - Let e_f be the body of f.
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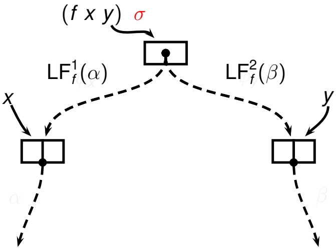
Liveness Analysis of Function Applications



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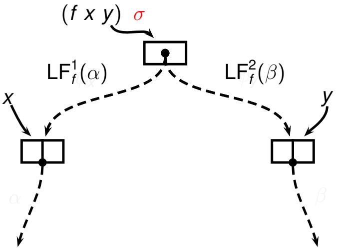
  - How to handle recursive calls? Use LF_f with appropriate demand !!

Liveness Analysis of Function Applications



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- We use LF_f: context independent summary of f.
- To find $\mathsf{LF}^i_f(\ldots)$:
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  - Let e_f be the body of f.
  - Set $\mathsf{LF}^i_f(\sigma_{sym})$ to $\mathcal{L}exp(e_f, \sigma_{sym})(x_i)$.
  - How to handle recursive calls? Use LF_f with appropriate demand !!

Liveness analysis – The big picture

π_{main}: (**let** `z` $\leftarrow$ ... **in**
(**let** `y` $\leftarrow$ ... **in**
π_9: (**let** `w` $\leftarrow$ (**append** `y z`) **in**
π_{10}: (**let** `a` $\leftarrow$ (**cdr** `w`) **in**
π_{11}: (**let** `b` $\leftarrow$ (**car** `a`) **in**
π_{12}: (**return** `b`))))))

(**define** (**append** `l1 l2`)
π_1: (**let** `test` $\leftarrow$ (**null?** `l1`) **in**
π_2: (**if** `test` π_3:(**return** `l2`)
π_4: (**let** `tl` $\leftarrow$ (**cdr** `l1`) **in**
π_5: (**let** `rec` $\leftarrow$ (**append** `tl l2`) **in**
π_6: (**let** `hd` $\leftarrow$ (**car** `l1`) **in**
π_7: (**let** `ans` $\leftarrow$ (**cons** `hd rec`) **in**
π_8: (**return** `ans`)))))))

Liveness environments:

$$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

...

$$\mathsf{L}_9^{\texttt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$$

Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – The big picture

π_{main}: (**let** `z` $\leftarrow$ ... **in**
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π_6: (**let** `hd` $\leftarrow$ (**car** `l1`) **in**
π_7: (**let** `ans` $\leftarrow$ (**cons** `hd rec`) **in**
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Liveness environments:

$$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

...

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Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – The big picture

π_{main}: (let z $\leftarrow$ ... in
(let y $\leftarrow$ ... in
π_9: (let w $\leftarrow$ (**append** y z) in
π_{10}: (let a $\leftarrow$ (cdr w) in
π_{11}: (let b $\leftarrow$ (car a) in
π_{12}: (return b)))))))

(**define** (**append** l1 l2)
π_1: (let test $\leftarrow$ (null? l1) in
π_2: (if test π_3:(**return** l2)
π_4: (let tl $\leftarrow$ (cdr l1) in
π_5: (let rec $\leftarrow$ (**append** tl l2) in
π_6: (let hd $\leftarrow$ (car l1) in
π_7: (let ans $\leftarrow$ (cons hd rec) in
π_8: (return ans)))))))

Liveness environments:

$$\mathsf{L}_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

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...

$$\mathsf{L}_9^{\mathtt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{1}\mathbf{0}\sigma_{all})$$

Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – The big picture

π_{main}: (let z $\leftarrow$ ... in
(let y $\leftarrow$ ... in
π_9: (let w $\leftarrow$ (**append** `y z`) in
π_{10}: (let a $\leftarrow$ (cdr w) in
π_{11}: (let b $\leftarrow$ (car a) in
π_{12}: (return b)))))))

(**define** (**append** `l1 l2`)
π_1: (let test $\leftarrow$ (null? l1) in
π_2: (if test π_3:(**return** `l2`)
π_4: (let tl $\leftarrow$ (cdr l1) in
π_5: (let rec $\leftarrow$ (**append** `tl l2`) in
π_6: (let hd $\leftarrow$ (car l1) in
π_7: (let ans $\leftarrow$ (**cons** `hd rec`) in
π_8: (**return** `ans`)))))))

σ $\bar{\mathbf{1}}\sigma$ σ

Liveness environments:

$\mathsf{L}_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

$\mathsf{L}_1^{\mathtt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

...

