CS618: Program Analysis
2016-17 Ist Semester

Sparse Conditional Constant Propagation

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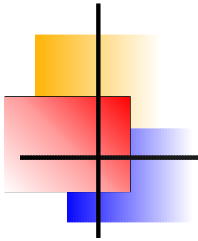
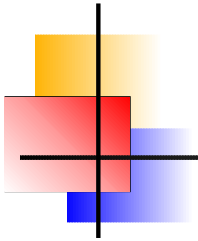


Sparse Simple Constant Propagation (SSC)

- Improved analysis time over Simple Constant Propagation
- Finds all simple constant
  - Same class as Simple Constant Propagation



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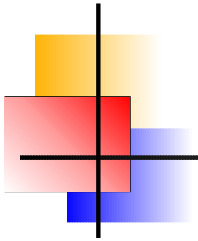

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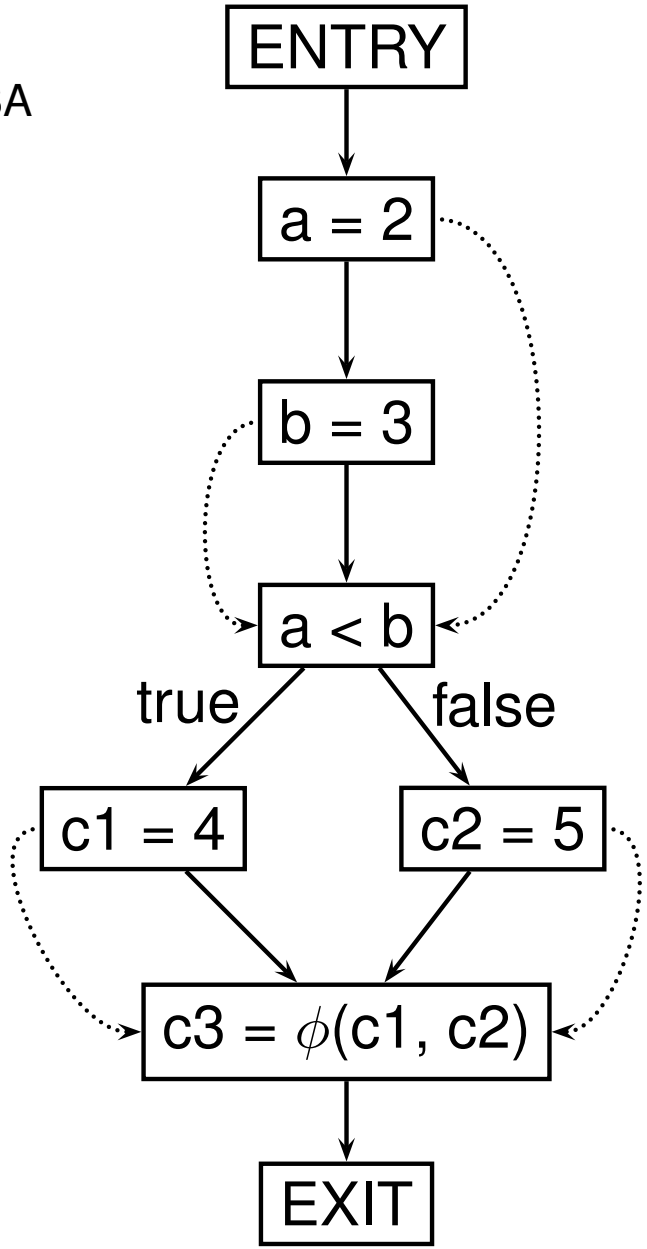
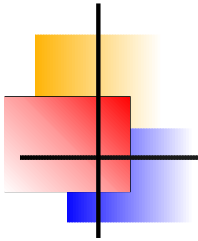
Motivating Example

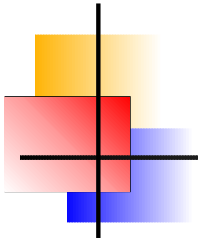
Dashed edges denote SSA def-use chains





Preparations for SSC Analysis

- Convert the program to SSA form
- One statement per basic block
- Add connections called *SSA edges*
  - Connect (unique) definition point of a variable to its use points
  - Same as def-use chains



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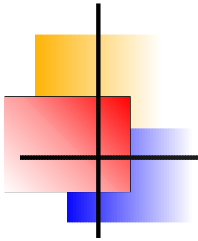

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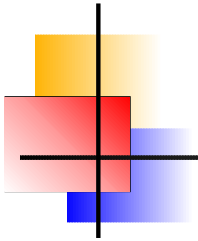

