Program Analysis

https://www.cse.iitb.ac.in/~karkare/cs618/

Static Single Assignment (SSA)

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SSA Form

- Developed by Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck,
  - in 1980s while at IBM.
- Static Single Assignment – A variable is assigned only once in program text
  - May be assigned multiple times if program is executed

SSA Form

- Intermediate representation
- Sparse representation
  - Definitions sites are directly associated with use sites
- Advantage
  - Directly access points where relevant data flow information is avaliable

SSA Form

- In SSA Form
  - Each variable has exactly one definition
    - Each use of a variable is reached by exactly one definition
  - Control flow like traditional programs
  - Some magic is needed at join nodes

SSA Form: Examples

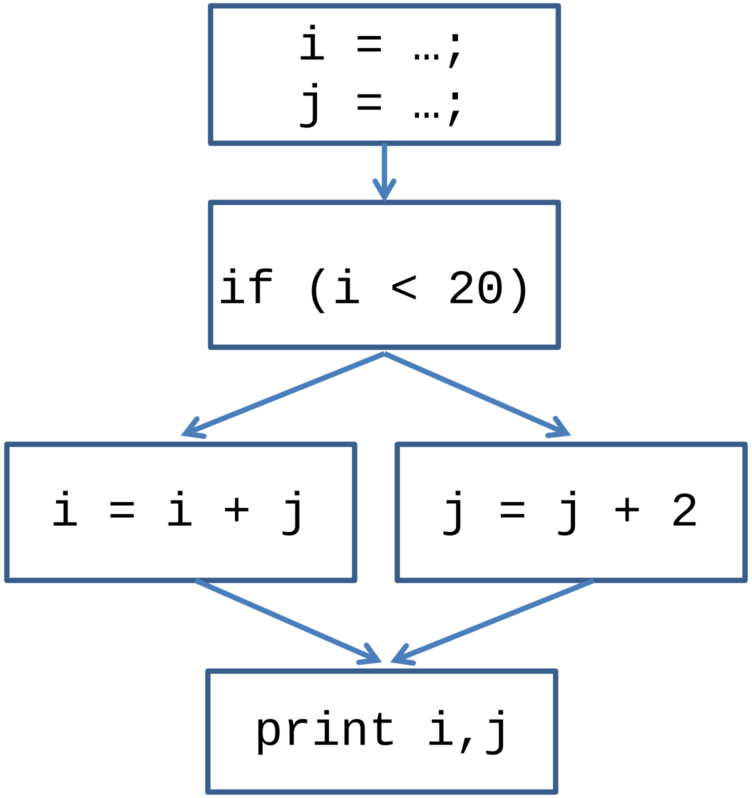
```
i = 0;
…
i = i + 1;
…
j = i * 5;
…
```

```
i1 = 0;
…
i2 = i1 + 1;
…
j1 = i2 + 1;
…
```