$\mathsf{L}_9^{\mathtt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$

Demand summaries:

Function summaries:

$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

Liveness analysis – The big picture

π_{main}: (let z ← ... in
(let y ← ... in
π_9: (let w ← (**append** y z) in
π_{10}: (let a ← (cdr w) in
π_{11}: (let b ← (car a) in
π_{12}: (return b))))))

(**define** (**append** l1 l2)
π_1: (let test ← (null? l1) in
π_2: (if test π_3:(**return** l2)
π_4: (let tl ← (cdr l1) in σ
π_5: (let rec ← (**append** tl l2) in
π_6: (let hd ← (car l1) in $\mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$
π_7: (let ans ← (**cons** hd rec) in $\mathbf{1}\sigma$
π_8: (**return** ans))))))) σ

Liveness environments:

$\mathsf{L}_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

$\mathsf{L}_1^{\mathtt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

...

$\mathsf{L}_9^{\mathtt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{1}\mathbf{0}\sigma_{all})$

Demand summaries:

Function summaries:

$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

Liveness analysis – The big picture

π_{main}: (**let** z ← ... **in**
(**let** y ← ... **in**
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
π_{11}: (**let** b ← (**car** a) **in**
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(**define** (**append** l1 l2)
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π_2: (**if** test π_3:(**return** l2)
π_4: (**let** tl ← (**cdr** l1) **in**
π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in** $\mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$
π_7: (**let** ans ← (**cons** hd rec) **in** $\mathbf{1}\sigma$
π_8: (**return** ans))))))))

σ σ

Liveness environments:

$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

...

$\mathsf{L}_9^{\texttt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{1}\mathbf{0}\sigma_{all})$

Demand summaries:

Function summaries:

$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

Liveness analysis – Demand Summary

$\sigma_{\textbf{main}} = \sigma_{all}$

π_{main}: (**let** z ← ... **in**
(**let** y ← ... **in**
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
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π_1: (**let** test ← (**null?** l1) **in**
π_2: (**if** test π_3:(**return** l2)
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π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in**
π_7: (**let** ans ← (**cons** hd rec) **in**
π_8: (**return** ans)))))))

Liveness environments:

$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup$
$\quad \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$

...

$\mathsf{L}_9^{\texttt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$

Demand summaries:

Function summaries:

$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup$
$\quad \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$

Liveness analysis – Demand Summary

$\sigma_{\textbf{main}} = \sigma_{all}$ σ_1

π_{main}: (**let** z ← ... **in**
(**let** y ← ... **in**
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
π_{11}: (**let** b ← (**car** a) **in**
π_{12}: (**return** b))))))

(**define** (**append** l1 l2)
π_1: (**let** test ← (**null?** l1) **in**
π_2: (**if** test π_3:(**return** l2)
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π_5: (**let** rec ← (**append** tl l2) **in**
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π_7: (**let** ans ← (**cons** hd rec) **in**
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Liveness environments:

$$\mathsf{L}_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$\mathsf{L}_1^{\mathtt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

...

$$\mathsf{L}_9^{\mathtt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$$

Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – Demand Summary

$\sigma_{\textbf{main}} = \sigma_{all}$ $\sigma_{\textbf{append}} = \sigma_1 \cup \ldots$

π_{main}: (**let** z ← . . . **in**
(**let** y ← . . . **in** σ_1
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
π_{11}: (**let** b ← (**car** a) **in**
π_{12}: (**return** b))))))

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π_1: (**let** test ← (**null?** l1) **in**
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π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in**
π_7: (**let** ans ← (**cons** hd rec) **in**
π_8: (**return** ans))))))))

Liveness environments:

$$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

. . .