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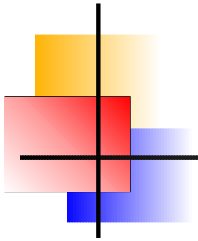

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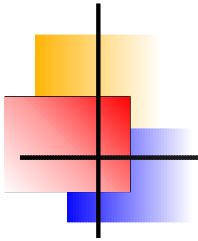
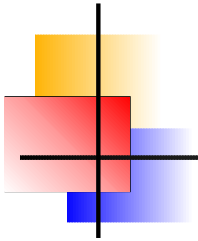


SSC Algorithm: Initialization

- Evaluate expressions involving constants only and assign the value (c) to variable on LHS
- If expression can not be evaluated at compile time, assign $\bot$
- Else (for expression contains variables) assign $\top$
- Initialize worklist WL with SSA edges whose def is not $\top$
- Algorithm terminates when WL is empty



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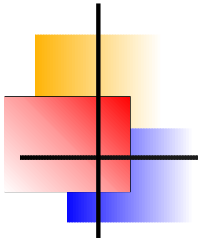

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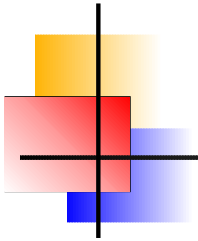

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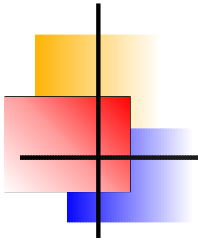

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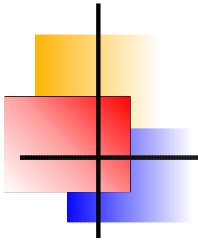
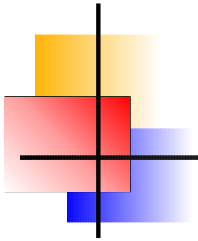


SSC Algorithm: Iterative Actions

- Take an SSA edge E out of WL
- Take meet of the value at def end and the use end of E for the variable defined at def end
- If the meet value is different from use value, replace the use by the meet
- Recompute the def d at the use end of E
- If the recomputed value is *lower* than the stored value, add all SSA edges originating at d



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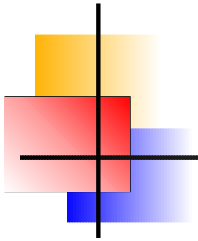

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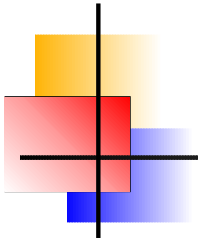

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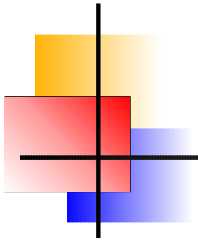

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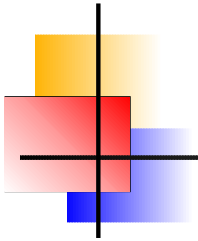
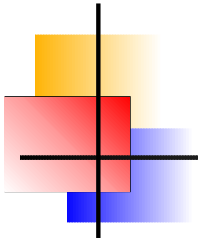


Meet for ϕ-function

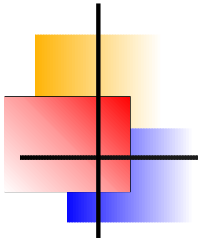
$$v = \phi(v_1, v_2, \ldots, v_k)$$

$$\Rightarrow \text{ValueOf}(v) = v_1 \wedge v_2 \wedge \ldots \wedge v_n$$



SSC Algorithm: Complexity

- Height of CP lattice = 2
- Each SSA edge is examined at most twice, for each lowering
- Theoritical size of SSA graph: $O(V \times E)$
- Practical size: linear in the program size



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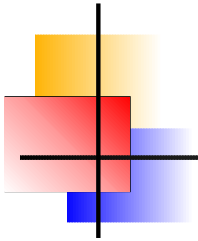

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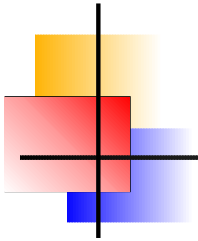