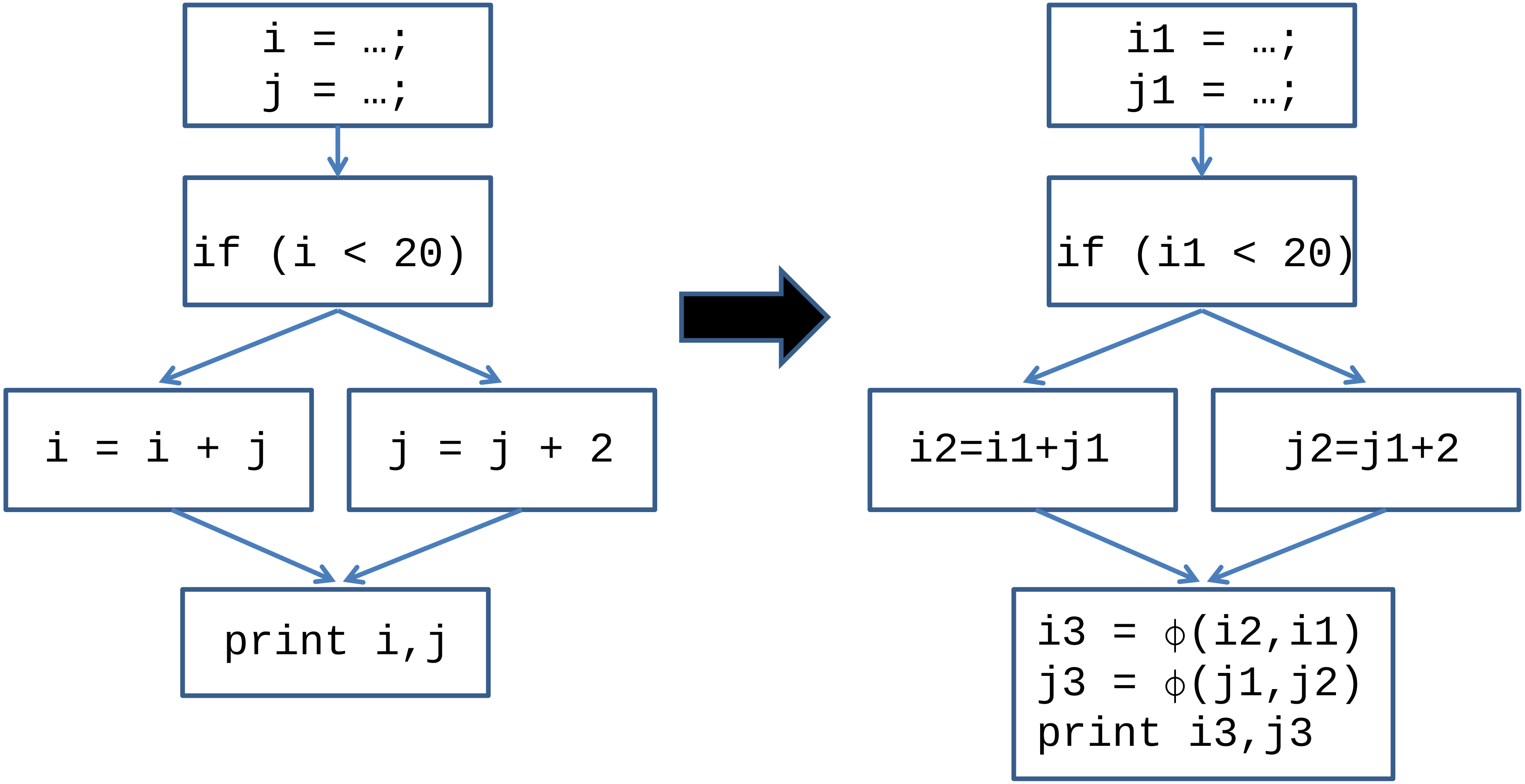
SSA Form: Examples

```
i = …;
j = …;
if (i < 20)
   i = i + j;
else
   j = j + 2;

print i,j;
```



SSA Form: Examples



```
i = …;
j = …;
```

```
if (i < 20)
```

```
i = i + j
```

```
j = j + 2
```

```
print i,j
```

```
i1 = …;
j1 = …;
```

```
if (i1 < 20)
```

```
i2=i1+j1
```

```
j2=j1+2
```

```
i3 = ϕ(i2,i1)
j3 = ϕ(j1,j2)
print i3,j3
```

SSA Form: Examples

```
i = …;
j = …;
if (i < 20)
  i = i + j;
else
  j = j + 2;

print i,j;
```

→

```
i1 = …;
j1 = …;
if (i1 < 20)
  i2 = i1 + j1;
else
  j2 = j1 + 2;
i3 = ϕ(i2, i1);
j3 = ϕ(j1, j2);
print i3, j3;
```

The “magic” : φ-function

- φ is used for selection
  - One out of multiple values at join nodes
- Not every join node needs a φ
  - Needed only if multiple definitions reach the node

But what does φ operation mean in a machine code?

- φ is a conceptual entity
- No direct translation to machine code
  - typically mimicked using “copy” in predecessors
  - Inefficient
  - Practically, the inefficiency is compensated by dead code elimination and register allocation passes