$$\mathsf{L}_9^{\texttt{y}} = \mathsf{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$$

Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – Demand Summary

$\sigma_{\textbf{main}} = \sigma_{all}$ $\qquad$ $\sigma_{\textbf{append}} = \sigma_1 \cup \ldots$

π_{main}: (**let** z ← . . . **in**
(**let** y ← . . . **in**
π_9: (**let** w ← (**append** y z) **in** $\quad \sigma_1$
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π_2: (**if** test π_3:(**return** l2)
π_4: (**let** tl ← (**cdr** l1) **in** $\quad \sigma_2$
π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in**
π_7: (**let** ans ← (**cons** hd rec) **in**
π_8: (**return** ans)))))))

Liveness environments:

$$L_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathrm{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$L_1^{\texttt{l2}} = \sigma \cup \mathrm{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

. . .

$$L_9^{\texttt{y}} = \mathrm{LF}^1_{\textbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$$

Demand summaries:

Function summaries:

$$\mathrm{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathrm{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathrm{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathrm{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – Demand Summary

$\sigma_{\textbf{main}} = \sigma_{all}$ $\qquad\qquad\qquad\qquad$ $\sigma_{\textbf{append}} = \sigma_1 \cup \sigma_2$

π_{main}: (**let** z ← ... **in**
(**let** y ← ... **in** σ_1
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
π_{11}: (**let** b ← (**car** a) **in**
π_{12}: (**return** b))))))

(**define** (**append** l1 l2)
π_1: (**let** test ← (**null?** l1) **in**
π_2: (**if** test π_3: (**return** l2)
π_4: (**let** tl ← (**cdr** l1) **in** σ_2
π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in**
π_7: (**let** ans ← (**cons** hd rec) **in**
π_8: (**return** ans))))))))

Liveness environments:

$$\mathsf{L}_1^{\texttt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}} \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$\mathsf{L}_1^{\texttt{l2}} = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

...

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Demand summaries:

Function summaries:

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathsf{LF}^2_{\textbf{append}}(\sigma) = \sigma \cup \mathsf{LF}^2_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

Liveness analysis – Demand Summary

π_{main}: (let z ← ... in
(let y ← ... in
π_9: (let w ← (**append** y z) in
π_{10}: (let a ← (cdr w) in
π_{11}: (let b ← (car a) in
π_{12}: (return b))))))

(**define** (**append** l1 l2)
π_1: (let test ← (null? l1) in
π_2: (if test π_3:(return l2)
π_4: (let tl ← (cdr l1) in
π_5: (let rec ← (**append** tl l2) in
π_6: (let hd ← (car l1) in
π_7: (let ans ← (cons hd rec) in
π_8: (return ans)))))))

Liveness environments:

$$L_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\mathbf{append}} \cup \mathbf{1}\mathrm{LF}^1_{\mathbf{append}}(\bar{\mathbf{1}}\sigma_{\mathbf{append}})$$

$$L_1^{\mathtt{l2}} = \sigma \cup \mathrm{LF}^2_{\mathbf{append}}(\bar{\mathbf{1}}\sigma_{\mathbf{append}})$$

...

$$L_9^{\mathtt{y}} = \mathrm{LF}^1_{\mathbf{append}}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$$

Demand summaries:

$$\sigma_{\mathbf{main}} = \sigma_{all}$$

$$\sigma_{\mathbf{append}} = \{\epsilon,\ \mathbf{1}\} \cup \mathbf{10}\sigma_{all} \cup \bar{\mathbf{1}}\sigma_{\mathbf{append}}$$

Function summaries:

$$\mathrm{LF}^1_{\mathbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}\mathrm{LF}^1_{\mathbf{append}}(\bar{\mathbf{1}}\sigma)$$

$$\mathrm{LF}^2_{\mathbf{append}}(\sigma) = \sigma \cup \mathrm{LF}^2_{\mathbf{append}}(\bar{\mathbf{1}}\sigma)$$