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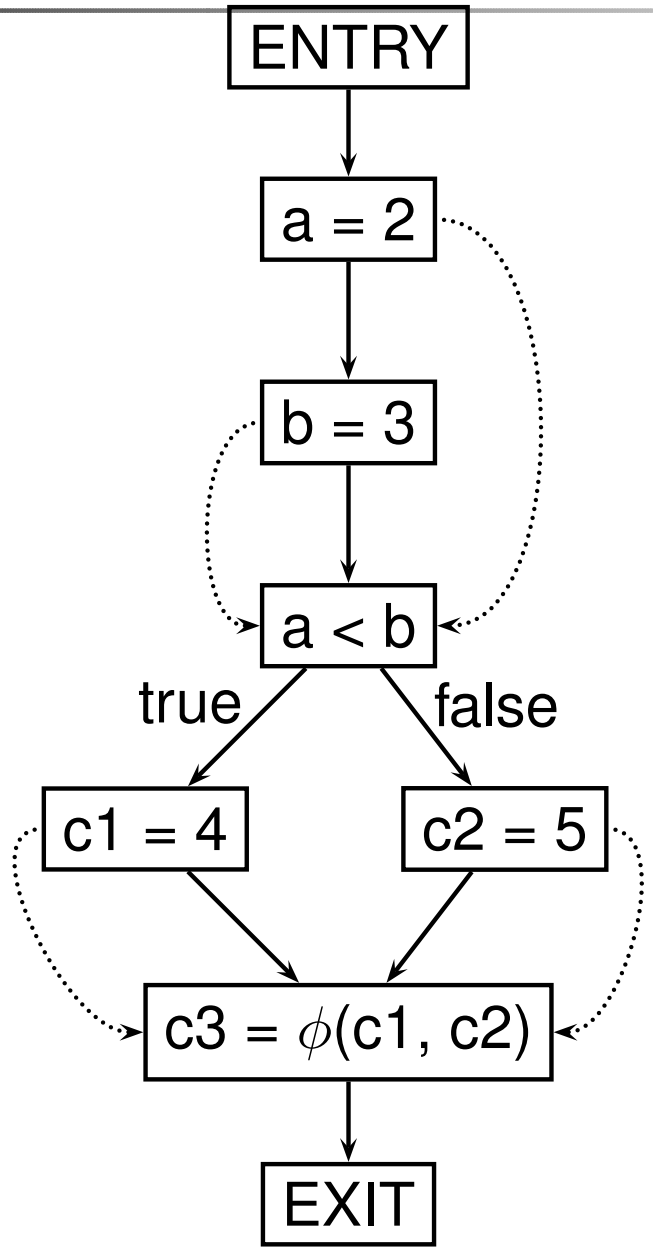
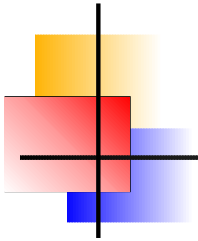
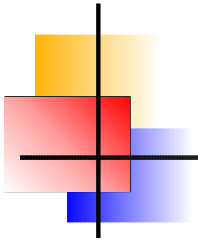




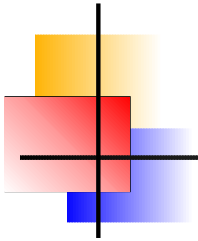
SSC: Practice Example

What if we change “c1 = 4” to “c1 = 5”?



Sparse Condtional Constant Propagation (SCC)

- Constant Propagation with *unreachable code elimination*
- Ignore definitions that reach a use via a non-executable edge



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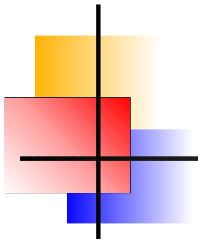
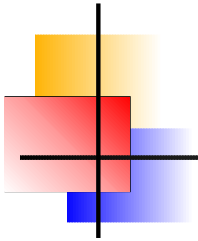


SCC Algorithm: Key Idea

$$v = \phi(v_1, v_2, \ldots, v_k)$$

$$\Rightarrow \text{ValueOf}(v) = \bigwedge_{i \in ExecutablePath} v_i$$

We ignore paths that are not “yet” marked executable



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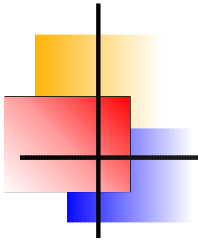
- Two Worklists
  - Flow Worklist (*FWL*)
    - Worklist of flow graph edges
  - SSA Worklist (*SWL*)
- Execution Halts when **both** worklists are empty
- Associate a flag, the *ExecutableFlag*, with every flow graph edge to control the evaluation of ϕ-function in the destination node



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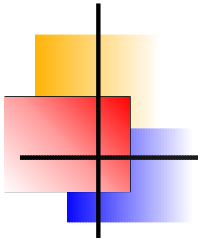

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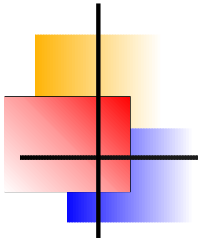

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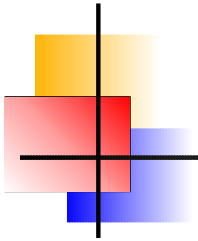

