



Program Analysis  
<https://www.cse.iitb.ac.in/~karkare/cs618/>

## Static Single Assignment (SSA)

Amey Karkare  
 Dept of Computer Science and Engg  
 IIT Kanpur  
 Visiting IIT Bombay  
[karkare@cse.iitk.ac.in](mailto:karkare@cse.iitk.ac.in)  
[karkare@cse.iitb.ac.in](mailto:karkare@cse.iitb.ac.in)

## SSA Form

- Developed by Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck,
  - in 1980s while at IBM.
- Static Single Assignment – A variable is assigned only once in program text
  - May be assigned multiple times if program is executed

2

## SSA Form

- Intermediate representation
- Sparse representation
  - Definitions sites are directly associated with use sites
- Advantage
  - Directly access points where relevant data flow information is available

3

## SSA Form

- In SSA Form
  - Each variable has exactly one definition
    - Each use of a variable is reached by exactly one definition
  - Control flow like traditional programs
  - Some **magic** is needed at **join** nodes

4

## SSA Form: Examples

```

i = 0;           i1 = 0;
...             ...
i = i + 1;      i2 = i1 + 1;
...             ...
j = i * 5;      j1 = i2 + 1;
...             ...

```



5

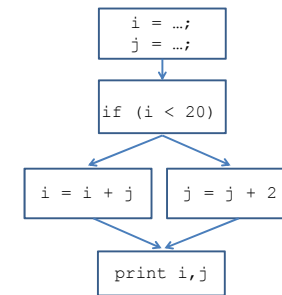
## SSA Form: Examples

```

i = ...;
j = ...;
if (i < 20)
    i = i + j;
else
    j = j + 2;

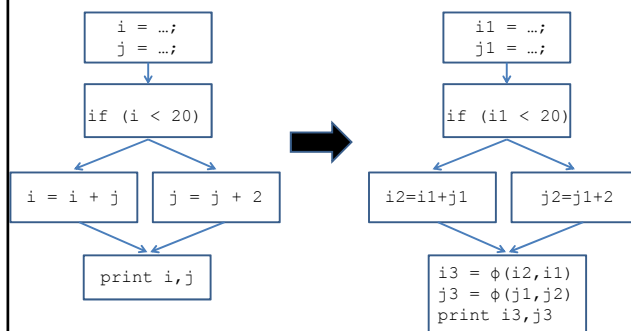
print i,j;

```



6

## SSA Form: Examples



7

## SSA Form: Examples

```

i = ...;           i1 = ...;
j = ...;           j1 = ...;
if (i < 20)        if (i1 < 20)
    i = i + j;      i2 = i1 + j1;
else                else
    j = j + 2;      j2 = j1 + 2;
                    i3 = phi(i2, i1);
                    j3 = phi(j1, j2);
print i,j;          print i3, j3;

```



8

## The “magic” : $\phi$ -function

- $\phi$  is used for selection
  - One out of multiple values at join nodes
- Not every join node needs a  $\phi$ 
  - Needed only if multiple definitions reach the node

9

## But what does $\phi$ operation mean in a machine code?

- $\phi$  is a conceptual entity
- No direct translation to machine code
  - typically mimicked using “copy” in predecessors
  - Inefficient
  - Practically, the inefficiency is compensated by dead code elimination and register allocation passes

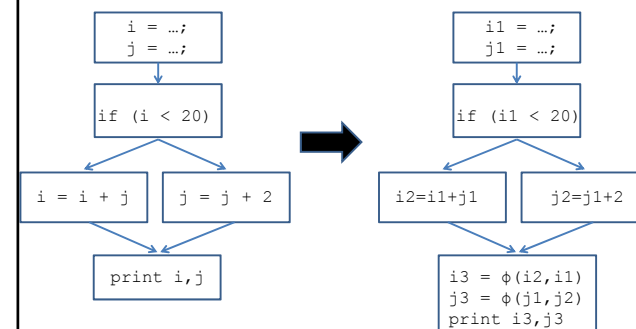
10

## $\phi$ Properties

- Placed only at the entry of a join node
- Multiple  $\phi$ -functions could be placed
  - for multiple variables
  - all such  $\phi$  functions execute concurrently
- n-ary  $\phi$  function at n-way join node
 
$$xm = \phi(x1, x2, \dots, xi, \dots, xn)$$
- $xm$  gets the value of i-th argument  $xi$  if control enters through i-th edge
  - Ordering of edges is important