Obtaining a closed form solution for LF

- Function summaries will always have the form:

$$\mathsf{LF}^i_f(\sigma) = \mathsf{I}^i_f \cup \mathsf{D}^i_f\sigma$$

- Consider the equation for $\mathsf{LF}^1_{\textbf{append}}$

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \ \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\bar{\mathbf{1}}\sigma)$$

- Substitute the assumed form in the equation:

$$\mathsf{I}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\sigma = \{\epsilon\} \ \cup \ \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}(\mathsf{I}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}\sigma)$$

- Equating the terms without and with σ, we get:

$$\mathsf{I}^1_{\textbf{append}} = \{\epsilon\} \ \cup \ \mathbf{1}\mathsf{I}^1_{\textbf{append}}$$
$$\mathsf{D}^1_{\textbf{append}} = \mathbf{0}\bar{\mathbf{0}} \cup \mathbf{1}\mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}$$

Obtaining a closed form solution for LF

- Function summaries will always have the form:

$$\mathsf{LF}_f^i(\sigma) = \mathsf{I}_f^i \cup \mathsf{D}_f^i\sigma$$

- Consider the equation for $\mathsf{LF}^1_{\textbf{append}}$

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\overline{\mathbf{0}}\sigma \ \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\overline{\mathbf{1}}\sigma)$$

- Substitute the assumed form in the equation:

$$\mathsf{I}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\sigma = \{\epsilon\} \ \cup \ \mathbf{0}\overline{\mathbf{0}}\sigma \cup \mathbf{1}(\mathsf{I}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\overline{\mathbf{1}}\sigma)$$

- Equating the terms without and with σ, we get:

$$\mathsf{I}^1_{\textbf{append}} = \{\epsilon\} \ \cup \ \mathbf{1}\mathsf{I}^1_{\textbf{append}}$$

$$\mathsf{D}^1_{\textbf{append}} = \mathbf{0}\overline{\mathbf{0}} \cup \mathbf{1}\mathsf{D}^1_{\textbf{append}}\overline{\mathbf{1}}$$

Obtaining a closed form solution for LF

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- Consider the equation for $\mathsf{LF}^1_{\textbf{append}}$

$$\mathsf{LF}^1_{\textbf{append}}(\sigma) = \{\epsilon\} \cup \mathbf{0}\overline{\mathbf{0}}\sigma \ \cup \mathbf{1}\mathsf{LF}^1_{\textbf{append}}(\overline{\mathbf{1}}\sigma)$$

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$$\mathsf{D}^1_{\textbf{append}} = \mathbf{0}\overline{\mathbf{0}} \cup \mathbf{1}\mathsf{D}^1_{\textbf{append}}\overline{\mathbf{1}}$$

Summary of Analysis Results

Liveness at program points:

$$L_1^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma \cup \mathbf{1}(\mathsf{l}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}\sigma_{\textbf{append}})$$

$$L_1^{\mathtt{l2}} = \{\epsilon\} \cup \mathsf{l}^2_{\textbf{append}} \cup \mathsf{D}^2_{\textbf{append}}\bar{\mathbf{1}}\sigma_{\textbf{append}}$$

$$L_5^{\mathtt{l1}} = \{\epsilon\} \cup \mathbf{0}\bar{\mathbf{0}}\sigma_{\textbf{append}}$$

$$L_5^{\mathtt{tl}} = \mathsf{l}^1_{\textbf{append}} \cup \mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}\sigma_{\textbf{append}}$$

$$L_5^{\mathtt{l2}} = \mathsf{l}^2_{\textbf{append}} \cup \mathsf{D}^2_{\textbf{append}}\bar{\mathbf{1}}\sigma_{\textbf{append}}$$

$\ldots$

Demand summaries:

$$\sigma_{\textbf{append}} = \{\epsilon,\ \mathbf{1}\} \cup \bar{\mathbf{1}}\sigma_{\textbf{append}} \cup \mathbf{10}\sigma_{all}$$