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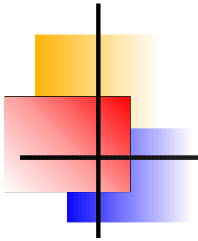

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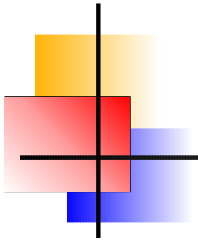

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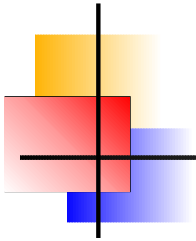

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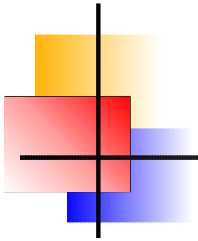

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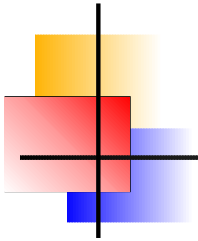

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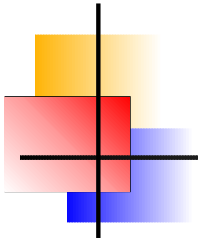

SCC Algorithm: Initialization

- Initialize *FWL* to contain edges leaving ENTRY node
- Initialize *SWL* to empty
- Each *ExecutableFlag* is false initially
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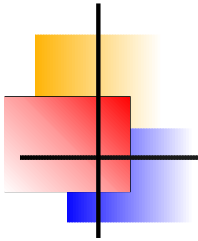
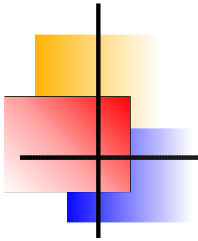


SCC Algorithm: Iterations

- Remove an item from either worklist
- process the item (described next)



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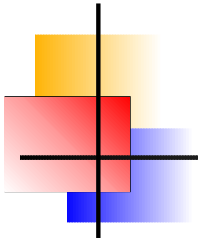

SCC Algorithm: Processing FWL *Item*

- Item is flow graph edge
- If *ExecutableFlag* is true, do nothing
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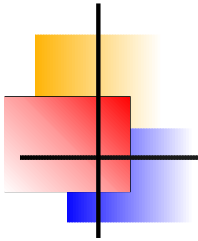
  - Mark the *ExecutableFlag* as true
  - Visitϕ for all ϕ-functions in the destination
  - If only one of the *ExecutableFlags* of incoming flow graph edges for dest is true (dest visited for the first time), then VisitExpression for all expressions in dest
  - If the dest contains only one outgoing flow graph edge add that edge to *FWL*



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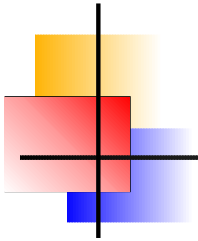

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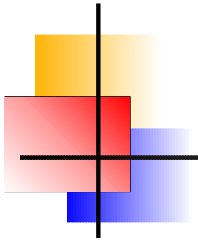

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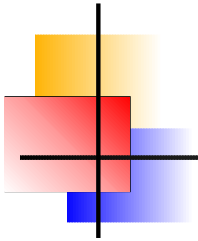

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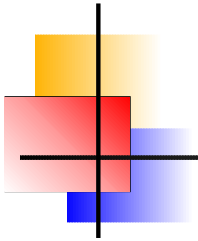

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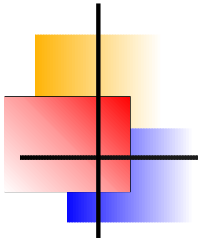

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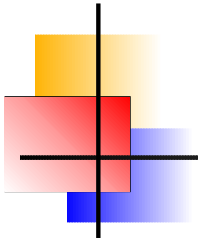
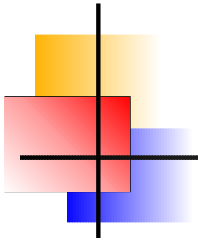


SCC Algorithm: Processing SWL *Item*

- Item is SSA edge
- If dest is a ϕ-function, **Visit-ϕ**
- If dest is an expression and any of *ExecutableFlags* for the incoming flow graph edges of dest is true, perform **VisitExpression**



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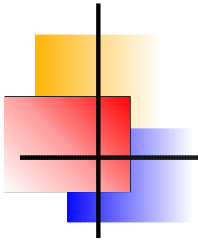