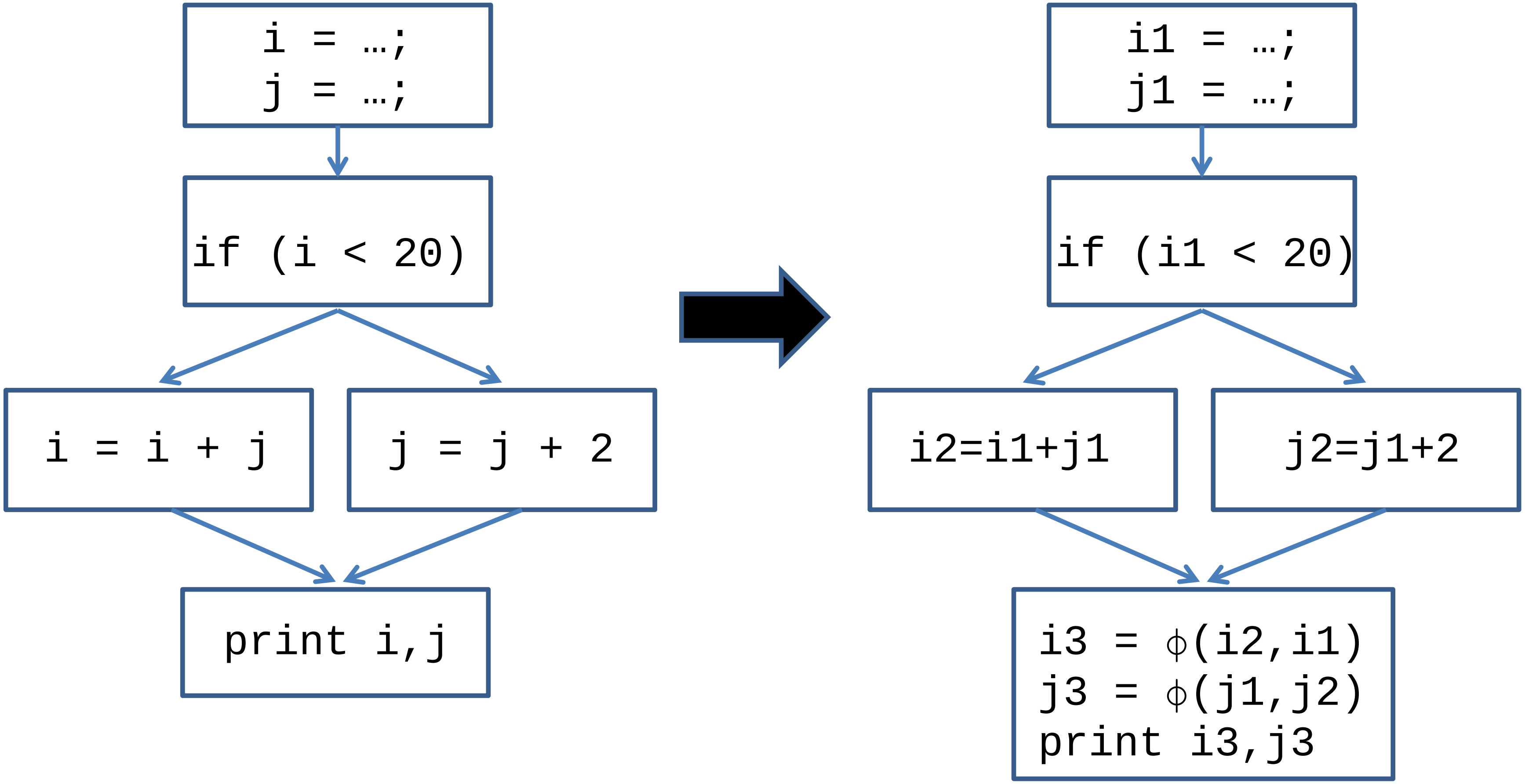
φ Properties

- Placed only at the entry of a join node
- Multiple φ-functions could be placed
  - for multiple variables
  - all such φ functions execute concurrently
- n-ary φ function at n-way join node

$$xm = \phi(x1, x2, \ldots, xi, \ldots, xn)$$

- xm gets the value of i-th argument xi if control enters through i-th edge
  - Ordering of edges is improtant

SSA Form: Example (revisit)



```
i = …;
j = …;
```

```
if (i < 20)
```

```
i = i + j
```

```
j = j + 2
```

```
print i,j
```

```
i1 = …;
j1 = …;
```

```
if (i1 < 20)
```

```
i2=i1+j1
```

```
j2=j1+2
```

```
i3 = ϕ(i2,i1)
j3 = ϕ(j1,j2)
print i3,j3
```

Construction of SSA Form

Assumptions

- Only scalar variables
  - Structures, pointers, arrays could be handled
  - Refer to publications

Dominators

- Nodes x and y in flow graph
- x dominates y if every path from ENTRY to y go through x
  - x dom y
  - partial order?
- x strictly dominates y if x dom y and $x \neq y$
  - x sdom y

Computing Dominators

$$DOM(n) = \{n\} \cup \left(\bigcap_{m \in preds(n)} DOM(m) \right)$$

Initial Conditions:

$$DOM(n_0) = \{n_0\}$$
$$\forall n \neq n_0 \; DOM(n) = N$$

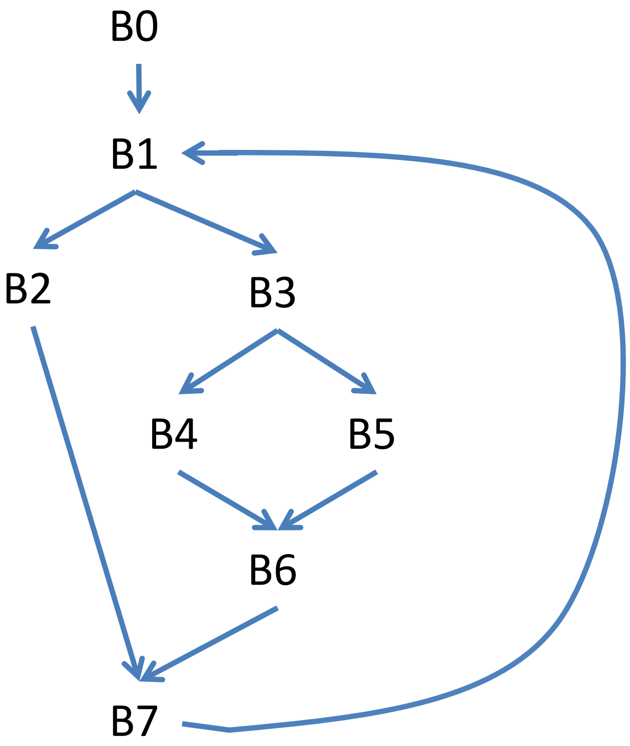
N is the set of all nodes, n_0 is ENTRY

NOTE: Efficient methods exist for computing dominators

Immediate Dominators and Dominator Tree

- x is immediate dominator of y if x is the *closest* strict dominator of y
  - unique, if it exists
  - denoted idom[y]
- Dominator Tree
  - A tree showing all immediate dominator relationships

Dominator Tree



B0

B1

B2

B3

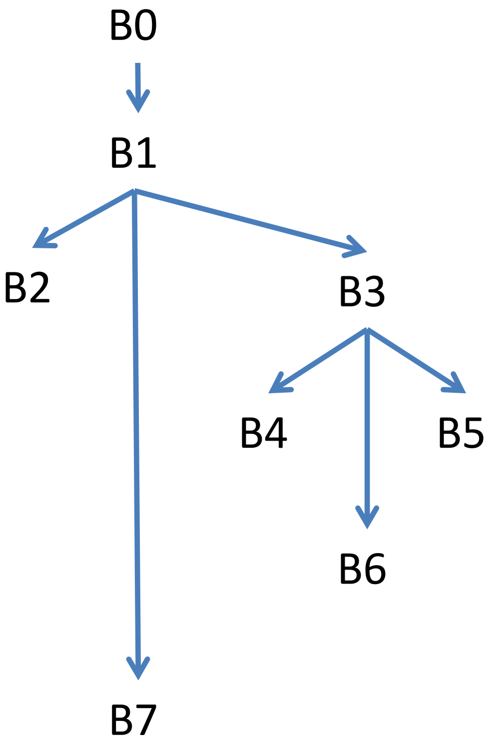
B4

B5

B6

B7

Control Flow Graph



B0

B1

B2

B3

B4

B5

B6

B7

Dominator Tree

Dominance Frontier

- Dominance Frontier of x is set of all nodes y s.t.
  - x dominates a predecessor of y AND
  - x does not strictly dominate y
- Denoted DF(x)
- Why do you think DF(x) is important for any x?
  - Think about information *originated* in x

Computing Dominance Frontier

$$DF(x) = DF_{local}(x) \cup \bigcup_{z \in \mathrm{chil}dren(x)} DF_{up}(z)$$

$$DF_{local}(x) = \{y \in succ(x) | idom(y) \neq x\}$$
$$DF_{up}(z) = \{y \in DF(z) | idom(y) \neq parent(z)\}$$

* parent, children in dominator tree, succ in CFG

* parent(z) = x above

Iterated Dominance Frontier

- $DF^{+}(S)$: Transitive closure of Dominance frontiers on a set of nodes

$$DF(S) = \bigcup_{x \in S(x)} DF(x)$$

$$DF^{1}(S) = DF(S)$$

$$DF^{i+1}(S) = DF(S \cup DF^{i}(S))$$

Minimal SSA Form Construction

- Compute DF+ set for each flow graph node
- Place trivial φ-functions for each variable in the node
- Rename variables

- Why DF+? Why not only DF?

Inserting φ-functions

```
foreach variable v {
    S = ENTRY ∪ {n | v defined in n}
    Compute DF+(S)
    foreach n in DF+(S) {
        insert ϕ-function for v at start of n
    }
}
```

Renaming Variables (Pseudo Code)

- Rename from the ENTRY node recursively
  - maintain a *rename* stack of $var \rightarrow var_{version}$ mapping
- For node n
  - For each assignment (x = …) in n
    - If non-phi assignment, Rename any use of x with the Top mapping of x from the rename stack
    - Push the x → x_i on rename stack
    - i = i + 1
- For successors of n
  - Rename φ operands through succ edge index
- Recursively rename for all child nodes in the dominator tree
- For each assignment (x = …) in n
  - Pop x $\rightarrow \cdots$ from the rename stack