Function summaries:

$$\mathsf{l}^1_{\textbf{append}} = \{\epsilon\} \cup \mathbf{1}\mathsf{l}^1_{\textbf{append}}$$

$$\mathsf{D}^1_{\textbf{append}} = \mathbf{0}\bar{\mathbf{0}} \cup \mathbf{1}\mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}$$

$$\mathsf{l}^2_{\textbf{append}} = \mathsf{l}^2_{\textbf{append}}$$

$$\mathsf{D}^2_{\textbf{append}} = \{\epsilon\} \cup \mathsf{D}^2_{\textbf{append}}\bar{\mathbf{0}}$$

Solution of the equations

View the equations as grammar rules:

$$
\begin{array}{rcl}
\mathsf{L}_1^{\mathtt{11}} & \rightarrow & \epsilon \mid \mathbf{0}\overline{\mathbf{0}}\sigma \ \mid \mathbf{1}(\mathsf{I}^1_{\textbf{append}} \mid \mathsf{D}^1_{\textbf{append}}\overline{\mathbf{1}}\sigma_{\textbf{append}}) \\
\mathsf{I}^1_{\textbf{append}} & \rightarrow & \epsilon \mid \mathbf{1}\mathsf{I}^1_{\textbf{append}} \\
\mathsf{D}^1_{\textbf{append}} & \rightarrow & \mathbf{0}\overline{\mathbf{0}} \mid \mathbf{1}\mathsf{D}^1_{\textbf{append}}\overline{\mathbf{1}}
\end{array}
$$

The solution of $\mathsf{L}_1^{\mathtt{11}}$ is the language $\mathscr{L}(\mathsf{L}_1^{\mathtt{11}})$ generated by it.

Working of Liveness-based GC (Mark phase)

- GC invoked at a program point π
- GC traverses a path α starting from a root variable x.
- GC consults L_π^x:
  - Does $\alpha \in \mathscr{L}(L_\pi^x)$?
  - If yes, then mark the current cell
- Note that α is a *forward*-only access path
  - consisting only of edges **0** and **1**, but not $\bar{\mathbf{0}}$ or $\bar{\mathbf{1}}$
  - But $\mathscr{L}(L_\pi^x)$ has access paths marked with $\bar{\mathbf{0}}/\bar{\mathbf{1}}$ for **0**/**1** removal arising from the **cons** rule.

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  - But $\mathscr{L}(\mathsf{L}_\pi^x)$ has access paths marked with $\bar{\mathbf{0}}$/$\bar{\mathbf{1}}$ for **0**/**1** removal arising from the **cons** rule.

$\overline{\mathbf{0}}/\overline{\mathbf{1}}$ *handling*

- $\mathbf{0}$ removal from a set of access paths:

$$\alpha_1\overline{\mathbf{0}}\mathbf{0}\alpha_2 \hookrightarrow \alpha_1\alpha_2$$

$$\alpha_1\overline{\mathbf{0}}\mathbf{1}\alpha_2 \hookrightarrow \text{ drop } \alpha_1\overline{\mathbf{0}}\mathbf{1}\alpha_2 \text{ from the set}$$

- $\mathbf{1}$ removal from a set of access paths:

$$\alpha_1\overline{\mathbf{1}}\mathbf{1}\alpha_2 \hookrightarrow \alpha_1\alpha_2$$

$$\alpha_1\overline{\mathbf{1}}\mathbf{0}\alpha_2 \hookrightarrow \text{ drop } \alpha_1\overline{\mathbf{1}}\mathbf{0}\alpha_2 \text{ from the set}$$

GC decision problem

- Deciding the membership in a CFG augmented with a fixed set of unrestricted productions.

$$\bar{\mathbf{0}}\mathbf{0} \rightarrow \epsilon$$
$$\bar{\mathbf{1}}\mathbf{1} \rightarrow \epsilon$$

- The problem shown to be undecidable[1].
  - Reduction from Halting problem.

[1] Prasanna, Sanyal, and Karkare. *Liveness-Based Garbage Collection for Lazy Languages*, ISMM 2016.

Practical $\overline{\mathbf{0}}/\overline{\mathbf{1}}$ simplification

- The simplification is possible to do on a finite state automaton.
- Over-approximate the CFG by an automaton (Mohri-Nederhoff transformation).
- Perform **0**/**1** removal on the automaton.