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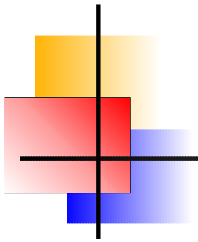
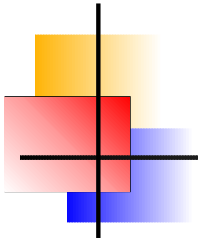


SCC Algorithm: Visit-ϕ

$$v = \phi(v_1, v_2, \ldots, v_k)$$

- If i^{th} incoming edge's *ExecutableFlag* is true, $val_i = \text{ValueOf}(v_i)$ else $val_i = \top$
- $\text{ValueOf}(v) = \bigwedge_i val_i$



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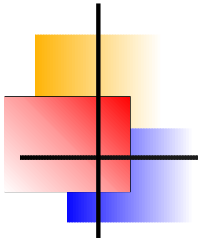
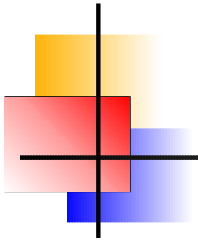


SCC Algorithm: VisitExpression

- Evaluate the expression using values of operands and rules for operators
- If the result is same as old, nothing to do
- Otherwise
  - If the expression is part of assignment, add all outgoing SSA edges to SWL
  - If the expression controls a conditional branch, then



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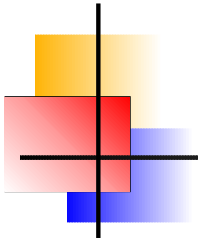

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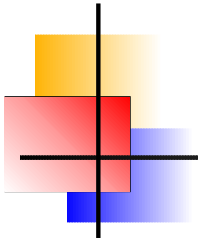
    - If the result is ⊤, add all outgoing flow edges to *FWL*
    - If the value is constant c, only the corresponding flow branch edge is added to *FWL*
    - Value can not be ⊤ (why?)



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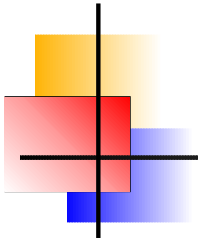

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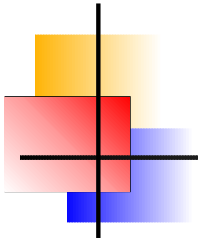

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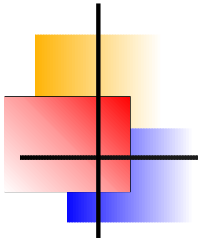

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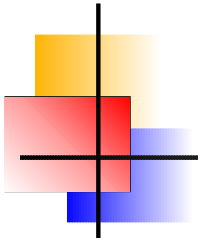

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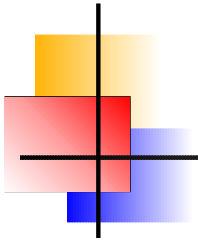
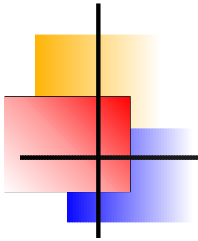


SCC Algorithm: Complexity

- Each SSA edge is examined twice
- Flow graph nodes are visited once for every incoming edge
- Complexity = O(# of SSA edges + # of flow graph edges)



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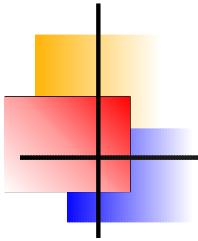

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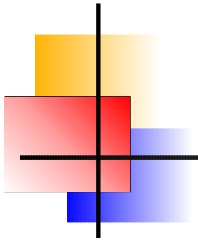
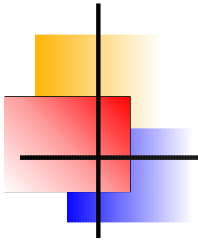


SCC Algorithm: Correctness and Precision

- SCC is conservative
  - Never labels a variable value as a constant
- SCC is at least as powerful as Conditional Constant Propagation (CC)
  - Finds all constants as CC does
- PROOFs: In paper **Constant propagation with conditional branches** by **Mark N. Wegman, F. Kenneth Zadeck**, ACM TOPLAS 1991.



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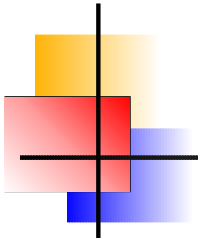

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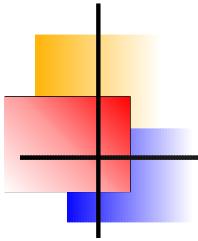

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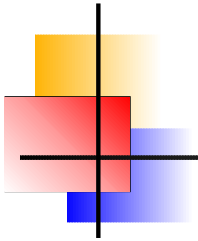

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