11

## SSA Form: Example (revisit)



12

## Construction of SSA Form

13

## Assumptions

- Only scalar variables
  - Structures, pointers, arrays could be handled
  - Refer to publications

14

## Dominators

- Nodes  $x$  and  $y$  in flow graph
- $x$  **dominates**  $y$  if **every** path from ENTRY to  $y$  go through  $x$ 
  - $x \text{ dom } y$
  - partial order?
- $x$  **strictly dominates**  $y$  if  $x \text{ dom } y$  and  $x \neq y$ 
  - $x \text{ sdom } y$

15

## Computing Dominators

$$DOM(n) = \{n\} \cup \left( \bigcap_{m \in \text{preds}(n)} DOM(m) \right)$$

Initial Conditions:

$$DOM(n_0) = \{n_0\}$$

$$\forall n \neq n_0 \quad DOM(n) = N$$

$N$  is the set of all nodes,  $n_0$  is ENTRY

NOTE: Efficient methods exist for computing dominators

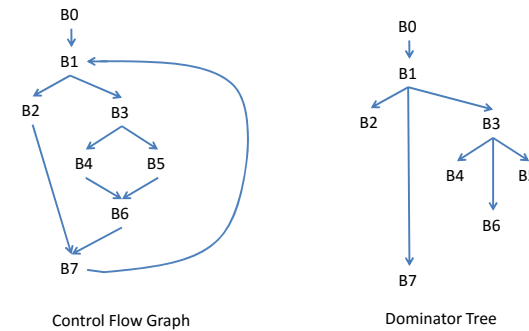
16

## Immediate Dominators and Dominator Tree

- x is **immediate dominator** of y if x is the *closest* strict dominator of y
  - unique, if it exists
  - denoted  $\text{idom}[y]$
- Dominator Tree
  - A tree showing all immediate dominator relationships

17

## Dominator Tree



18

## Dominance Frontier

- Dominance Frontier of x is set of all nodes y s.t.
  - x **dominates** a **predecessor** of y AND
  - x **does not strictly dominate** y
- Denoted  $DF(x)$
- Why do you think  $DF(x)$  is important for any x?
  - Think about information *originated* in x

19

## Computing Dominance Frontier

$$DF(x) = DF_{local}(x) \cup \bigcup_{z \in \text{children}(x)} DF_{up}(z)$$

$$DF_{local}(x) = \{y \in \text{succ}(x) \mid \text{idom}(y) \neq x\}$$

$$DF_{up}(z) = \{y \in DF(z) \mid \text{idom}(y) \neq \text{parent}(z)\}$$

\* parent, children in dominator tree, succ in CFG

\*  $\text{parent}(z) = x$  above

20

## Iterated Dominance Frontier

- $DF^+(S)$ : Transitive closure of Dominance frontiers on a set of nodes

$$DF(S) = \bigcup_{x \in S(x)} DF(x)$$

$$DF^1(S) = DF(S)$$

$$DF^{i+1}(S) = DF(S \cup DF^i(S))$$

21

## Minimal SSA Form Construction

- Compute  $DF^+$  set for each flow graph node
- Place **trivial**  $\phi$ -functions for each variable in the node
- Rename variables
- **Why  $DF^+$ ? Why not only  $DF$ ?**

22

## Inserting $\phi$ -functions

```

foreach variable v {
  S = ENTRY  $\cup$  {n | v defined in n}
  Compute  $DF^+(S)$ 
  foreach n in  $DF^+(S)$  {
    insert  $\phi$ -function for v at start of n
  }
}

```

23

## Renaming Variables (Pseudo Code)

- Rename from the ENTRY node recursively
  - maintain a *rename* stack of  $var \rightarrow var_{version}$  mapping
- For node n
  - For each assignment ( $x = \dots$ ) in n
    - If non-phi assignment, Rename any use of x with the Top mapping of x from the rename stack
    - Push the  $x \rightarrow x_i$  on rename stack
    - $i = i + 1$
- For successors of n
  - Rename  $\phi$  operands through succ edge index
- Recursively rename for all child nodes in the dominator tree
- For each assignment ( $x = \dots$ ) in n
  - Pop  $x \rightarrow \dots$  from the rename stack

24