Example

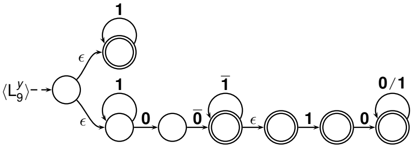
Grammar for L_9^{y}

$$
\begin{array}{rcl}
L_9^y & \rightarrow & I^1_{\mathbf{append}} \mid D^1_{\mathbf{append}}(\epsilon \mid \mathbf{1} \mid \mathbf{10}\sigma_{all}) \\
I^1_{\mathbf{append}} & \rightarrow & \epsilon \mid \mathbf{1}I^1_{\mathbf{append}} \\
D^1_{\mathbf{append}} & \rightarrow & \mathbf{0}\overline{\mathbf{0}} \mid \mathbf{1}D^1_{\mathbf{append}}\overline{\mathbf{1}} \\
\sigma_{all} & \rightarrow & \epsilon \mid \mathbf{0}\sigma_{all} \mid \mathbf{1}\sigma_{all}
\end{array}
$$

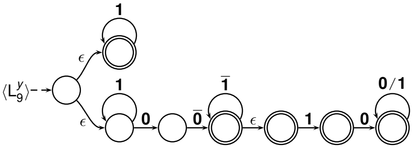
After Mohri-Nederhoff transformation

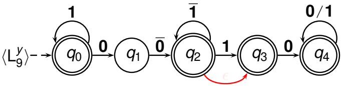
$$
\begin{array}{rcl}
L_9^y & \rightarrow & I^1_{\mathbf{append}} \mid D^1_{\mathbf{append}}(\epsilon \mid \mathbf{1} \mid \mathbf{10}\sigma_{all}) \\
I^1_{\mathbf{append}} & \rightarrow & \epsilon \mid \mathbf{1}I^1_{\mathbf{append}} \\
D^1_{\mathbf{append}} & \rightarrow & \mathbf{0}\overline{\mathbf{0}}\widehat{D}^1_{\mathbf{append}} \mid \mathbf{1}D^1_{\mathbf{append}} \\
\widehat{D}^1_{\mathbf{append}} & \rightarrow & \overline{\mathbf{1}}\widehat{D}^1_{\mathbf{append}} \mid \epsilon \\
\sigma_{all} & \rightarrow & \epsilon \mid \mathbf{0}\sigma_{all} \mid \mathbf{1}\sigma_{all}
\end{array}
$$

Automaton for L_9^y



Automaton for L_9^y





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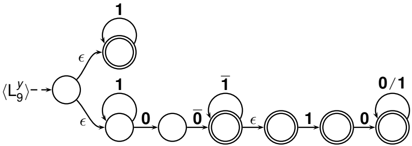

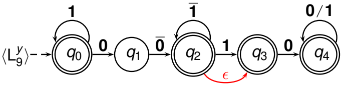

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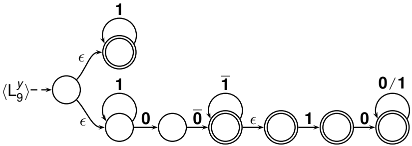

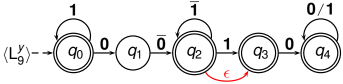

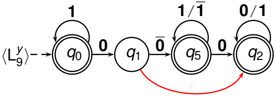

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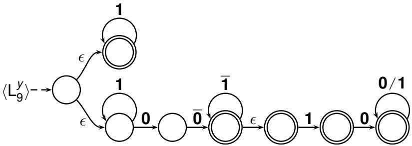

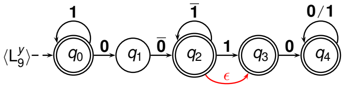

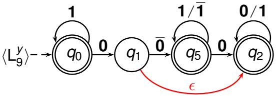

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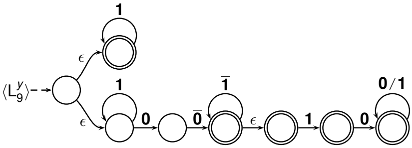

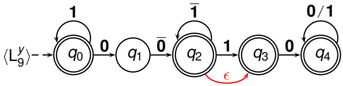

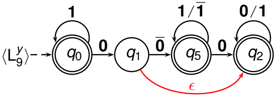

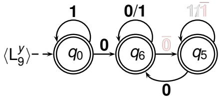

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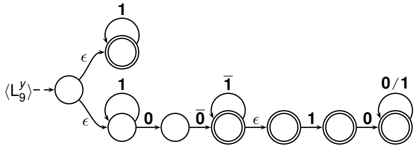

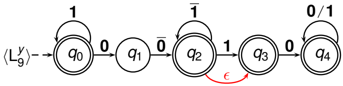

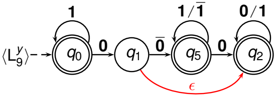

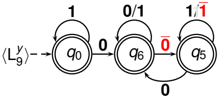

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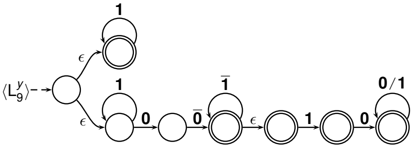
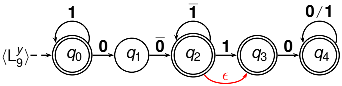
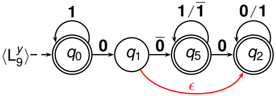
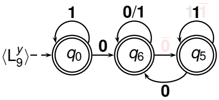
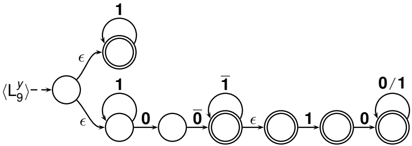


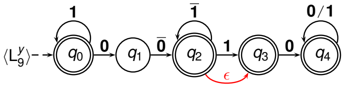


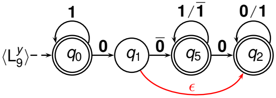


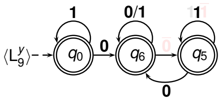


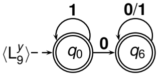
Automaton for L_9^y







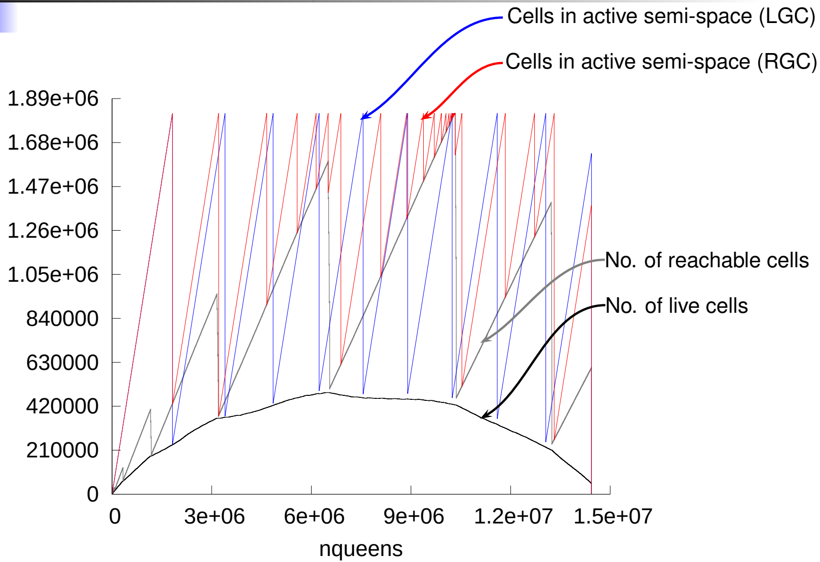




Experimental Setup

- Built a prototype consisting of:
  - An ANF-scheme interpreter
  - Liveness analyzer
  - A single-generation copying collector.
- The collector optionally uses liveness
  - Marks a link during GC only if it is live.
- Benchmark programs are mostly from the no-fib suite.

GC behavior as a graph



Results as Tables

Analysis Performance:

Program	sudoku	lcss	gc_bench	knightstour	treejoin	nqueens	lambda
Time (msec)	120.95	2.19	0.32	3.05	2.61	0.71	20.51
DFA size	4251	726	258	922	737	241	732
Precision(%)	**87.5**	**98.8**	**99.9**	**94.3**	**99.6**	**98.8**	**83.8**

Results as Tables

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
sudoku	**490**	**1306**	22	9	1704	589	.028	.122
lcss	**46522**	**51101**	8	7	52301	1701	.045	.144
gc_bench	**129179**	**131067**	9	9	131071	6	.086	.075
nperm	**47586**	**174478**	14	4	202597	37507	1.406	.9
fibheap	**249502**	**251525**	1	1	254520	13558	.006	.014
knightstour	**2593**	**314564**	1161	10	508225	307092	464.902	14.124
treejoin	**288666**	**519943**	2	1	525488	7150	.356	.217
nqueens	**283822**	**1423226**	46	9	1819579	501093	70.314	24.811
lambda	**205**	**556**	23	8	966	721	.093	2.49

- LGC collects more garbage than RGC.

Results as Tables

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
sudoku	490	1306	**22**	**9**	1704	589	.028	.122
lcss	46522	51101	**8**	**7**	52301	1701	.045	.144
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lambda	205	556	**23**	**8**	966	721	.093	2.49

- # collections of LGC no higher than RGC. Often, smaller.

Results as Tables

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
sudoku	490	1306	22	9	**1704**	**589**	.028	.122
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lambda	205	556	23	8	**966**	**721**	.093	2.49

- Programs require smaller heaps to execute with LGC.

Results as Tables

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
sudoku	490	1306	22	9	1704	589	.028	.122
lcss	46522	51101	8	7	52301	1701	.045	.144
gc_bench	129179	131067	9	9	131071	6	**.086**	**.075**
nperm	47586	174478	14	4	202597	37507	**1.406**	**.9**
fibheap	249502	251525	1	1	254520	13558	.006	.014
knightstour	2593	314564	1161	10	508225	307092	**464.902**	**14.124**
treejoin	288666	519943	2	1	525488	7150	**.356**	**.217**
nqueens	283822	1423226	46	9	1819579	501093	**70.314**	**24.811**
lambda	205	556	23	8	966	721	.093	2.49

- GC time is smaller for LGC in some cases. . .

Results as Tables

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
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- An evaluation strategy in which evaluation of an expression is postponed until its value is needed
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Laziness: Example

```
(define (length l)
  (if (null? l)
    return 0
    return (+ 1 (length (cdr l)))))
```

```
(define (main)
  (let a ← ( a BIG closure ) in
    (let b ← (+ a 1) in
      (let c ← (cons b nil) in
        (let w ← (length c) in
          (return w))))))
```

Handling lazy semantics: Challenges

- Laziness complicates liveness analysis itself.
  - Data is made live by evaluation of closures
  - In lazy languages, the place in the program where this evaluation takes place cannot be statically determined
- Liveness-based garbage collector significantly more complicated than that for an eager language.

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Handling possible non-evaluation

- Liveness no longer remains independent of demand σ
  - If (**car** x) is not evaluated at all, it does not generate any liveness for x
- Require a new terminal **2** with following semantics

$$\mathbf{2}\sigma \hookrightarrow \begin{cases} \emptyset & \text{if } \sigma = \emptyset \\ \{\epsilon\} & \text{otherwise} \end{cases}$$

$$\mathcal{L}app((\textbf{car}\ \mathrm{x}), \sigma) = \mathrm{x}.\{\mathbf{2}, \mathbf{0}\}\sigma$$

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Scope for future work

- Reducing GC-time.
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- Increasing the scope of the method.
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Conclusions

- Proposed a liveness-based GC scheme.
- Not covered in this talk:
  - The soundness of liveness analysis.
  - Details of undecidability proof.
  - Details of handling lazy languages.
- A prototype implementation to demonstrate:
  - the precision of the analysis.
  - reduced heap requirement.
  - reduced GC time for a majority of programs.
- Unfinished agenda:
  - Improving GC time for a larger fraction of programs.
  - Extending scope of the method.